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Direct Spatiotemporal Interpolation of Reservoir Flow Responses

by

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Dedication

To Dr. Sanjay Srinivasan
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Abstract

Direct Spatiotemporal Interpolation of Reservoir Flow Responses

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Abstract: The traditional reservoir modeling workflow consists of first developing a reservoir model, performing flow simulation on that model, verifying the model by performing history matching and finally using the history matched model to make predictions of future performance. In contrast, this research focused on two approaches to directly analyze the spatio-temporal variations of dynamic responses such as pressure and well flowrates and perform interpolation. Both these techniques are anchored to the data at the wells. Therefore, the resultant spatio-temporal predictions of dynamic response are history matched by construction. Interpolation or extrapolation of dynamic response to locations away from wells is possible using both the approaches. Therefore, the proposed approaches can be used to quickly determine optimal location to drill additional wells and to gauge the influence of reservoir management decisions.

In the first approach, dynamic responses such as pressure transients are treated as time series data. They are analyzed using wavelets that facilitate multiscale decomposition of
pressure signals. Using a wavelet-lifting scheme, the transient signal is decomposed into averages and residuals. The corresponding filter coefficients defining the wavelets are treated as spatial random variables and estimated using geostatistics at locations away from wells. A pressure response is reconstructed at unsampled locations by employing the inverse wavelet transform.

In an alternate approach, direct spatiotemporal extrapolation of pressure is performed. The transient pressure data at the wells are first analyzed using correlation measures such as semivariograms. For simplicity, time is taken as another spatial dimension and semivariogram values corresponding to the resultant lag-vectors are inferred and subsequently modeled. Spatiotemporal extrapolation is then performed to obtain the response at any location in space and at any instant in time. The robustness of both these approaches is verified on several case examples.
# Table of Contents

List of Tables .................................................................................................................... xi

List of Figures ................................................................................................................... xii

Chapter 1: Introduction .................................................................................................... 1

Chapter 2: Literature Review .......................................................................................... 5
  2.1 Regression Methods for Analysis of Pressure data ........................................... 5
  2.2 History Matching ................................................................................................. 6
  2.3 Geostatistical Space-time Models ..................................................................... 11
  2.4 Spatiotemporal Covariance Functions ............................................................... 12
  2.5 Application of Wavelets to Pressure Transient Data ..................................... 13

Chapter 3: Time Series Analysis ..................................................................................... 16
  3.1 Fourier transforms ............................................................................................... 16
  3.2 Wavelet theory .................................................................................................... 18
    3.2.1 Basic Concept .............................................................................................. 18
    3.2.2 Multiresolution Analysis ............................................................................ 20
    3.2.3 Construction of Scaling and Wavelet Functions ....................................... 20
    3.2.4 Dilation Equations ..................................................................................... 23
    3.2.5 Discrete Wavelet Transform ..................................................................... 23
    3.2.6 Implementation of Wavelet Analysis Schemes ........................................... 25

Chapter 4: Analysis of Wavelet Coefficients ................................................................ 27
  4.1 1 x 3 Synfield ...................................................................................................... 27
    4.1.1 Case 1 ........................................................................................................... 28
    4.1.2 Case 2 .......................................................................................................... 30
    4.1.3 Observations ............................................................................................... 31
  4.2 1 x 1 Synfield ...................................................................................................... 31
    4.2.1 Case Description ......................................................................................... 31
    4.2.2 Inference ...................................................................................................... 32
  4.3 10 x 15 Synfield .................................................................................................. 33
    4.3.1 Case Description ......................................................................................... 33
Chapter 5: Lifting Scheme and Application to Synthetic fields

5.1 Wavelet Implementation with a Lifting Scheme

5.2 Synthetic Case 1

5.2.1 Lifting Scheme Implementation Approach

5.2.2 Observations

5.2.3 Inference

5.3 Modeling Spatial Variability of Coefficients

5.4 Sequential Gaussian Simulation

5.5 Reconstruction of Pressure Signal at Unsampled Location

5.6 Synthetic Case 2

5.6.1 Wavelet Decomposition

5.6.2 Kriging

5.6.3 Reconstruction Using Inverse Lifting Scheme

5.6.4 Inference

Chapter 6: Joint Space-time Analysis Using Spatiotemporal Covariances

6.1 Spatiotemporal Covariances

6.2 Spatiotemporal Covariance models

6.3 Synthetic Case 1

6.3.1 Inferring Spatiotemporal Semivariogram

6.3.2 Sequential Gaussian Simulation

6.4 Synthetic Case 2

6.4.1 Spatiotemporal Semivariogram

6.4.2 Spatiotemporal Kriging

6.4.3 Inference

6.5 Linear Model of Coregionalization

6.5.1 Product-sum Models

6.5.2 Implementation Approach

6.6 Synthetic One Dimensional Case

6.6.1 Observations
6.6.2 Modeling Spatiotemporal Semivariogram Surface.....................75
6.6.3 Space-time Kriging .....................................................................78
6.7 Discussion...............................................................................................79

