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2004
INTERPRETATION OF BOREHOLE MEASUREMENTS
ACQUIRED IN LAMINATED CLASTIC SEQUENCES
SUBJECT TO MUD FILTRATE INVASIÓN

by

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ACQUIRED IN LAMINATED CLASTIC SEQUENCES
SUBJECT TO MUD FILTRATE INVASION

APPROVED BY

SUPERVISING COMMITTEE:

_____________________________
Carlos Torres-Verdín, Supervisor

_____________________________
Steven L. Bryant
DEDICATION

To my wife Suspa.
ACKNOWLEDGMENTS

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ABSTRACT

INTERPRETATION OF BOREHOLE MEASUREMENTS ACQUIRED IN LAMINATED CLASTIC SEQUENCES SUBJECT TO MUD FILTRATE INVASION

Subhadeep Chowdhury, M.S.E.
The University of Texas at Austin, 2004

Supervisor: Carlos Torres-Verdín

The effect of mud-filtrate invasion on resistivity, nuclear, and formation tester wireline measurements is poorly understood when the formation of interest is laminated on a scale below the vertical resolution of borehole logging instruments. This thesis describes a quantitative study to assess the effect of mud-filtrate invasion on well-log measurements acquired in thinly laminated clastic sequences.

Synthetic models are constructed with varying proportions of shale and two types of sand. These models are used to simulate the process of mud-filtrate invasion. Each type of sand is assumed isotropic, homogeneous, water wet, and saturated with oil to the level of irreducible water saturation. Simulations of the process of mud-filtrate invasion yield 2D cross-sections of water saturation, salt concentration, and electrical resistivity in the invaded rock formations. These
cross-sections are then used to simulate wireline resistivity, density, neutron, and formtion tester measurements. Using standard interpretation techniques, saturation of original oil in place is calculated with all the simulated log measurements.

Results indicate that for both laminated sandy and shaly sand rock formations, induction (electrical resistivity), nuclear (porosity), and formation pressure tester (pressure) measurements are significantly affected by both relative proportion of lithology and invasion of mud-filtrate. For a thinly laminated sandy rock formation, invasion of mud-filtrate gives rise to an average increase of bulk density of 5%. A decrease of approximately 24,000 ppm in the salinity of the mud filtrate gives rise to a 10% decrease in water saturation if the water saturation is calculated with resistivity readings acquired with conventional induction logging instruments. Water saturation in a laminated sandy rock formation can vary from 10% to 23%, when calculated with saturation and porosity exponents perpendicular and parallel to the bedding plane, respectively, and with the vertical resistivity reading acquired with a tri-axial induction instrument. Permeability calculated from dual-packer formation tester measurements can significantly depart from the effective average permeability in a composite and laminated sandy formation. The error in the estimated permeability is 12% in a laminated sandy formation with equal proportion of high and low permeability sands.

In laminated shaly sand rock formations, the error in the estimation of water saturation is 25% lower when calculated with the vertical resistivity measured by a tri-axial induction instrument than when calculated with the resistivity measured by a conventional induction instrument. Conventional induction resistivity instruments cannot distinguish between resistivities in the flushed and invaded zones when the volume of shale is higher than 50%. Presence of shale also causes a significant error in the estimated permeability. For a laminated shaly sand
formation with 50% volume of shale, the error in the estimated permeability is approximately 33%.
TABLE OF CONTENTS

ACKNOWLEDGEMENTS........................................................................................................v
ABSTRACT.........................................................................................................................vii
TABLE OF CONTENTS........................................................................................................x
LIST OF TABLES..................................................................................................................xii
LIST OF FIGURES..............................................................................................................xiii
CHAPTER 1: INTRODUCTION..............................................................................................1
  1.1 Background..................................................................................................................1
  1.2 Objective....................................................................................................................3
  1.3 Methodology..............................................................................................................4
CHAPTER 2: NUMERICAL SIMULATION OF WATER-BASED MUD FILTRATE INVASION.........................................................................................8
  2.1 Background ................................................................................................................8
  2.2 Simulation with INVADE............................................................................................9
  2.3 Simulation with UTCHEM ..........................................................................................11
CHAPTER 3: CAPILLARY PRESSURE AND RELATIVE PERMEABILITY MODEL .................................................................................................12
  3.1 Theory .......................................................................................................................12
  3.2 Description of Models ..............................................................................................13
CHAPTER 4: CONSTRUCTION OF LITHOLOGY AND RESISTIVITY MODELS.................................................................................................16
  4.1 Description of lithology models..................................................................................16
  4.2 Radial Profiles of Electrical Resistivity.................................................................19
  4.3 Simulation of HDIL and 3DEX resistivity readings .................................................24
    4.3.1 Effect of Salt Concentration..............................................................................33
CHAPTER 5: NUCLEAR LOGGING ..................................................................................36
CHAPTER 6: CALCULATION OF SATURATION OF ORIGINAL FLUID INPLACE

6.1 Sand No. 1 – Sand No. 2 composite rock formation
6.2 Sand No.1 – Shale composite rock formation

CHAPTER 7: FORMATION TESTER RESPONSES

7.1 Methodology
7.2 Effective Permeability

CHAPTER 8: DISCUSSION

CHAPTER 9: CONCLUSIONS

NOMENCLATURE
REFERENCES
VITA
LIST OF TABLES

Table 4.1- Summary of the petrophysical properties of Sand No. 1, Sand No. 2, and mud properties assumed in the construction of synthetic rock formations models.................................................................18
LIST OF FIGURES

Figure 1.1: Schematic diagram of the two synthetic composite rock formations considered in this thesis. (A) Sand No.1 – Shale. (B) Sand No. 1 – Sand No. 2. The light vertical bar to the left of each figure indicates the location of the borehole. Individual lamina thickness is 1.5-inches………………………………………………………5

Figure 2.1: Calculated Flow rate of mud-filtrate invasion as a function of time of invasion in Sand No. 1…………………………………….10

Figure 3.1: Water–oil capillary pressure and relative permeability curves. The black curve with cyan dots identifies the capillary pressure function for Sand No. 1 and the cyan curve with dark dots identifies the capillary pressure function for Sand No. 2. The left- and right-hand diagrams of relative permeability correspond to Sand No. 1 and Sand No. 2, respectively, and the light and dark color curves correspond to relative permeability of water and oil, respectively……………………………………………………..15

Figure 4.1: Radial profiles of water saturation ($S_w$) and water resistivity ($R_w$) as a function of distance away from the borehole wall. The dark and light curves identify radial profiles for Sand No. 1 and Sand No. 2, respectively. Mud filtrate salinity is 30,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days………20

Figure 4.2: Radial profiles of water saturation ($S_w$) and water resistivity ($R_w$) as a function of distance away from the borehole wall. The dark and light curves identify Sand No. 1 and Sand No. 2 respectively. Mud filtrate salinity is 6,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days…………………………. 21
Figure 4.3: Radial profiles of formation resistivity for Sand No. 1 and Sand No. 2 as a function of distance away from the borehole wall. The dark curve identifies Sand No. 1 and the light curve identifies Sand No. 2. Mud filtrate salinity is 30,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days. ………23

Figure 4.4: Radial profiles of electrical resistivity for Sand No. 1 and Sand No. 2 as a function of distance away from the borehole wall. The dark curve identifies Sand No. 1 and the light curve identifies Sand No. 2. Mud filtrate Salinity is 6,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days………………….…………….24

Figure 4.5: Synthetic electrical resistivity model of 50:50 Sand No. 1 – Sand No. 2 rock formations. An enlarged 5-ft thick section is shown to display the spatial distribution of electrical resistivity. The dark line identifies deep and true formation resistivity ($R_t$), and the light curve identifies shallow and invaded zone resistivity ($R_{xo}$). The salinity of mud-filtrate is 30,000 ppm and the time of invasion is 4 days………………………………….…………25

Figure 4.6: Tool Mnemonics associated with the readings of the HDIL instrument for different lengths of investigation. These curves are generated from the raw field data through HDIL processing. The M2R1 and M2RX mnemonics identify the shallowest and deepest resistivity readings, respectively. ………………………………..27

Figure 4.7: Simulated HDIL and 3DEX resistivity readings in Sand No. 1 - Sand No. 2 rock formations. The corresponding relative proportions of Sand No. 1 and Sand No. 2 are shown in the box below each plot. Salinity of the mud-filtrate is 30,000 ppm. For HDIL resistivity readings, refer to Figure. 4.6 for a description of tool mnemonics. For 3DEX instrument, resistivity readings are
identified with a thin red curve (vertical resistivity $R_v$) and a dashed cyan curve (horizontal resistivity $R_h$). Two pairs of thick and dashed curves at the left-most part of the plots identify the vertical resistivity obtained through the inversion of 3DEX measurements.