Chapter 7: Conclusions and Future Course of Work...............................80

References.............................................................................................................84

Vita .......................................................................................................................89
List of Tables

Table 4.1.1: Wavelet coefficients for case 1 .........................................................29
Table 4.1.2: Correlation coefficients for case 1 ....................................................29
Table 4.1.3: Wavelet coefficients for case 2 .........................................................30
Table 4.1.4: Correlation coefficients for case 2 ....................................................30
Table 5.1: Parameters of modeled semivariograms ...........................................47
Table 6.1: Semivariogram model parameters ......................................................61
Table 6.2: Parameters of spatiotemporal semivariogram ....................................64
Table 6.3: Parameters of spatiotemporal semivariogram ....................................75
Table 6.4: Variance of spatiotemporal semivariogram surface .........................78
List of Figures

Figure
3.1: Multiresolution analysis of wavelets ................................................22
3.2: Wavelet decomposition of a typical pressure transient signal.......25
4.1.1: Well locations for case 1..............................................................29
4.1.2: Well locations for case 2..............................................................30
4.2: Maps of wavelet coefficients ..........................................................33
4.3: Well model....................................................................................34
4.3: Semivariogram of underlying geology .............................................34
4.5: Coefficient maps .........................................................................35
4.6: Comparison between geostatistical and actual profiles .................36
5.1: The lifting scheme ........................................................................37
5.2: Inverse transform of the lifting scheme .........................................38
5.3: Well model with underlying permeability field............................39
5.4: Pressure transients at three locations in a reservoir .................41
5.5: Maps of averages and details at different scales.........................42
5.6: Integer translates at resolution '3'..................................................44
5.7: Experimental semivariograms in azimuth 0 and 90 directions ......45
5.8: Semivariogram models for coefficients ........................................46
5.9: Scheme illustrating splitting and reconstruction of pressure signal..48
5.10: Interpolated maps of coefficients at different scales.................49
5.11: Reconstructed pressure profiles at indicated locations .............50
5.12: Well model with underlying permeability field........................51
5.13: Coefficients of maps at different scales 'm' ............................53
5.14: Experimental and modeled semivariograms at one scale ..........54
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.15</td>
<td>Kriged maps of averages and details at different scales ..........55</td>
</tr>
<tr>
<td>5.16</td>
<td>Reconstructed pressure profiles at (990 ft, 990 ft)................56</td>
</tr>
<tr>
<td>6.1</td>
<td>Experimental semivariograms and corresponding fitted models.....61</td>
</tr>
<tr>
<td>6.2</td>
<td>Anisotropy of spatiotemporal semivariogram .......................62</td>
</tr>
<tr>
<td>6.3</td>
<td>Simulated vs predicted pressure profile at fourth grid block........63</td>
</tr>
<tr>
<td>6.4</td>
<td>Three dimensional Spatiotemporal semivariogram for two dimensional reservoir .........................................................64</td>
</tr>
<tr>
<td>6.5</td>
<td>Pressure response generated at different grid blocks ..............67</td>
</tr>
<tr>
<td>6.6</td>
<td>Pressure maps at various time instants................................68</td>
</tr>
<tr>
<td>6.7</td>
<td>Spatiotemporal semivariogram surface for one dimensional case....73</td>
</tr>
<tr>
<td>6.8</td>
<td>Truncated semivariogram surface.........................................74</td>
</tr>
<tr>
<td>6.9</td>
<td>Sample semivariograms and fitted models .................................76</td>
</tr>
<tr>
<td>6.10</td>
<td>Experimental and product sum models....................................77</td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

The objective in a number of earth science applications is to predict the dynamic response of a system to external impulses. This prediction could be either at locations different from monitoring stations, where data is available, or at temporal instants different from those at which historic data has been sampled. The traditional approach has been to attribute the spatial and temporal variations in the response variable to a set of physical variables. For example, terrain topology and conductivity of channels, in the case of hydrologic network problems and petrophysical variables, such as permeability and porosity, in the case of hydrocarbon reservoirs.

The spatial distribution of attributes is modeled as a random function and the multivariate distribution characterizing the random function (RF) is synthesized using the available measurements. Outcomes of the RF (Chiles and Delfiner, 1999) are sampled from the multivariate distributions and these realizations depict the pattern of spatial variability of the attribute of interest. The spatially varying attributes and the dynamic response variable are, in most cases, related to each other through a complex, non-linear transfer function. Numerical flow simulators incorporate this transfer function as illustrated by the case of hydrocarbon reservoirs (Aziz and Settari, 1975). The stochastically generated random fields are processed through the transfer function model to yield the dynamic response at all locations of the spatial domain and at time instants both past and future.

The traditional method suffers from at least two major drawbacks:
Frequently, calibration of the spatial model is performed using dynamic data recorded at select monitoring stations. In most modeling scenarios, the task of anchoring spatial random fields to dynamic information is severely ill posed. Adjusting the spatial model to reflect the historic dynamic data without really understanding the link between patterns of variability exhibited by the spatial model. The observed dynamic response also results in a suite of spatial models that erroneously predict the dynamic behavior of the reservoir.

The process of adjusting the RF model to reproduce historic data is typically posed as an inverse problem requiring repeated solutions of a complex transfer function model. These repeated solutions add significantly to computational costs. This thwarts attempts to quantify the uncertainty in reservoir performance.

An alternate stochastic, spatiotemporal approach to understand and analyze the dynamic characteristics of petroleum reservoirs is herein proposed. In this approach, the dynamic response of interest (e.g. the fluid pressure at any location and time) is treated as a RF $Z(u,t)$ in the $(n + 1)$-dimensional space $\mathbb{R}^n \times T$ where $n$ is the spatial dimensionality of $u$ (typically 3) and $T$ is the additional temporal dimension. If the link between the spatiotemporal statistics of the RF $Z(u,t)$ and the physics of fluid flow in reservoirs is established then the procedure will yield a method to infer spatiotemporal statistics such as the semivariogram $\gamma(u - u', t - t')$. The manner in which the pattern of geological heterogeneity affects the observed dynamic responses $Z(u,t)$ and $Z(u',t')$ at pairs of locations $u$ and $u'$ and times $t$ and $t'$ can thus be represented through the semivariogram
function. Dynamic response at unsampled locations or instants can then be obtained using interpolation procedures, such as kriging (Deutsch and Journel, 1998), that use the spatiotemporal semivariogram model together with the data recorded at monitoring stations.

The above spatiotemporal modeling approach, though consistent with traditional geostatistics, requires sufficient data to be available to infer the requisite spatiotemporal statistics. Typically, reservoir production data are densely sampled in time but are available at only a few sparse well locations. To exploit the dense temporal sampling, an alternate proposal is to fit a time series to the transient pressure profile at different well locations. The parameters of the time series can then be extrapolated to locations away from the conditioning well.

Spatiotemporal variations in reservoir response can thus be generated using the proposed approach. The function used for interpolating the parameters of the time series can be modeled to mimic the relationship between fluid flow and the underlying pattern of reservoir heterogeneity. Prediction of future reservoir performance will use such a function and will also be constrained to the historic information available at monitoring locations. There are, thus, no trial-and-error adjustments to be made to the spatial models to replicate the observed response. In addition, by treating the dynamic response at all locations within the reservoir as manifestations of a spatiotemporal random function, the joint spatiotemporal uncertainty in dynamic response is explicitly modeled. Hence, there is no need to generate multiple realizations of the underlying spatial reservoir/aquifer
attributes and subsequently simulate flow using computationally expensive flow simulators.
Chapter 2: Literature Review

2.1 Regression Methods for Analysis of Pressure data

Regression is a mathematical technique used to infer model parameters by minimizing the difference between the measured data and a proposed model response. The most popular approach is to use the least squares method in which an objective function to be minimized is the sum of the squares of the differences between the measurements and the model response. Athichanagorn and Horne (1999) implemented the non-linear regression technique to analyze and predict pressure transient data. They primarily addressed the issue of incomplete flow history. Because of dynamic conditions exhibited by the reservoir, there could be several changes induced in the measurements of well response and many times the measured flow rate may not indicate the true flow rate.

The linear relationship between flow rate and pressure was exploited and the ratio of pressure difference to flow rate difference was used to identify inconsistencies between measurement and true response. In other words, unknown flow rates (incomplete flow history) were used as the nonlinear regression model parameters to be estimated to match the pressure response.

An objective function was defined in terms of the dimensionless pressure drop during a multiple rate production test. This objective function (pressure equation in terms of the flow rates) was differentiated with respect to each of the unknown flow rates to characterize a Hessian matrix, which is a matrix of second-order partial derivatives of the objective function. The model parameters were obtained by inverting this Hessian matrix and multiplying it with the gradient matrix, which is the matrix of first-order partial
derivatives of the objective function with respect to model parameters. This technique is termed the Gauss-Marquardt method. Chang and Ershagi (1986) used this method in tandem with a direct search algorithm, which locates the neighborhood of a point where the objective function is minimum among points that are uniformly distributed in space.

The Gauss-Marquardt method was proposed in an attempt to solve the problem of ill-conditioning of the Hessian matrix, which is observed in the Gauss-Newton method. Marquardt (1963) rectified this ill-conditioning by adding a small constant to the diagonals of the matrix. Greenstadt (1967) further improved the conditioning by modifying the smaller eigen values of the Hessian. Barua et al. (1988) summarized a comparison of all these regression techniques and concluded that Gauss-Marquardt method was the most reliable of all but Greenstadt’s eigen value modification works better when more than one parameter is ill-defined.