**Figure 4.8:** Simulated 3DEX and HDIL resistivity readings in Sand No. 1-shale rock formations. The salinity of mud-filtrate is 30,000 ppm. Relative proportions of shale are shown in the box inserts. The 3DEX resistivity readings are shown on the left-hand panels. Vertical resistivity ($R_v$) readings are identified with a red dotted curve. The smooth vertical resistivity reading is identified with a thick magenta curve, and the horizontal resistivity reading ($R_h$) is identified with a blue dashed curve. HDIL resistivity readings are shown on the right-hand panel. Refer to **Figure 4.6** for a description of HDIL tool mnemonics.

**Figure 4.9:** Comparison of HDIL resistivity readings for two types of Sand No. 1 - Sand No. 2 rock formations. Relative proportions are 90% of Sand No. 1, and 10% of Sand No. 2. (A) Salinity of mud-filtrate is 6,000 ppm. (B) Salinity of mud-filtrate is 30,000 ppm.

**Figure 4.10:** Comparison of HDIL resistivity readings for two types of Sand No. 1 - Shale rock formations. Relative proportions are 90% of Sand No. 1, and 10% of Shale. The thin dark dashed line identifies the deepest resistivity reading. (A) Salinity of mud-filtrate is 6,000 ppm. (B) Salinity of mud-filtrate is 30,000 ppm.

**Figure 5.1:** Two-dimensional bulk density response map. This map is used as a spatial filter to calculate density tool readings. The two peaks in the response map indicate higher count rates near the detectors.
Figure 5.2: Bulk density response of Sand No. 1 - Sand No. 2 rock formations. Density measurements simulated for invaded rock formations are identified with the grey line and the dark dots. Density measurements for non-invaded rock formations are identified by a dark line with grey dots.

Figure 5.3: Bulk density response of Sand No. 1 - shale rock formations. Density measurements simulated for invaded rock formations are identified with the grey line and the dark dots. Density measurements for non-invaded rock formations are identified by a dark line with grey dots.

Figure 5.4: Neutron response of Sand No. 1 - Sand No. 2 rock formations. Neutron measurements are simulated with an average 2D spatial filter. A linear relationship is observed between the neutron responses and the relative fraction of rocks.

Figure 5.5: Neutron response in Sand No. 1 - shale rock formations. Neutron measurements are simulated with an average 2D spatial filter. A linear relationship is observed between the neutron responses and the relative fraction of rocks.

Figure 6.1: Water saturation ($S_w$) calculated with 3DEX-derived vertical and horizontal resistivities. The dark curves with grey dots identify $S_w$ values calculated with porosity ($m$) exponent parallel to the bedding plane. The grey curves with dark dots identify $S_w$ values calculated with $m$ exponent perpendicular to the bedding plane. The dash lines identify $S_w$ values calculated with the 3DEX-derived vertical resistivity ($R_v$), whereas the bold line identifies water saturation ($S_w$) values calculated with the true formation resistivity. The straight bold red line shows the water saturation calculated with constant $m$ (=1.9) and $n$ (=2) exponents.
Figure 6.2: Water saturation ($S_w$) calculated for different values of salt concentration of mud-filtrate. Differences in $S_w$ calculated with HDIL resistivity readings and true connate water saturation are small when Sand No. 1 - Sand No. 2 rock formations are invaded with a mud filtrate of 6,000 ppm salinity……………………………..51

Figure 6.3: Differences between the calculated and true connate water saturations ($S_w$) for different values of sand fraction. Saturations are calculated in Sand No. 1 - Sand No. 2 sequences with HDIL resistivity readings for two values of mud filtrate salinity. The thick black horizontal line identifies the zero difference between calculated and original values of $S_w$. Differences between the calculated and original water saturation when the salinity of mud-filtrate is 6,000 ppm……………………………………………...52

Figure 6.4: Water saturation ($S_w$) calculated in Sand No. 1 - shale rock formations. The vertical resistivity simulated for the 3DEX instrument is used to calculate water saturation using five standard interpretation models of resistivity-water saturation. The thick dark line at the bottom of the figure identifies the actual connate water saturation in Sand No. 1………………………………………………….54

Figure 6.5: Water saturation ($S_w$) calculated in Sand No. 1 - Shale rock formations. The simulated HDIL deep resistivity reading is used to calculate water saturation using five standard interpretation models of resistivity-water saturation. The thick dark line at the bottom of the figure identifies the actual connate water saturation in Sand No. 1. Salinity of the mud filtrate is 30,000 ppm. ……………….55

Figure 6.6: Water saturation ($S_w$) calculated in Sand No. 1 - Shale rock formations. The simulated HDIL deep resistivity reading is used to calculate water saturation using five standard interpretation models
of resistivity-water saturation. The thick dark line at the bottom of
the figure identifies the actual connate water saturation in Sand
No. 1. Salinity of mud-filtrate is 6,000 ppm.......................56

Figure 7.1: Simulated dual-packer pressure transients in Sand No.1- Sand No.
2 rock formations. The differential pressure (ΔP) increases with an
increase in the relative proportion of Sand No.2....................59

Figure 7.2: Simulated dual-packer pressure transients in Sand No.1 - shale
rock formations. The differential pressure (ΔP) increases with an
increase in the relative proportion of shale in the formation. Six
values of shale fractions are considered in the simulations........60

Figure 7.3: Effective permeability of Sand No. 1 – Sand No. 2 and Sand No. 1
– shale rock formations. The dark and grey dash lines identify the
effective permeability obtained from the inversion of dual-packer
pressure transient data. The dark and grey bold lines identify
volume-averaged permeability values for sandy and shaly sand
rock formations, respectively..............................................62
CHAPTER 1:
INTRODUCTION

1.1 Background

The evaluation of thinly laminated clastic formations has been an outstanding petrophysical problem for many years. Laminated clastic sequences are usually associated with low values of electrical resistivity because of presence of shale. In most instances, hydrocarbon reserves are underestimated in laminated clastic reservoirs due their relatively low values of electrical resistivity. Extensive work has been undertaken to address this problem. Recently, Shray and Borbas (2001) advanced a procedure to assess water saturation in laminated sand formations based on the use of nuclear magnetic resonance (NMR) measurements and estimates of resistivity anisotropy. Accurate calculation of fluid saturation is only possible with a methodology that combines resistivity anisotropy and high-resolution borehole measurements such as NMR and borehole imaging.

Forsyth et al. (1993) proposed a parallel resistivity model to assess water saturation in a laminated clastic hydrocarbon field located off the coast of Brunei. Their interpretation method combines rock-core data (or borehole image logs), production data, and resistivity measurements acquired with conventional induction instruments. Even though their interpretation increased the value of in-
place hydrocarbon reserves by 10%, Forsyth et al.’s (1993) method is relatively less successful for lamina thinner than the vertical resolution of resistivity instruments. Andreani (1989) described a methodology that incorporates both attenuation and transit time of electromagnetic propagation tools to evaluate the hydrocarbon potential of laminated formations. This method proved adequate to quantify the relative proportions of rock constituents in composite clastic sequences in the offshore Nile delta. Gill et al. (1991) described the use of layered resistivity models to derive the true resistivity of laminated clastic reservoirs. The derived values of electrical resistivity were subsequently used to condition the estimation of water saturation from induction resistivity readings.

To date, none of the interpretation methods proposed for the petrophysical evaluation of laminated clastic reservoirs has quantified the effect of mud-filtrate invasion on well logs. Borehole logging measurements acquired in laminated clastic formations are not only conditioned by relatively poor vertical resolution but also by the presence of mud-filtrate invasion in the porous sand components of the lamination. A highly relevant situation occurs in cases of deep invasion and large contrasts in salt concentration between mud filtrate and connate water. The central objective of this thesis is to assess the effect of water-based mud-filtrate invasion in clastic laminated sequences with various proportions of sand and shale constituents.
The effect of mud-filtrate invasion on the spatial distribution of fluids and salt concentration in porous formations penetrated by an overbalanced well has been documented in the literature (Dewan et al., 2001 and Wu et al., 2004). George et al., 2004 describe a field example that quantifies the impact of mud-filtrate invasion on the spatial distribution of electrical resistivity within a sand invaded with mud filtrate. These influential research projects revolve primarily on the petrophysical interpretation of uniform, isotropic, and homogeneous sands. The process of mud-filtrate invasion is governed not only by the properties of mud but also by the petrophysical properties of the formation. Rate of mudcake growth, an important factor that controls the length of invasion, depends on both permeability and porosity of the formation. In a composite rock formation, where a mixture of two different rocks constitutes a layered alternation, the spatial distribution of fluids resulting from the process of mud-filtrate invasion is a function of depth. Presence of shale hinders the invasion of mud-filtrate.

1.2 Objectives

The central objective of this thesis is to study the effect of mud-filtrate invasion on several borehole logging measurements acquired in sand-sand and sand-shale composite rock formations. A change in the relative proportions of shale and sand in a composite rock formation gives rise to various measurements of non-invaded and invaded zone resistivity, resistivity anisotropy ($R_v/R_h$), bulk
density, neutron, and formation tester. All of these measurements are different from the measurements that would be acquired in homogeneous sands or shales. As a result, the estimated hydrocarbon saturation departs from the actual in-situ saturation.