2.2 History Matching

History matching is the exercise of developing a robust reservoir model by repetitive perturbations of a prior geologic reservoir model until consistency with measured dynamic production data at wells is realized.

(i) Geostatistical methods

Caers (2002) presented applications of two geostatistical methods using sequential simulation for history matching. The first method involves perturbation of the conditional probabilities and the second involves perturbation of random numbers while maintaining a fixed random path for searching nodes. The first method uses the dual covariance (data-
data and data to unknown) reproduction characteristic of sequential simulation to perturb a prior training image and integrate soft data such as seismic data into this prior model using cokriging or block kriging to develop an update.

Realizing the incompatibility of time variant production data directly into reservoir models, this paper presents a novel approach to turn production data into a spatial constraint and use sequential simulation to integrate the constraint into a realization that is consistent with the geology depicted by the prior training image.

The second approach generates a vector of random numbers from a uniform distribution by Gaussian transforming a random vector $Y(r_D)$, which is a linear combination of random vectors $Y_1 \cos(r_D)$ and $Y_2 \sin(r_D)$, and perturbs this prior random number vector repetitively until a history match is obtained. The random path selected for sequential simulation is fixed during the entire procedure as opposed to the first approach where this path is variable. The paper claims that the second approach is slower and less efficient in terms of retrieval of full space of realizations than the first.

(ii) Assisted History Matching

The idea of assisted history matching originally developed by Schiozer et al. (1997) uses parallel computing and a traditional reservoir flow simulator. Individual objective functions are defined for each parameter associated with the reservoir such as water rate, oil rate or pressure. These objective functions are generally expressed as the sum of squares deviations of the predictions made using the modified model from the measured history data. Considering the fact that several associated reservoir attributes affect the
well and reservoir performance, a global objective function is defined as a weighted linear combination of subsidiary objective functions. Each attribute has associated with it a modifier and spans an axis in n-dimensional space, ‘n’ representing the number of attributes.

The assisted history matching algorithm performs a series of alternating linear and exploratory searches along each identified direction. An exploratory search helps vary a parameter and identify the direction of approach to the minima based on values of objective functions at neighboring nodes of the n-dimensional grid. A linear search is then performed along this identified direction to obtain a minimum for the selected parameter. This minimum is now used as a point for a new exploratory search and the whole exercise is repeated until a global minimum is attained.

An attempt to match history globally can often distort the local minima, which calls for frequent calibration of parameters in local regions. This is one of the most cumbersome and time consuming exercises. The idea of streamline simulation in tandem with assisted history matching ensures consistency of the global optimum solution with the local optimum. Streamline simulation is used to identify regions of influence between producer-injector pairs. The streamlines are subsequently updated by incorporating measured field pressure data. By identifying these local regions as a function of flow data, parameters could be varied locally to comply with the global objective function.
(iii) Automatic History Matching with Variable Metric Methods

Assisted history matching uses a univariate method (linear search) for approaching the minimum function. Contrary to this approach several first derivative approaches such as steepest descent and conjugate gradient methods utilizing optimal control theory have been proposed in the literature. (Chen, 1974). As these methods suffer from some drawbacks like slow progress in the vicinity of minimum, variable metric methods (Yang et al., 1988) were proposed. Variable metric methods allow for inclusion of parameter-inequality constraints which offer them superiority over conventional steepest descent and conjugate gradient methods. By performing constrained minimization, desirable convergence properties in the selected search directions are preserved.

(iv) History Matching by Spline Approximation and Regularization

One of the common problems in history matching is the ill-posed nature of the inverse problem. The inverse problem can be understood as estimating reservoir parameters given a prior reservoir model and the observed production response. This problem is termed well posed if for every production response data set, there exists a unique solution of reservoir parameters and stable if small perturbations in production response imply small changes in reservoir parameters. Otherwise, it is considered ill-posed. The ill-posed nature of the problem stems from the large dimensionality of the history matching problem, in other words, estimation of several parameters simultaneously. In many history matching exercises, the spatially varying parameters render the parabolic governing equations of a reservoir to be unstable and as a consequence, the inverse
problem is ill-posed. Thus, Tai-Yong et al. (1986) introduced the regularization approach, where an augmented objective function comprising of the traditional least squares and a stabilizing function is minimized. The stabilizing function is a proposed smooth bicubic spline function whose coefficients are to be estimated. The desired reservoir properties can be expressed as a complex linear combination of these weighting coefficients of the bicubic spline. Spline representation imparts a degree of smoothness to the parameter distribution that could eliminate some of the ill-conditioning inherent in the finite-difference simulation. The grid on which the reservoir model is based may differ from the grid defining the bicubic spline function. The history matching exercise now boils down to estimating the coefficients of the proposed bicubic spline subject to available production data. This requires the estimation of partial derivatives of the objective function w.r.t the coefficients which constitutes the most time consuming part of the exercise. The paper claims that an optimal control formulation as in conjugate gradient methods can enable direct estimation of these partial derivatives and expedite the process of obtaining a history match.