1.3 Methodology

This thesis considers synthetic 2D models of two types of rock formations: (a) 20-ft thick alternations of thin (1.5-inches), homogeneous sand (Sand No. 1) and shale (1.5-inches), here referred to as Sand No. 1 – shale, and (b) 20-ft thick laminated sandy rock formations comprising two alternating thin (1.5-inches) sand layers that differ in petrophysical properties, here referred to as Sand No. 1 - Sand No. 2. **Figure 1.1** is a schematic diagram of each type of composite rock formation. As indicated in **Figure 1.1**, each sand lamina is homogeneous with respect to porosity, permeability, capillary pressure, and relative permeability. In both types of sequences, relative proportions of shale and sand vary from 0 to 100%.
Figure 1.1: Schematic diagram of the two synthetic composite rock formations considered in this thesis. (A) Sand No.1 – Shale. (B) Sand No. 1 – Sand No. 2. The light vertical bar to the left of each figure indicates the location of the borehole. Individual lamina thickness is 1.5-inches.
Initially, all the sands are imbued with oil up to the level of irreducible water saturation. We assume a borehole of 4 inches in diameter drilled with a water-based mud in overbalanced condition. Salinity of the mud is lower than that of the connate water. Spatial distributions of saturation and salt concentration in the invaded sequence are simulated using a 2D multi-phase, multi component, finite-difference simulator – UTCHEM. Using Archie’s equation, a spatial distribution of electrical resistivity is constructed and taken as input to simulate the response of High Density Induction Logging (HDIL), and 3D Explorer (3DEX) instruments using proprietary eXpress software.

Nuclear log measurements are simulated for the Sand No. 1 - shale and Sand No. 1 - Sand No. 2 rock formations using 2D response maps in the form of spatial filters. For density, such response map is generated via Monte Carlo simulations (Ellis, 2003). A linear spatial filter is used to simulate neutron log measurements. The density responses for the Sand No. 1 - Sand No. 2 and Sand No. 1 - shale rock formations are simulated for both invaded and non-invaded conditions. Neutron responses are simulated only for the non-invaded condition with the assumption that invasion (salt concentration) of mud-filtrate does not affect the neutron responses. A final exercise is performed to simulate dual-packer formation tester measurements in the same composite rock formations. Pressure

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transient analysis assumes the spatial distribution of fluid saturation and invaded formation pressure as initial condition. Finally, using the simulated results, inversion is carried out to assess original oil in place with different petrophysical models of water saturation.
CHAPTER 2:
NUMERICAL SIMULATION OF WATER-BASED MUD FILTRATE INVASION

2.1 Background

Only water-based mud with low salinity is considered in this thesis for the simulation of mud-filtrate invasion. Fluid phases are immiscible, but salt mixing between mud filtrate and connate water is considered in the numerical simulation (Dewan and Chenevert, 2001). The flow rate of mud-filtrate invasion is controlled by the gradual thickening and compaction of mudcake (Wu et al., 2004). Flow rate of mud-filtrate invasion can be described with Darcy’s law, namely,

$$Q_f = \frac{kA \Delta P}{\mu h_{mc}},$$  

(2.1)

where $Q_f$ is the flow rate of mud-filtrate across the borehole wall, $k$ is the permeability of mudcake, $A$ is the cross-sectional area, $\mu$ is the viscosity of filtrate, $h_{mc}$ is the thickness of mudcake, and $\Delta P$ is the pressure drop across the mudcake.
2.2 Simulation with INVADE

In this thesis, the flow rate of invasion is simulated with a 2D, multi-component and multi-phase simulator - INVADE\(^3\) (Wu et al., 2004). Figure 2.1 displays the flow rate of mud-filtrate invading a 1.5-inches-thick Sand No. 1 lamina, over a period of four days. Within a short period (\(10^{-4}\) days or 0.144 minutes), the rate of invasion decreases to a steady-state value. There are five assumptions considered for the invasion of the mud-filtrate: (a) incompressible and immiscible fluid, (b) two-phase flow comprising water and oil, (c) Fickian dispersion, (d) Darcy’s law, and (e) local thermodynamic equilibrium.

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Figure 2.1: Calculated Flow rate of mud-filtrate invasion as a function of time of invasion in Sand No. 1.

Equation (2.1) indicates that the numerical simulation with INVADE gives rise to different average flow rate of mud-filtrate invasion over time for Sand No. 1 and Sand No. 2. No flow rate of mud-filtrate invasion is considered in shales. The average flow rates for mud filtrate invasion are entered into UTCHEM for the numerical simulation of the spatial distribution of water saturation and salt concentration in the invaded rock formations.
2.3 Simulation with UTCHEM

In this work, radial cross-sections of synthetic rock formations are discretized into several nodes for the numerical simulation of mud-filtrate invasion with UTCHEM. There are 120 logarithmically spaced nodes in the radial direction and 160 nodes in the vertical direction in a 2D cross-section of area equal to 6x20-ft$^2$. Axial-symmetric flow is assumed in the simulation of mud-filtrate invasion. The numerical simulation generates values of water saturation ($S_w$) and salt concentration ($C_w$) at each node. Electrical resistivity of water ($R_w$) is calculated at each node from the spatial distribution of salt concentration using the conversion formula (Bigelow, 1992)

$$R_w = \left[ 0.0123 + \frac{3647}{C_w^{0.955}} \right] \frac{82}{1.8T + 39}, \quad (2.2)$$

where $C_w$ is salt concentration in ppm, and $T$ is the temperature in degrees Celsius.
CHAPTER 3:

CAPILLARY PRESSURE AND RELATIVE PERMEABILITY MODEL

3.1 Theory

We assume a simple modified Brooks-Corey model (Corey, 1994) to calculate capillary pressure and relative permeability in Sand No. 1 and Sand No. 2. Accordingly, the relationship between capillary pressure and water saturation of a two-phase fluid mixture is given by

\[ P_c = P_e \cdot (S_{wt}^*)^{-1/k}, \]  

(3.1)

where \( P_c \) is capillary pressure, \( P_e \) is the entry capillary pressure, \( k \) is a pore-shape factor, and \( S_{wt}^* \) is the effective wetting phase saturation, given by

\[ S_{wt}^* = \frac{S_{u} - S_{sw}}{1 - S_{swr} - S_{nwr}}, \]  

(3.2)

where \( S_{swr} \) is the irreducible water saturation and \( S_{nwr} \) is the irreducible oil saturation.
The relationship between relative permeability and water saturation for each fluid phase can be constructed from the Brooks and Corey model (Corey, 1994), i.e.

\[ k_{rw} = k_{rwo} \cdot \left( S_{wr}^* \right)^{\frac{2}{\lambda}}, \]  

(3.3)

and

\[ k_{rnw} = (1 - S_{nr}^*)^2 \cdot \left[ 1 - \left( S_{nr}^* \right)^{\frac{2}{\lambda}} \right], \]  

(3.4)

where \( k_{rw} \) and \( k_{rnw} \) are the wetting phase and non-wetting phase relative permeability, respectively.

### 3.2 Description of Models

**Figure 3.1** displays the water-oil capillary pressure and relative permeability curves assumed for Sand No. 1 and Sand No. 2. Sand No. 1 is characterized by a higher absolute permeability and a higher porosity than Sand No. 2. Displacement pressures for Sand No. 1 and Sand No. 2 are 0.3 and 0.8 psi,
respectively. Both sands consist of well sorted grains, thus giving rise to a constant slope (horizontal) in the capillary pressure curves for high values of water saturation.

Figure 3.1 also depicts important basic parameters for the saturating fluids. Both sands are water wet. A comparison of the relative permeability curves for the two sands indicates that oil exhibits a higher end-point relative permeability for Sand No. 1 (0.80) than for Sand No. 2 (0.69). End-point relative permeability for water decreases from 0.35 in Sand No. 1 to 0.30 in Sand No. 2. All rock and fluid properties assumed in the numerical simulation of mud-filtrate invasion are based on well documented rock-core laboratory measurements (Amyx et al., 1960).
Figure 3.1: Water–oil capillary pressure and relative permeability curves. The black curve with cyan dots identifies the capillary pressure function for Sand No. 1 and the cyan curve with dark dots identifies the capillary pressure function for Sand No. 2. The left- and right-hand diagrams of relative permeability correspond to Sand No. 1 and Sand No. 2, respectively, and the light and dark color curves correspond to relative permeability of water and oil, respectively.
CHAPTER 4:

CONSTRUCTION OF LITHOLOGY AND RESISTIVITY MODELS

4.1 Lithology Models

Lithology models are constructed using an algorithm written in MATLAB\textsuperscript{4}. Each clastic sequence is 20-ft thick and is bounded at the top and bottom by impermeable shale layers. The initial synthetic model of rock formation contains 100% pure Sand No. 1. For simplicity in the numerical simulation, the smallest thickness of the numerical layer is 1.5-inches, which is also the thickness of the thinnest geological layer. There are a total of 160 numerical layers stacked vertically in the 20-ft thick rock formation. Consequently, in the composite Sand No. 1 – shale and Sand No. 1 – Sand No. 2 rock formations, relative proportions of shale and Sand No. 2 increase, from 0 to 100%. Increments in the relative proportion of constituents are considered at uniform incremental intervals of 1.5-inches, or multiples of it. Taking the entire relative proportions into consideration, there are a total of ten sequences of the types Sand No. 1 – shale and Sand No. 1 – Sand No. 2. Relative proportions of

\footnote{Mark of The Mathworks INC.}
lithology considered in this thesis are 0, 10, 20, 25, 33, 50, 66, 75, 80, 90, and 100%.

Spatial distributions of water saturation and salt concentration for each composite sequence are simulated with UTCHEM after an uninterrupted time of invasion of four days. **Table 4.1** summarizes the mud properties, and the petrophysical and fluid parameters assumed for Sand No. 1 and Sand No. 2. In two separate cases, simulations for mud filtrate invasion are carried out assuming values of salt concentration of 30,000 ppm and 6,000 ppm, respectively, whereas the salinity of connate water remains at 100,000 ppm.
Table 4.1: Summary of the petrophysical properties of Sand No. 1, Sand No. 2, and mud properties assumed in the construction of synthetic rock formations models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Sand No. 1</th>
<th>Sand No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid system</td>
<td>Water/Oil</td>
<td>Water/Oil</td>
</tr>
<tr>
<td>Porosity (p.u.)</td>
<td>0.2</td>
<td>0.08</td>
</tr>
<tr>
<td>Horizontal permeability (md)</td>
<td>700</td>
<td>70</td>
</tr>
<tr>
<td>Vertical permeability (md)</td>
<td>700</td>
<td>70</td>
</tr>
<tr>
<td>Displacement pressure (psia)</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>End point relative-permeability for water (dimensionless)</td>
<td>0.35</td>
<td>0.3</td>
</tr>
<tr>
<td>End point relative-permeability for oil (dimensionless)</td>
<td>0.82</td>
<td>0.69</td>
</tr>
<tr>
<td>Pore shape factor ($\lambda$)</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>0.11</td>
<td>0.23</td>
</tr>
<tr>
<td>Irreducible oil saturation</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>Density of formation water ($g/cm^3$)</td>
<td>1.06</td>
<td>1.06</td>
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<tr>
<td>Density of oil ($g/cm^3$)</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Viscosity of formation water (cp)</td>
<td>0.78</td>
<td>0.78</td>
</tr>
<tr>
<td>Viscosity of oil (cp)</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Type of mud (water/oil-based)</td>
<td>Water-based</td>
<td>Water-based</td>
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<tr>
<td>Mud weight ($g/cm^3$)</td>
<td>1.86</td>
<td>1.86</td>
</tr>
<tr>
<td>Salinity of formation water (ppm)</td>
<td>100,000</td>
<td>100,000</td>
</tr>
<tr>
<td>Salinity of mud-filtrate (ppm)</td>
<td>30,000; 6,000</td>
<td>30,000; 6,000</td>
</tr>
<tr>
<td>Archie's porosity exponent</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Archie's saturation exponent</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Archie's tortuosity factor</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Connate water resistivity (ohm-m)</td>
<td>0.0465</td>
<td>0.0465</td>
</tr>
<tr>
<td>True formation resistivity (ohm-m)</td>
<td>77.173</td>
<td>286.52</td>
</tr>
<tr>
<td>Length of mud-filtrate invasion after 4 days (ft)</td>
<td>4.2</td>
<td>4.8</td>
</tr>
<tr>
<td>Average flow rate of mud-filtrate in 1.5-inches thick lamina(ft$^3$/day)</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>Borehole diameter (inch)</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
4.2 Radial Profiles of Electrical Resistivity

**Figures 4.1 and 4.2** show radial profiles of water saturation (Sw) and water resistivity (Rw) in Sand No. 1 and Sand No. 2 for values of mud-filtrate salinity of 30,000 ppm and 6,000 ppm, respectively. Each radial profile represents one numerical layer. After four days of invasion, the saturation profiles exhibit a length of invasion of 4.2-ft and 4.8-ft for Sand No. 1 and Sand No. 2, respectively. Figures 4.1 and 4.2 also indicate that a change in the salinity of mud-filtrate from 30,000 ppm to 6,000 ppm produces a significant difference in the radial profiles of water resistivity for Sand No. 1 and Sand No. 2. The simulated cross-sections of water saturation and water resistivity (salt concentration) are now used to calculate the corresponding cross-section of electrical resistivity in the synthetic laminated rock formations.
Figure 4.1: Radial profiles of water saturation ($Sw$) and water resistivity ($Rw$) as a function of distance away from the borehole wall. The dark and light curves identify radial profiles for Sand No. 1 and Sand No. 2, respectively. Mud filtrate salinity is 30,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days.
Figure 4.2: Radial profiles of water saturation (Sw) and water resistivity (Rw) as a function of distance away from the borehole wall. The dark and light curves identify Sand No. 1 and Sand No. 2 respectively. Mud filtrate salinity is 6,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days.

Electrical resistivity of the invaded rock formations is calculated using Archie’s equation,

\[ S_w^n = \frac{R_w}{R_t} \frac{a}{\phi^m}, \]  

(4.1)

where \( S_w \) is water saturation, \( a \) is the tortuosity factor, \( m \) is the cementation or porosity exponent, \( n \) is the saturation exponent, \( R_w \) is the resistivity of connate water, \( R_t \) is true formation resistivity, and \( \phi \) is porosity. In this thesis, \( m \) is
referred to as the porosity exponent. The tortuosity factor $a$ and the saturation exponent $n$ are the same for both Sand No. 1 and Sand No. 2. However, different values are assigned to the porosity exponents of Sand No. 1 and Sand No. 2 because of the large difference in porosity between the two sands (Table 4.1). We refer to the resistivity calculated using equation (4.7) as the original resistivity of the synthetic rock formation model.

**Figure 4.3** shows the simulated radial profiles of electrical resistivity for Sand No. 1 and Sand No. 2 layers when both sands are invaded with mud filtrate of 30,000 ppm salinity. There is a shallow low-resistivity annulus in the resistivity profile of Sand No. 2. As shown in **Figure 4.4**, the simulated radial profile of electrical resistivity for Sand No. 2 exhibits a wider and deeper low resistivity annulus when the salinity of mud-filtrate is 6,000 ppm.
Figure 4.3: Radial profiles of formation resistivity for Sand No. 1 and Sand No. 2 as a function of distance away from the borehole wall. The dark curve identifies Sand No. 1 and the cyan curve identifies Sand No. 2. Mud filtrate salinity is 30,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days.
Figure 4.4: Radial profiles of electrical resistivity for Sand No. 1 and Sand No. 2 as a function of distance away from the borehole wall. The dark curve identifies Sand No. 1 and the light cyan curve identifies Sand No. 2. Mud filtrate salinity is 6,000 ppm and connate water salinity is 100,000 ppm. The time of invasion is 4 days.

Based on the simulated radial profiles of electrical resistivity, we construct a summary table of results for each composite sequence. We refer to this table as the resistivity table for the synthetic model. The table comprises the following variables: resistivity of the non-invaded zone \( R_t \), resistivity of the invaded zone \( R_{xo} \), length of invasion \( L_{xo} \), resistivity of mud-filtrate \( R_m \), and borehole diameter \( B_d \). In all of the simulation examples, the borehole diameter is kept constant at 4 inches. The resistivity table is entered to the proprietary eXpress software to simulate the response of High Density Induction Logging (HDIL) and 3D Explorer (3DEX) instruments. We remark that in each clastic sequence the
smallest lamina thickness is 1.5-inches, i.e. below the vertical resolution of resistivity logging instruments. **Figure 4.5** displays the distribution of resistivity with depth in the invaded 50:50 Sand No. 1 – Sand No. 2 rock formation.

**Figure 4.5:** Synthetic electrical resistivity model of 50:50 Sand No. 1 – Sand No. 2 rock formations. An enlarged 5-ft thick section is shown to display the spatial distribution of electrical resistivity. The dark line identifies deep and true formation resistivity (Rt), and the light curve identifies shallow and invaded zone resistivity (Rxo). The salinity of mud-filtrate is 30,000 ppm and the time of invasion is 4 days.
4.3 Simulation of HDIL and 3DEX Resistivity Readings

Simulation of electrical resistivity readings is performed in three steps. First, the resistivity table of the synthetic model is entered into eXpress. Second, two input resistivity models are generated separately for HDIL and 3DEX raw data. The last and third step consists of running the HDIL processing software and the 3DEX multi-frequency focusing inversion to simulate the instrument resistivity readings across the composite rock formation. In this work, both HDIL and 3DEX responses are plotted together for the Sand No. 1 – Sand No. 2 sequences. For Sand No. 1 – shale rock formations, HDIL and 3DEX resistivity readings are plotted in separate panels. The objective of using different panels is to visually assess the sensitivity of HDIL readings to both flushed and non-invaded zones in the presence of shale laminations.