The approaches to history matching for developing complete reservoir models are time-consuming and demand frequent manual adjustments to simulator input parameters depending upon difference between simulator predictions and observed field production and pressure data. To obtain a quicker technique for reservoir model validation, few authors developed the idea of geostatistical interpolation and basing their future judgements on geostatistical space-time models. However, there is a tradeoff between time and the completeness of the model. Geostatistical space-time models are not complete.
They only reconstruct the dynamics of reservoirs at unsampled locations thus enabling decision making about drilling future well locations.

2.3 Geostatistical Space-time Models

The variation of pressure as a function of time jointly at multiple locations within the reservoir is an example of space-time data. Analysis of data considering either spatial or temporal lags is common in traditional geostatistics. Joint analysis of space and time data is challenging and requires definition of spatiotemporal lags in order to explore the patterns of variability. A surrogate for such complex spatiotemporal modeling (Dimitrakopoulos and Luo, 1994) schemes is using smooth interpolation of the maps of an attribute over specific time instants or by spatially correlating the parameters of time series at various locations over a spatial domain. Kyriakidis (1999) provides an exhaustive review of geostatistical space-time models stating each of their contributions and listing their limitations. He established a link between two approaches; one involving a single spatiotemporal random function and another involving estimation of vectors of space random functions or that of time series. The paper discusses the decomposition of random variables into trends and residuals and modeling each component separately using either stochastic or deterministic techniques.

Higdon (1998) coined a novel approach by employing process convolutions to build space-time models that are flexible enough to accommodate vast amounts of data. The primary advantage offered over other conventional approaches is to allow for non-standard features such as non-stationarity, edge effects, dimension reduction, non-Gaussian fields and alternative space-time models. A simple discrete process is defined...
over space and time and subsequently smoothened by utilizing kernels, which are mathematical functions based on the convolution theorem. The paper also talks about incorporating fine scale information into the prior multiresolution model as the measured data are capable of providing information only up to a limited resolution. It proposes to delve deeper into the determination of temporal dependence at different spatial resolutions. For example coarser resolutions depict a distinct temporal dependence while finer resolutions are temporally independent.

Bogaert (1996) investigated the performance and limitations of the decomposition of space and time correlation functions based on space-time separability hypothesis and traditional geostatistical theory in the space-time domain which uses a coregionalization of spatially correlated time series. They concluded that the use of space-time semivariogram has the advantage to allow prediction at any space or time as opposed to the Linear Model of Coregionalization which is defined for fixed time or space increments.

2.4 Spatiotemporal Covariance Functions

Dynamic reservoir attributes such as pressure and flow rates may be regarded as realizations of space-time random fields. Spatiotemporal geostatistics; an extension to traditional geostatistics, provides a number of approaches to develop space-time correlation structures, one of which is an obvious extension of existing spatial techniques into temporal domain. As straightforward as this extension appears, several issues of concern have to be addressed before its successful implementation. De Cesare et al. (2001) gave a brief account of few approaches that have been used in the area of
spatiotemporal geostatistics. Bogaert (1996) concluded that the use of time as another spatial coordinate relies on geometric or zonal anisotropy modeling. This method of covariance modeling can lead to an invalid kriging system for a few data configurations. (Rouhani and Myers, 1990). Cressie and Huang (1999) introduced a special class of integral non separable spatiotemporal covariance models which were subsequently generalized by De Iaco et al (2001) to a more general class of product-sum models, which depend only on one parameter to be estimated from data. Admissible values of this parameter are discussed and thereby the necessary condition for positive definiteness is developed. A brief account of space-time kriging, the next step after semivariogram modeling is provided in the paper by Rouhani and Hall (1989).

The main objectives of the time series analyses presented in this report are to provide a compact description of the spatiotemporal data estimate the parameters of the spatiotemporal model, check / validate the model and decipher the physics of the underlying process generating the data. Time series data may contain trends, abrupt changes, random errors and outliers that may represent complex influence of geology and flow on pressure response and may not be ignored.

2.5 Application of Wavelets to Pressure Transient Data

Wavelets (Debnath, 2002) are categorized as a class of functions that are constructed from dilations and translations of a single function called the “mother wavelet”. They are identified with two parameters viz. a scaling parameter, which measures the degree of compression and a translation parameter, which determines the time location of the wavelet. Wavelets capture essential trends, discontinuities and noise in a signal with
minimal number of coefficients. The scaling parameter can zoom into local regions and capture almost all frequency components, while the time localization window is handled by the translation parameter. Dijkerman and Mazumdar (1994) employed wavelets to develop a multiresolution representation of stochastic processes in signal processing. They primarily analyzed correlation structure between discrete wavelet coefficients resulting from the representations of the process signals. This paper is the primary motivation behind application of wavelets for multiresolution decomposition of well bottomhole pressure signals thereby analyzing the decay in correlation between wells along the spatial domain and across multiple scales.