There are a total of six curves output by HDIL processing (Figure 4.6). These curves exhibit different lengths of investigation. Due to the lack of space in the plots of HDIL logs, Figure 4.6 is helpful to interpret the HDIL resistivity readings. For illustration of the 3DEX-derived resistivity readings, two important curves – horizontal resistivity ($R_h$) and vertical resistivity ($R_v$) are shown in the corresponding log display for each composite rock formation. Figure 4.7 shows the simulated 3DEX and the HDIL resistivity readings for a total of nine
composite Sand No. 1 – Sand No. 2 rock formations. Among these nine cases, only two are isotropic, consisting of either 100% Sand No. 1 or 100% Sand No. 2. For the rest of the composite rock formations, resistivity anisotropy is readily identified from the separation between vertical resistivity $R_v$ (thin red line) and horizontal resistivity $R_h$ (dashed cyan line). The salinity of mud-filtrate assumed in this example is 30,000 ppm. An increase in the relative proportion of Sand No. 2 causes an increase in both the vertical resistivity $R_v$ and HDIL resistivity readings.

![Figure 4.6: Tool Mnemonics associated with the readings of the HDIL instrument for different lengths of investigation. These curves are generated from the raw field data through HDIL processing. The M2R1 and M2RX mnemonics identify the shallowest and deepest resistivity readings, respectively.](image)
Figure 4.7: Simulated HDIL and 3DEX resistivity readings in Sand No. 1 - Sand No. 2 rock formations. The corresponding relative proportions of Sand No. 1 and Sand No. 2 are shown in the box below each plot. Salinity of the mud-filtrate is 30,000 ppm. For HDIL resistivity readings, refer to Figure 4.6 for a description of tool mnemonics. For 3DEX instrument, resistivity readings are identified with a thin red curve (vertical resistivity $R_v$) and a dashed cyan curve (horizontal resistivity $R_h$). Two pairs of thick and dashed curves at the left-most part of the plots identify the vertical resistivity obtained through the inversion of 3DEX measurements.
Sand No. 1 = 100%, Sand No. 2 = 0%

Sand No. 1 = 90%, Sand No. 2 = 10%

Sand No. 1 = 80%, Sand No. 2 = 20%

Sand No. 1 = 66%, Sand No. 2 = 33%

Sand No. 1 = 50%, Sand No. 2 = 50%

Sand No. 1 = 33%, Sand No. 2 = 66%

Sand No. 1 = 20%, Sand No. 2 = 80%

Sand No. 1 = 10%, Sand No. 2 = 90%

Sand No. 1 = 0%, Sand No. 2 = 100%
Figure 4.8 illustrates the 3DEX and HDIL resistivity readings for four Sand No. 1 – shale rock formations. The relative proportions of shale considered are 10, 25, 50, and 75%, respectively. The assumed salinity of mud-filtrate is 30,000 ppm. Plots in the left-hand panel display the vertical resistivity ($R_v$), identified with a red dotted curve, and the horizontal resistivity ($R_h$), identified with a blue dashed curve. There is also a smooth vertical resistivity identified with a thick bold magenta curve. The smooth vertical resistivity curves are generated only in the Sand No. 1 – shale rock formations due to shoulder bed effects of low resistivity, laminar shale (1.5 ohm-m). For interpretation purposes, the smooth vertical resistivity curve is ignored in this study. The HDIL resistivity readings are plotted on the right-hand panel. Mnemonics associated with the HDIL resistivity readings are defined in Figure 4.6. The vertical resolution of the HDIL instrument decreases with an increase in the relative proportion of shale. HDIL measurements cannot distinguish between the shallow and deep resistivity readings if the proportion of shale ($V_{sh}$) is higher than 50%.
Figure 4.8: Simulated 3DEX and HDIL resistivity readings in Sand No. 1-shale rock formations. The salinity of mud-filtrate is 30,000 ppm. Relative proportions of shale are shown in the box inserts. The 3DEX resistivity readings are shown on the left-hand panels. Vertical resistivity ($R_v$) readings are identified with a red dotted curve. The smooth vertical resistivity reading is identified with a thick magenta curve, and the horizontal resistivity reading ($R_h$) is identified with a blue dashed curve. HDIL resistivity readings are shown on the right-hand panel. Refer to Figure. 4.6 for a description of HDIL tool mnemonics.
4.3.1 Effect of Salt Concentration of Mud-filtrate

As mud filtrate invades the formation, salt mixing between mud and connate water causes a variable radial distribution of resistivity anisotropy \((R_v/R_h)\) away from the borehole wall. However, for the sake of simplicity, the resistivity anisotropy is only calculated in the non-invaded zone. As a result, salt concentration of mud-filtrate does not affect the resistivity anisotropy of Sand No. 1 - Sand No. 2 composite rock formations.

The effect of a contrast in salt concentration between mud filtrate and connate water is significant on HDIL resistivity readings. Figures 4.9 and 4.10 show a graphical comparison between the HDIL resistivity readings simulated for the same formation subject to invasion with two different values of salt concentration in the mud filtrate. Salinity of connate water remains at 100,000 ppm in each case. The HDIL resistivity reading in a Sand No. 1 – Sand No. 2 rock formation increases from 60 ohm-m when invaded with a 30,000 ppm saline mud filtrate, to 200 ohm-m when invaded with a 6,000 ppm saline mud filtrate (Figure 4.9). Similarly, a decrease in the salt concentration of mud filtrate causes an increase in the HDIL resistivity readings for the Sand No. 1 – shale rock formations. Figure 4.10 shows that a decrease in the salt concentration of mud-filtrate, from 30,000 ppm to 6,000 ppm, increases the deepest HDIL resistivity reading from 10 ohm-m to 15 ohm-m. For both types of composite rock formations, a change in the
The salinity of mud-filtrate does not cause an appreciable change in the vertical resistivity derived from 3DEX measurements.

**Figure 4.9:** Comparison of HDIL resistivity readings for two types of Sand No. 1 - Sand No. 2 rock formations. Relative proportions are 90% of Sand No. 1, and 10% of Sand No. 2. (A) Salinity of mud-filtrate is 6,000 ppm. (B) Salinity of mud-filtrate is 30,000 ppm.
Figure 4.10: Comparison of HDIL resistivity readings for two types of Sand No. 1- Shale rock formations. Relative proportions are 90% of Sand No. 1, and 10% of Shale. The thin dark dashed line identifies the deepest resistivity reading. (A) Salinity of mud-filtrate is 6,000 ppm. (B) Salinity of mud-filtrate is 30,000 ppm.
CHAPTER 5:

NUCLEAR LOGGING RESPONSES

We assume that density and neutron logging measurements are performed by moving the instruments against the formation and by averaging the corresponding counting rates. The calculation porosity in the Sand No.1 – Sand No. 2 rock formations is based on the equation

\[ \rho_b = \rho_f \phi + \rho_{ma} (1 - \phi), \]  

(5.1)

The calculation porosity in the Sand No.1 – shale formation is based on the equation

\[ \rho_b = \rho_f \phi + \rho_{ma} (1 - \phi - V_{sh}) + \rho_{sh} V_{sh}, \]  

(5.2)

where \( \rho_b \) is bulk density, \( \rho_f \) is fluid density, \( \phi \) is porosity, \( \rho_{ma} \) is matrix density (assumed 2.65 g/cc), \( \rho_{sh} \) is density of shale (assumed 2.60 g/cc) and \( V_{sh} \) is the volume of shale. The fluid density for each simulation node of the 2D cross-section is calculated with the expression

\[ \rho_f = S_{wir} \rho_w + (S_w - S_{wir}) \rho_{mf} + (1 - S_w) \rho_{oil}, \]  

(5.3)
where $S_{wir}$ is irreducible water saturation (which is also the initial water saturation for the rock formation), $\rho_w$ is density of connate water, $S_w$ is the UTCHEM simulated water saturation, $\rho_{mf}$ is density of mud-filtrate, and $\rho_{oil}$ is density of oil.

Bulk density responses for the Sand No. 1 – Sand No. 2 and Sand No. 1 – shale sequences are calculated with the 2D density response map shown in Figure 5.1 that assumes two gamma-ray detectors separated by a distance of 10 centimeters. The density response map is obtained from a multitude of Monte-Carlo simulations (Ellis, 2003). Most of the instrument responses originate from the near borehole formation because of the small mean free path of gamma rays. The response map is normalized to the highest sensitivity value such that the total contribution of the response map on the density reading is equal to one.
Figure 5.1: Two-dimensional bulk density response map. This map is used as a spatial filter to calculate density tool readings. The two peaks in the response map indicate higher count rates near the detectors.
Figure 5.2 illustrates that an increase in the proportion of Sand No. 2 causes an increase in the bulk density of the formation. The same trend is observed when the relative proportion of shale increases in the laminated rock formation (Figure 5.3). Density responses are simulated for the same rock formation in the absence of mud-filtrate invasion. Comparisons between the density responses simulated with and without the presence of invasion are shown in Figures 5.2 and 5.3 for the Sand No. 1 – Sand No. 2 and Sand No. 1 – shale rock formations, respectively. It is also important to mention that the density instrument exhibits a shorter length of investigation than both HDIL and 3DEX instruments. Because of its short length of investigation, the measurement performed by the density instrument corresponds to the bulk density in the flushed zone of the corresponding rock formation.
Figure 5.2: Bulk density response of Sand No. 1 - Sand No. 2 rock formations. Density measurements simulated for invaded rock formations are identified with the grey line and the dark dots. Density measurements for non-invaded rock formations are identified by a dark line with grey dots.
Figure 5.3: Bulk density response of Sand No. 1 - shale rock formations. Density measurements simulated for invaded rock formations are identified with the grey line and the dark dots. Density measurements for non-invaded rock formations are identified by a dark line with grey dots.
For the simulation of neutron instrument responses, we assume a linear spatial filter as a sensitivity response map (Ellis et al., 2000). In the construction of the neutron porosity model, porosity values for shale, Sand No. 1, and Sand No. 2 are equal to 0.45, 0.20, and 0.08 p.u., respectively. The effect of salt concentration on the neutron reading is not considered in the simulations.

**Figures 5.4 and 5.5** illustrate the effect of an increase in the relative proportion of shale and Sand No. 2 on the corresponding neutron responses of the composite rock formations. An increase in the relative proportion of shale causes an increase in the neutron response. On the other hand, an increase in the relative proportion of low porosity Sand No. 2 decreases the neutron response.

Equations (5.1) through (5.3) are used to calculate porosity in the Sand No.1 – Sand No.2 and the Sand No.1 – shale rock formations. The irreducible water saturation, density of formation water, mud-filtrate and oil are assumed to be the same values used in the synthetic model (Table 4.1). In the following sections, the porosity calculated from equations (5.1) and (5.2) is used for the calculation of water saturation.
Figure 5.4: Neutron response of Sand No. 1 - Sand No. 2 rock formations. Neutron measurements are simulated with an average 2D spatial filter. A linear relationship is observed between the neutron responses and the relative fraction of rocks.
Figure 5.5: Neutron response in Sand No. 1 - shale rock formations. Neutron measurements are simulated with an average 2D spatial filter. A linear relationship is observed between the neutron responses and the relative fraction of rocks.
CHAPTER 6:

CALCULATION OF SATURATION OF ORIGINAL FLUID IN PLACE

Water saturation is calculated iteratively from the water saturation equation, and the porosity equations (5.1) and (5.2) for Sand No. 1 – Sand No.2 and Sand No.1 – shale rock formations, respectively. Initially, fluid density is assumed to be equal 1 for the first calculation of porosity using equations (5.1) and (5.2). The resulting water saturation is then substituted into the porosity equations in order to obtain a new porosity value. This iterative process continues until both water saturation and porosity converge to a stable value. In the following sections, we describe the process of estimating water saturation in the composite rock formations using the resistivity readings of HDIL and 3DEX and porosity obtained from equations (5.1) and (5.2).

6.1 Sand No. 1 – Sand No. 2 composite rock formation

Water saturations for Sand No. 1 – Sand No. 2 rock formations are calculated using two different resistivity readings, i.e. the vertical resistivity ($R_v$) derived from 3DEX measurements and the deep resistivity ($R_d$) measured by the HDIL. In the water saturation equation, the formation resistivity factor $F (a/\phi^m)$ varies with the effective direction of the resistivity measurement. Tortuosity factor ($\alpha$) and
porosity ($\phi$) are essentially scalar quantities, whereupon the porosity exponent ($m$) varies with the direction of the resistivity measurement (Kennedy and Herrick, 2004). Apart from the two limiting cases, all composite rock formations are anisotropic from an electrical resistivity point of view. The saturation and porosity exponents depend not only on the direction of the measurement (orientation of the bedding plane) but also on the relative proportion of lithology. This remark is implicit in the following equations (Kennedy and Herrick, 2004):

\[
m_{\parallel} = \frac{\ln\left(\beta\phi_1^{m_1} + (1 - \beta)\phi_2^{m_2}\right)}{\ln \phi}, \tag{6.1}
\]

\[
m_{\perp} = \frac{\ln\left(\frac{\phi_1^{m_1} \phi_2^{m_2}}{\beta\phi_2^{m_2} + (1 - \beta)\phi_1^{m_1}}\right)}{\ln \phi}, \tag{6.2}
\]

\[
n_{\parallel} = \frac{\ln\left(\beta\phi_1^{m_1} S_{W_1}^{n_1} + (1 - \beta)\phi_2^{m_2} S_{W_2}^{n_2}\right)}{\ln S_W}, \tag{6.3}
\]

and

\[
n_{\perp} = \frac{\ln\left(\frac{1}{\phi} - \frac{\phi_1^{m_1} S_{W_1}^{n_1} \phi_2^{m_2} S_{W_2}^{n_2}}{\beta\phi_2^{m_2} S_{W_2}^{n_2} + (1 - \beta)\phi_1^{m_1} S_{W_1}^{n_1}}\right)}{\ln S_W}, \tag{6.4}
\]

where, $m_{\parallel}$ is the porosity exponent parallel to the bedding plane, $m_{\perp}$ is the porosity exponent perpendicular to the bedding plane, $n_{\parallel}$ is the saturation
exponent parallel to the bedding plane, \( n_{\perp} \) is the saturation exponent perpendicular to the bedding plane, \( \beta \) is the relative proportion of Sand No. 1, and \( \phi \) is porosity. In equations (6.1) through (6.4) \( \bar{\phi} \) and \( \bar{s}_w \) are the volume-average porosity and saturation, respectively.

In this thesis, we consider two types of water saturation: (a) actual water saturation: water saturation calculated using the original resistivity (horizontal or vertical) of the synthetic rock formation in Archie’s equation, and (b) simulated water saturation: water saturation calculated using the resistivity readings derived from the 3DEX and HDIL measurements in Archie’s equation. Figure 6.1 shows the water saturation \( (S_w) \) calculated using equations (6.1) through (6.2). The saturation exponent \( n \) is assumed constant (=2) for each composite sequence. The grey bold curve at the top of Figure 6.1 identifies the actual water saturation calculated in the direction perpendicular to the bedding plane, using the original vertical resistivity of the synthetic model. The grey dashed curve at the top identifies the simulated water saturation calculated in the direction perpendicular to the bedding plane using the vertical resistivity \( (R_v) \) derived from 3DEX measurements. The ‘bedding perpendicular’ porosity exponent \( (m_{\perp}) \) calculated from equation (6.2) is used for the calculation of the actual and simulated water saturation. An increase in the relative proportion of porous and permeable Sand
No. 1 decreases the separation between the simulated and the actual water saturation curves.

**Figure 6.1:** Water saturation ($S_w$) calculated with 3DEX-derived vertical and horizontal resistivities. The dark curves with grey dots identify $S_w$ values calculated with porosity exponent ($m$) parallel to the bedding plane. The grey curves with dark dots identify $S_w$ values calculated with $m$ exponent perpendicular to the bedding plane. The saturation exponent is assumed constant (= 2). The dash lines identify $S_w$ values calculated with the 3DEX-derived vertical resistivity ($R_V$), whereas the bold line identifies water saturation ($S_w$) values calculated with the true formation resistivity. The straight bold red line shows the water saturation calculated with constant $m$ (=1.9) and $n$ (=2) exponents.
At the bottom of Figure 6.1, the dark dashed and bold curves describe, respectively, the simulated and actual water saturation, calculated in the direction parallel to the bedding plane. The ‘bedding parallel’ porosity exponent ($m_{||}$), calculated from equation (6.1) is used for the calculation of the actual and simulated water saturation. An increase in the relative proportion of Sand No. 1 in the Sand No. 1 – Sand No. 2 rock formations causes a decrease in the difference between simulated and actual water saturation values. Figure 6.1 also indicates that depending on the orientation of the bedding plane, water saturation ranges from 0.227 to 0.10 in the 50:50 Sand No. 1 – Sand No. 2 rock formation. In the middle of Figure 6.1 the straight bold red line represents the water saturation calculated using constant values of the porosity and saturation exponents ($m = 1.9, n = 2$).