Measured pressure data are inherently multiscale owing to diverse contributions resulting from events occurring at different well locations over a spatial reservoir domain with different localization in time and frequency. The pressure at any location within a reservoir away from a data location is uncertain owing to the uncertainty in the underlying geology. The transient pressure variations in the reservoir may therefore be treated as random variables. In this research, the transient pressure variations at the drilled well locations are expressed in the form of a time series using wavelets (Akbaryan and Bishnoi, 2000). The wavelet coefficients are regionalized (Oehlert, 1993) in space over the entire two dimensional reservoir domain. Conventional geostatistical interpolation techniques such as stochastic simulation; help establish the wavelet coefficient set at a location away from the existing wells. Bakshi (1999) has shown how to decorrelate stochastic from deterministic components by exploiting the filtering nature
of wavelets. In his paper he has emphasized on a bunch of filtering methods as applied to multiscale data analysis.

One of the primary problems in geostatistics is to establish a semivariogram representative of the spatial distribution of the underlying heterogeneity. In this case, the semivariogram of wavelet coefficients is required for the spatial interpolation. It has been verified by rigorous mathematical analysis that the pressure response is directly correlated to the underlying permeability heterogeneity (Yortsos, 2000) and hence the semivariogram used for coefficient estimation is that of the underlying geology. Once the wavelet parameters have been interpolated / extrapolated to locations away from the wells, the pressure response at those locations is reconstructed using the coefficients and the chosen basis functions.
Chapter 3: Time Series Analysis

3.1 Fourier Transforms

Any transient signal can be decomposed into long range low frequency (mean) trends and short range high frequency vibrations about those mean trends. The short range high frequency vibrations are discernible only in the frequency domain. The task of modeling the temporal variations of the signal in the frequency domain is accomplished by a Fourier transform. The Fourier transform of a function $f(x)$ defined on an interval $(-l, l)$ is given by:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{in\pi t}{l}\right)$$

Where the Fourier coefficients are:

$$c_n = \frac{1}{2l} \int_{-l}^{l} f(t) \exp\left(-\frac{in\pi t}{l}\right) dt$$

The above transformation is true for continuous periodic signals with period $2l$. However, in all practical applications the signals are discrete in nature with measurements available only at specific instants in time. In such a case discrete Fourier transforms (Smith, 1999) are used. The DFT basis functions may be summarized as:

$$r_k[i] = \cos\left(\frac{2\pi ki}{N}\right)$$

$$s_k[i] = \sin\left(\frac{2\pi ki}{N}\right)$$
$r_k$ and $s_k$ represent the frequency components of the real and imaginary parts of the signal in the frequency domain; the index $k$ ranges between 0 and $N/2$; $N$ represents the number of discrete sample points and the index $i$ ranges from 0 to $N-1$.

The analysis equations may be represented as

$$
Re\left( X(k) \right) = \sum_{i=0}^{N/2} X[i] r_k[i] 
$$

$$
Im\left( X(k) \right) = \sum_{i=0}^{N/2} X[i] s_k[i] 
$$

$x[i]$ are the sample values of the transient signal, $N$ is the total number of sample points.

The synthesis equations are

$$
x[j] = \sum_{k=0}^{N/2} \text{Re}X[k] \cos \left( \frac{2\pi kj}{N} \right) + \sum_{k=0}^{N/2} \text{Im}X[k] \sin \left( \frac{2\pi kj}{N} \right)
$$

The point of synthesis ‘$j$’, may be different from the sample values ‘$i$’

**Disadvantages of Discrete Fourier Transforms:**

1. Fourier transforms is a non-linear complex transformation of temporal signals into the frequency domain. Operations such as smoothing/filtering are done in frequency domain and simultaneous verification of data conditioned in the time domain cannot be verified readily. Thus, they cannot operate simultaneously in both time and frequency domains.

2. They require a large number of coefficients to reproduce the original transient signal. Since subsequently the coefficients must be extrapolated to unsampled
locations, kriging or stochastic simulation of a large number of coefficients is a tedious task

3. They are inefficient for representing signals with discontinuities. The series cannot be localized and hence tailored to represent discontinuities.

3.2 Wavelet Theory

3.2.1 Basic Concept

The need to represent time and frequency components simultaneously was realized by Gabor, who first introduced the concept of windowed Fourier transform in 1946. The idea of multi-resolution analysis stemmed from this concept and Morlet came up with the notion of wavelets in 1982. Wavelets are characterized by two primary parameters; a translation parameter, that determines the time location of the wavelet and a scaling parameter measuring the degree of scaling.