The 3DEX measurements and their corresponding horizontal and vertical resistivity readings are not affected by the salinity of mud-filtrate. Therefore, provided that porosity, permeability, and the relative proportion of lithology are constant, a change in the salinity of the mud-filtrate from 30,000 ppm to 6,000 ppm does not cause an appreciable change in the water saturation of the Sand No. 1 – Sand No. 2 rock formations. However, a change in the salinity of the mud-filtrate does cause an appreciable change in the water saturation when calculated with the HDIL resistivity readings. Figure 6.2 describes the calculated values of water saturation corresponding to values of mud-filtrate salinity of 30,000 ppm.
and 6,000 ppm. Figure 6.3 indicates that the difference between actual and calculated values of water saturation is negligible when the salinity of mud-filtrate is 6,000 ppm and when the relative proportion of Sand No. 1 is above 30%.
Figure 6.2: Water saturation ($S_w$) calculated for different values of salt concentration of mud-filtrate. Differences in $S_w$ calculated with HDIL resistivity readings and true connate water saturation are small when Sand No. 1 - Sand No. 2 rock formations are invaded with a mud filtrate of 6,000 ppm salinity.
Figure 6.3: Differences between the calculated and true connate water saturations ($S_w$) for different values of sand fraction. Saturations are calculated in Sand No. 1 - Sand No. 2 sequences with HDIL resistivity readings for two values of mud-filtrate salinity. The thick black horizontal line identifies the zero difference between calculated and original values of $S_w$. Differences between the calculated and original water saturation when the salinity of mud-filtrate is 6,000 ppm.
6.2. Sand No. 1 – Shale sequences

A comparison is made between the values of water saturation obtained with different shaly-sand saturation equations for the Sand No. 1 – shale sequences. The equations considered in this thesis are (not necessarily in order) Poupon, Simandoux, Juhasz, Schlumberger, and Archie’s. We assume constant values for the porosity and saturation exponents \( m = 1.9 \) and \( n = 2 \) because there is only one type of sand present. Figure 6.4 shows the values of simulated water saturation calculated with the 3DEX-derived vertical resistivity \( R_v \). The salinity of mud-filtrate is 30,000 ppm. Figure 6.4 shows that the calculated water saturation is close to the actual water saturation when the calculation is performed with Poupon’s Indonesian formula.
Figure 6.4: Water saturation ($S_w$) calculated in Sand No. 1 - shale rock formations. The vertical resistivity simulated for the 3DEX instrument is used to calculate water saturation using five standard interpretation models of resistivity-water saturation. The thick dark line at the bottom of the figure identifies the actual connate water saturation in Sand No. 1.

Figure 6.5 displays the values of water saturation calculated with the HDIL resistivity readings for a salinity of mud-filtrate of 30,000 ppm. As in the case of 3DEX measurements, Poupon’s Indonesian formula provides the closest match to the actual water saturation.
Figure 6.5: Water saturation ($S_w$) calculated in Sand No. 1 - Shale rock formations. The simulated HDIL deep resistivity reading is used to calculate water saturation using five standard interpretation models of resistivity-water saturation. The thick dark line at the bottom of the figure identifies the actual connate water saturation in Sand No. 1. Salinity of the mud-filtrate is 30,000 ppm.

Figure 6.6 graphically describes the values of water saturation calculated from the HDIL resistivity readings for a salinity of mud-filtrate of 6,000 ppm. A decrease in the salt concentration of mud-filtrate from 30,000 ppm to 6,000 ppm closes the gap between the calculated and actual water saturation for relative proportions of shale below 30%.
Figure 6.6: Water saturation ($S_w$) calculated in Sand No. 1 - Shale rock formations. The simulated HDIL deep resistivity reading is used to calculate water saturation using five standard interpretation models of resistivity-water saturation. The thick dark line at the bottom of the figure identifies the actual connate water saturation in Sand No. 1. Salinity of mud-filtrate is 6,000 ppm.
CHAPTER 7:

DUAL-PACKER FORMATION TESTER MEASUREMENTS

7.1 Methodology

Dual-packer formation tester measurements are simulated for both Sand No. 1-Sand No. 2 and Sand No. 1 – shale rock formations. Initially, synthetic rock formations are invaded with mud filtrate for a period of 1.5 days. The lengths of invasion for Sand No.1 and Sand No. 2 after 1.5 days are 2.5 and 3 ft, respectively. These lengths of invasion are considered adequate for near wellbore pressure transient analysis. The invaded condition is then considered as the initial condition in the numerical simulations for pressure transient analysis performed with proprietary ECLIPSE-100\textsuperscript{5} software. A dual-packer module with two vertical probes is assumed in the numerical simulations. This corresponds to three pressure sensor points located directly against the composite rock formations. (Ayan et al., 2001). For simplicity, the pressure sensor located at the center of the probe is assumed fixed against Sand No. 1 in each sequence. The purpose of this study is to compare the transient pressure responses for a variety of composite rock formations. We neglect effects of instrument storage, cleaning, and contamination in the pressure transient analysis. Invasion of mud-filtrate continues uninterrupted for 1.5 days. Subsequently, a draw-down flow regime is

\textsuperscript{5} Mark of Schlumberger
enforced at a rate of 1.5 bbls/day, and continues until a pseudo-steady state pressure distribution is reached in approximately 1.7 days. A build up test begins immediately and continues until the pressure reaches a steady-state value (close to formation pressure).

**Figure 7.1** shows transient pressure measurements simulated with a progressive increase in the relative proportion of Sand No. 2 (permeability = 70 md) in the Sand No. 1 – Sand No. 2 rock formations. Only the pressure variation with time measured by the mid-packer sensor is plotted to assess the effect of lithology on the pressure transient data. An increase in the proportion of ‘low permeability’ Sand No. 2 in the composite rock formation causes an increase in the differential pressure ($\Delta P$). Pressure transient analysis for the Sand No. 1 – shale rock formations is performed assuming a very low value of shale permeability (0.001 md) in order to secure vertical fluid flow across the sand components of the laminated sequences. Six values of relative proportion of shale are considered in this study, from 10% to 66%. **Figure 7.2** shows that the simulated differential pressure ($\Delta P$) increases with an increase in the relative proportion of shale.
Figure 7.1: Simulated dual-packer pressure transients in Sand No.1 - Sand No. 2 rock formations. The differential pressure ($\Delta P$) increases with an increase in the relative proportion of Sand No. 2.
Figure 7.2: Simulated dual-packer pressure transients in Sand No.1 - shale rock formations. The differential pressure ($\Delta P$) increases with an increase in the relative proportion of shale in the formation. Six values of shale fractions are considered in the simulations.

7.2 Effective Permeability

Values of effective permeability of laminated composite rock formations are calculated using an inversion algorithm. Transient pressure data shown in
Figures 7.1 and 7.2 are entered into the inversion algorithm to estimate a value of absolute permeability assuming that the entire laminated sequence is a homogeneous and isotropic rock formation. The inversion algorithm provides the effective value of absolute permeability by minimizing the least-squares differences between the measured and simulated pressure transient data. Figure 7.3 shows the comparison between the estimated and the volume-average permeability of the composite rock formations. The volume-average permeability of a composite rock formation is calculated from the formula

$$\overline{k} = \beta \cdot k_1 + (1 - \beta) \cdot k_2,$$  \hspace{1cm} (7.1)

where $\beta$ is the relative proportion of Sand No. 1, $k_1$ is permeability of Sand No. 1, and $k_2$ is either the permeability of Sand No. 2 in the Sand No. 1 – Sand No. 2 composite rock formation, or the permeability of shale in the Sand No. 1 – shale composite rock formation.

Two types of curves are shown in Figure 7.3. The black bold line describes the volume-average permeability for the Sand No. 1 – Sand No. 2 composite rock formations. The grey bold line describes the effective permeability yielded by the inversion algorithm. For the Sand No. 1 – Shale rock formations, similar curves are shown with black and grey dashed lines, respectively. Effective permeability values yielded by the inversion algorithm for
the Sand No. 1 – Shale composite rock formations are shown only up to a value of 50% shale proportion. The inversion algorithm was unable to secure a good match between input and simulated pressure transient data for values of shale proportion less than 50% and hence the corresponding results are not reported in this thesis.

Results indicate that, for Sand No. 1 – Sand No. 2 rock formations, the separation between estimated and average permeability increases with an increase in the relative proportion of Sand No 1. In the Sand No.1 – shale rock formations. The difference between estimated and average permeability increases with an increase in the relative proportion of shale. For a 50:50 laminated sandy formation, a deviation of 50 md is observed in the estimated permeability from the average permeability (400 md). On the other hand, for a 50:50 shaly sand the difference increases to 100 md.
**Figure 7.3:** Effective permeability of Sand No. 1 – Sand No. 2 and Sand No. 1– shale rock formations. The dark and grey dash lines identify the effective permeability obtained from the inversion of dual-packer pressure transient data. The dark and grey bold lines identify volume-averaged permeability values for sandy and shaly sand rock formations, respectively.
CHAPTER 8: DISCUSSION OF RESULTS

For laminated sandy rock formations, HDIL resistivity readings are significantly more affected by the salt concentration of mud-filtrate than by a change in the relative proportion of lithology. A decrease in the salt concentration of mud-filtrate from 30,000 ppm to 6,000 ppm causes an increase of 140 ohm-m in the deepest HDIL resistivity reading. Invasion of mud-filtrate causes an average increase of 5% in the simulated bulk density values for the Sand No. 1 – Sand No. 2 rock formations. Therefore, the largest error in the estimation of water saturation occurs when the calculation is performed (a) in the presence of a large contrast in salt concentration between mud filtrate and connate water, and (b) with resistivity readings acquired with the HDIL instrument and with a porosity estimated from measurements of bulk density. Provided that the relative proportions of lithology, porosity, connate water salinity, and the length of mud-filtrate invasion are constant, a 10% decrease in the calculation of water saturation ensues when the contrast between the salinity of mud-filtrate and connate water is low.

When the calculation of water saturation is performed with values of vertical resistivity acquired with the 3DEX instrument, different results will be obtained depending on the orientation of the bedding plane. This is because the porosity and saturation exponents, used in the calculation of water saturation via
Archie’s equations, depend on the orientation of the bedding plane. One of the limitations of the present study is the inability of the eXpress software to incorporate the entire radial profile of the electrical resistivity anisotropy for the simulation of vertical resistivity readings. Another limitation of the analysis described in this thesis is that the effects of the salinity of mud-filtrate on neutron measurements have not been considered for the estimation of porosity.

Our study indicates that, for shaly sand laminated rock formations, errors in the calculation of water saturation are high (10-40%) if the HDIL resistivity readings are used in the various standard interpretation models for resistivity and water saturation. HDIL readings cannot distinguish between shallow and deep resistivity if the relative proportion of shale is above 50%. An error in the calculation of porosity occurs because of unaccounted presence of shale on the neutron responses. The error in the estimation of water saturation is low (5-10%) if the calculation is performed with the vertical resistivity derived from the 3DEX instrument. It is impossible to assess true water saturation in a laminated composite shaly sand rock formation. An increase in the relative proportion of shale causes an increase in the error of the estimated water saturation.

Permeability values yielded by the inversion of transient pressure data give rise to an error because of the assumption of a homogeneous and isotropic rock formation. The inverted value of permeability is consistently smaller than the
corresponding value of permeability calculated with a mixing law that takes into account the specific volumetric proportions of sand and shale constituents. This error could be decreased if the inversion were performed with an additional degree of freedom for vertical and horizontal permeability in the assumed rock formation model. However, it remains to be examined whether the pressure transient measurements embody equal sensitivity to both vertical and horizontal permeability to warrant a stable and accurate estimation. Additional work is needed to properly understand the effect of various levels of noise on the inverted permeability values of composite rock formations.
CHAPTER 8:

CONCLUSIONS

The following is a summary of the most important conclusions stemming from the study described in this thesis:

1. Proportion of lithology and invasion of mud-filtrate are the leading factors that control measurements of electrical resistivity, nuclear, and formation tester instruments acquired in thinly laminated, clastic formations.

2. For a constant composite lithology and length of mud-filtrate invasion, the HDIL instrument provides abnormally high resistivity readings for low values of salt concentration of mud-filtrate than for high values of salt concentration of mud-filtrate.

3. In laminated, shaly sand rock formations, HDIL measurements cannot distinguish between the shallow and deep resistivity of invaded formations if the relative proportion of shale is higher than 50%. This conclusion indicates that there is no measurable effect of mud-filtrate invasion on the HDIL readings.

4. The 3DEX instrument measures the vertical and horizontal resistivity of anisotropic rock formations. Even though the measurement is restricted to the non-invaded zones, it does remain affected by the presence of mud-filtrate invasion.
5. Unaccounted mud filtrate invasion causes an error in the estimation of porosity in laminated composite rock formations.

6. For the case of a homogeneous but invaded rock formation, conventional induction logging instruments do not provide a reliable reading of true formation resistivity if the salinity contrast between mud filtrate and connate water is as large as 70,000 ppm. On the other hand, if the formation is sand-sand or sand-shale laminated, the deep resistivity reading will be lower than the true formation resistivity. In turn, this will translate to low estimates of water saturation. A 35% increase in the salinity contrast causes a 10% decrease in water saturation.

7. Water saturation calculated with the vertical resistivity measured with the 3DEX instrument is lower than the value of water saturation calculated with deep HDIL resistivity readings. Assuming that both relative proportion of lithology and depth of invasion are constant, water saturation calculated with the vertical resistivity measured by the 3DEX instrument will be the closest to the actual water saturation.

8. In laminated rock formations, conventional interpretation methods that assume constant porosity and saturation exponents give erroneous values of water saturation. It is necessary that the two exponents be modified to account for the direction of the resistivity measurement relative to the orientation of the bedding plane. For electrically anisotropic rock formations, water saturation
calculated in the direction perpendicular to the bedding plane is higher than that the one calculated in the direction parallel to the bedding plane.

9. Errors in the estimation of water saturation are always higher for laminated, shaly sand rock formations than for laminated, sandy rock formations.

10. The error in the estimation of permeability from formation tester measurements is higher for laminated, shaly sand rock formations than for laminated, sandy rock formations.
NOMENCLATURE

\( A \) : Cross-sectional area \([\text{ft}^2]\)

\( a \) : Archie’s tortuosity factor \([\text{dimensionless}]\)

\( B_d \) : Borehole diameter \([\text{inch}]\)

\( C_w \) : Salinity of mud-filtrate \([\text{ppm}]\)

\( D_{kl} \) : Molecular diffusion coefficient \([\text{ft}^2/\text{sec}]\)

\( h_{mc} \) : Thickness of mudcake \([\text{inch}]\)

\( K \) : Dispersion (tensor) \([\text{ft}^2/\text{sec}]\)

\( \bar{\kappa} \) : Volume-average permeability

\( k \) : Permeability

\( k_{r_{nw}} \) : Non-wetting-phase relative permeability \([\text{dimensionless}]\)

\( k_{r_{nw}}^0 \) : Non-wetting phase end-point relative permeability \([\text{dimensionless}]\)

\( k_{r_w} \) : Wetting-phase relative permeability \([\text{dimensionless}]\)

\( k_{r_w}^0 \) : Wetting phase end-point relative permeability \([\text{dimensionless}]\)

\( L_{xo} \) : Length of invasion \([\text{inch}]\)

\( m \) : Archie’s porosity exponent \([\text{dimensionless}]\)

\( n \) : Archie’s saturation exponent \([\text{dimensionless}]\)

\( P_c \) : Capillary pressure \([\text{psia}]\)

\( P_e \) : Entry capillary pressure \([\text{psia}]\)

\( \Delta P \) : Pressure drop \([\text{psia}]\)

\( Q_f \) : Flow rate of mud-filtrate \([\text{ft}^3/\text{day}/\text{ft}]\)

\( R_t \) : True formation resistivity \([\text{ohm-m}]\)

\( R_w \) : Connate water resistivity \([\text{ohm-m}]\)

\( R_{xo} \) : Measured shallow resistivity \([\text{ohm-m}]\)

\( S_{\text{wt}}^* \) : Effective wetting-phase saturation \([\text{fraction}]\)

\( S_w \) : Water saturation \([\text{fraction}]\)

\( S_{wir} \) : Irreducible water saturation \([\text{fraction}]\)

\( S_{wnr} \) : Irreducible non-wetting phase saturation \([\text{fraction}]\)

\( S_w \) : Volumetric average of water saturation \([\text{fraction}]\)

\( u \) : Vector component of Darcy velocity \([\text{ft}/\text{sec}]\)

\( V_{\text{sand}} \) : Volume of sand \([\text{fraction}]\)

\( V_{sh} \) : Volume of shale \([\text{fraction}]\)

\( \alpha_L \) : Longitudinal dispersivity \([\text{ft}]\)

\( \alpha_T \) : Transverse dispersivity \([\text{ft}]\)

\( \delta \) : Kronecker delta function \([\text{dimensionless}]\)

\( \lambda \) : Pore shape factor \([\text{dimensionless}]\)

\( \bar{\sigma} \) : Volume-average porosity \([\text{fraction}]\)

\( \phi \) : Porosity \([\text{fraction}]\)
\( \mu \) : Viscosity [cp]
\( \rho_f \) : Fluid density \([\text{g/cm}^3]\)
\( \rho_b \) : Bulk density \([\text{g/cm}^3]\)
\( \rho_{ma} \) : Matrix density \([\text{g/cm}^3]\)
\( \rho_{sh} \) : Shale density \([\text{g/cm}^3]\)
\( \tau \) : Tortuosity in the dispersion equation \([\text{dyne/cm}^2]\)

**Subscripts and super scripts**

- \( h \) : Horizontal
- \( i \) : i-th Cartesian direction
- \( j \) : j-th Cartesian direction
- \( k \) : Component
- \( l \) : Phase
- \( L \) : Longitudinal
- \( rw \) : Related to wetting phase
- \( rnw \) : Related to non-wetting phase
- \( v \) : Vertical
- \( w \) : Water
- \( wir \) : Irreducible wetting phase
- \( wnr \) : Irreducible non-wetting phase
- \( sh \) : Shale
- \( T \) : Transverse
- \( xo \) : Related to shallow resistivity
- \( \parallel \) : Parallel to the bedding plane
- \( \perp \) : Perpendicular to the bedding plane
- \( 1 \) : Related to Sand No. 1
- \( 2 \) : Related to Sand No. 2
REFERENCES


VITA

Subhadeep Chowdhury was born in Bankura, West Bengal, India, on January 05, 1974. He received a Bachelor and Masters of Science degree in Geology, with honors, from Jadavpur University, Kolkata in 1997. He joined Enron Oil and Gas India Limited (later became BG India Limited) in 1998 as Geologist where he worked until 2002. He began graduate studies at the University of Texas at Austin in August 2002. Since spring 2003, Mr. Chowdhury has been a research assistant with the Department of Petroleum and Geosystems Engineering.

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