A “mother” wavelet is generally defined (Debnath, 2002) as follows:

\[ \psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi \left( \frac{t-b}{a} \right), \quad a, b > 0 \]  

\[ 3.6 \]

\( a \) – scaling parameter; \( b \)-shift parameter

For all practical applications the mother wavelet is represented by:

\[ \psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m} t - n) \]  

\[ 3.7 \]

Where \( m \) and \( n \) are the scaling and translation parameters respectively.

The parent signal may be expressed as a linear combination of many wavelets with the magnitude of each coefficient directly proportional to the amount of information carried
by it. As the time discretization progressively gets finer, more information is contained in
the coefficients and consequently the parent signal is reproduced better.

The parent signal may be represented in the following form:

$$f(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{m,n} \psi(2^m t - n) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} d_{m,n} \phi(2^m t - n)$$

(3.8)

$c_{m,n}$ are the wavelet coefficients, $d_{m,n}$ are the scaling coefficients, $\psi_{m,n}$ are the wavelet
basis functions and $\phi_{m,n}$ are the scaling functions corresponding to the wavelet basis
functions. The coefficients may be determined using conventional least squares or any
other numerical residual minimization technique where the objective is to fit the data at
their locations. The wavelets can be as simple as step functions (Haar wavelet) or a
complex combination of sinusoids or Gaussian functions, Dirac delta functions etc. The
Haar wavelet is applied for analysis in this research.

If the wavelets form an orthonormal basis in the $L^2$ sense, then the parent signal can be
reconstructed from its discrete wavelet transform as

$$f(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (f, \psi_{m,n}) \psi_{m,n}(t) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (f, \phi_{m,n}) \phi_{m,n}(t)$$

(3.9)

where $(f, \psi_{m,n}) = \int_0^\infty f(t) \psi_{m,n}(t) dt$; $(f, \phi_{m,n}) = \int_0^\infty f(t) \phi_{m,n}(t) dt$ are called the inner
product spaces. Orthonormality in an $L^2$ sense, implies independence or perpendicularity
between one wavelet and every other wavelet basis function. In other words, in n-
dimensional space each of these wavelet functions are oriented along independent
directions which are perpendicular to each other. Thus, the mutual inner product between
any combination of such basis functions is zero.
An example of a wavelet \( \psi(\bullet) \) constituting an orthonormal basis is the Haar wavelet:

\[
\psi(t) = \begin{cases} 
1, & 0 \leq t < \frac{1}{2} \\
-1, & \frac{1}{2} \leq t < 1 \\
0, & \text{otherwise}
\end{cases}
\]

3.2.2 Multiresolution Analysis

Fourier transforms decompose a signal into a set of harmonic components of different frequencies while wavelet transforms decompose it into wavelets at different scales. A signal \( f(t) \) may be expressed as a linear combination of wavelet (and scaling) components at different scales:

\[
f(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} c_{m,n} \psi(2^m t - n) + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} d_{m,n} \phi(2^m t - n)
\]

In the simplest sense, the concept of multi-resolution can be viewed as zooming into localized regions of any signal. This is explained in Fig. 3.1. At level 2, there are two cycles of a wavelet in a unit interval. At level 1, there is one cycle of a wavelet in a unit interval. At level \( n \), there will be \( 2^{n-1} \) wavelets per interval. This explains the representation of any signal as an aggregation of wavelets and scaling functions at several levels.

3.2.3 Construction of Scaling and Wavelet Functions

The scaling and wavelet functions are constructed using the following relations:
\[ \phi(t) = \sqrt{2} \sum_n h_n \phi(2t - n) \]
\[ \psi(t) = \sqrt{2} \sum_n \tilde{h}_n \phi(2t - n) \]  

The desired filter coefficients \( h_n \) and \( \tilde{h}_n \) when subjected to Fourier transformation will yield the following definitions:

\[ H(\xi) = \sum_n h_n e^{2\pi in\xi} \]  

\[ \tilde{H}(\xi) = \sum_n \tilde{h}_n e^{2\pi in\xi} \]

Vetterli et al. (1992) introduced the following two conditions, which should be solved simultaneously to construct filter coefficients \( H(\xi) \) and \( \tilde{H}(\xi) \).

\[ |H(\xi)|^2 + |\tilde{H}(\xi)|^2 = 2 \]  

\[ \tilde{H} \left( \xi + \frac{1}{2} \right) H(\xi) + \tilde{H} \left( \xi + \frac{1}{2} \right) \tilde{H}(\xi) = 0 \]

Other definitions essential to estimate the filter coefficients are:

\[ \tilde{H}(\xi) = -e^{2\pi i\xi} H \left( \xi + \frac{1}{2} \right) \]

For the case of the Haar wavelets, the filters \( h_n \) and its corresponding conjugate filter \( \tilde{h}_n \) are estimated by employing inverse Fourier transform of the estimated filters \( H(\xi) \) and \( \tilde{H}(\xi) \) along similar lines as pointed out in eqn. 3.5.
\[ h_n = \begin{cases} 
\frac{1}{\sqrt{2}}, & n = 0, 1 \\
0, & \text{otherwise} 
\end{cases} \]

\[ \tilde{h}_n = \begin{cases} 
\frac{1}{\sqrt{2}}, & n = 0 \\
-\frac{1}{\sqrt{2}}, & n = 1 \\
0, & \text{otherwise} 
\end{cases} \]

‘n’ represents dimensionless time.

Fig. 3.1: Multiresolution analysis of wavelets
3.2.4 Dilation Equations

Dilation implies expansion and compression of functions over a given interval shown in Fig. 3.1. This can be elucidated by the following equations for the scaling function:

\[
\phi(x) = \phi(2x) + \phi(2x - 1)
\]

\[
\phi(2x) = \phi(4x) + \phi(4x - 1) + \phi(4x - 2) + \phi(4x - 3)
\]

Generalizing, we get

\[
\phi(2^m x) = \sum_{n=0}^{2^{m+1} - 1} \phi(2^{m+1} x - n)
\]

Thus, if we sum up all the scaling functions from \(m=0\) to \(m=\infty\) we obtain the scaling function at the coarsest scale, \(\phi(x)\).

\[
\sum_{m=0}^{\infty} \phi(2^m x) = \sum_{m=0}^{\infty} \sum_{n=0}^{2^{m+1} - 1} \phi(2^{m+1} x - n) = \phi(x)
\]

3.2.5 Discrete Wavelet Transform

It is often convenient to express all quantities in their dimensionless forms so that all variables are defined on the interval 0 to 1. Combining equations 3.22 and 3.23, in matrix form, the parent signal may be represented as

\[
f(x) = a_0 \phi(x) + a_1 \psi(x) + [a_2 \ a_3] \begin{bmatrix} \psi(2x) \\ \psi(2x - 1) \end{bmatrix} + [a_4 \ a_5 \ a_6 \ a_7] \begin{bmatrix} \psi(4x) \\ \psi(4x - 1) \\ \psi(4x - 2) \\ \psi(4x - 3) \end{bmatrix} + \ldots
\]

\[
+ [a_{2^n} \ a_{2^n+1} \ \ldots \ a_{2^n+n}] \begin{bmatrix} \psi(2^n x) \\ \psi(2^n x - 1) \\ \vdots \\ \psi(2^n x - n) \end{bmatrix} + \ldots
\]

3.24
Where \( x = \frac{t}{T} \) is the dimensionless time. Thus, all the scaling functions summed up together are held in the first term of equation 3.24.

Such a zooming process, shown in Fig. 3.1, is called wrapping. The subsequent terms represent wavelets at scales 0, 1, 2 and so on respectively. As the scales increase, the wavelets carry more detail; however, more coefficients must be estimated. The number of coefficients associated with ‘\( n \)’ scales is \( 2^{n+1} - 1 \).

As the wavelets and scaling functions are orthogonal, the coefficients may be evaluated as follows:

\[
a_{2^n, x} = 2^n \int_0^1 f(x) \psi(2^n x - n) \, dx \approx 2^n \sum_{i=1}^{N} f_i(x_i) \psi(2^n x_i - n) \Delta x_i
\]

\[3.25\]

Also, since the scaling function is orthogonal to the original signal,

\[
a_0 = \int_0^1 f(x) \phi(x) \, dx \approx \sum_{i=0}^{N} f_i(x_i) \phi(x_i) \Delta x_i
\]

\[3.26\]

The bounds of each of the above integrals are between 0 and 1 because they are defined on the dimensionless variable ‘\( x \)’. Fig. 3.2 shows the wavelet decomposition of a typical pressure transient signal ‘s’. The integer translates ‘\( n \)’ have been shown in blue boxes, which represent specific time intervals. The decomposition is only up to a scale of 2 indicated by parameter ‘\( m \)’. The original signal is shown in red at \( m=0 \).
3.2.6 Implementation of Wavelet Analysis Schemes

The above equations may be summarized in the form of an algorithm constituting the following steps

**Step 1:** Convert the pressure versus time data measured at wells into dimensionless form
Select an appropriate wavelet basis function. Scaling function can be derived from the wavelet function based on Vetterli conditions illustrated by eqns. 3.13 to 3.18.

**Step 2:** Fix the number of scales ‘m’ for decomposition and estimate the coefficients from equations 4.8 and 4.9

**Step 3:** At each scale, we now have a vector of coefficients of varying lengths as in Eqn. 4.7. Using equation 4.7, reconstruct the signal and compare with the original signal to check for error.
**Step 4:** If error is too large then change the value of m and repeat steps 1-3. Carry out this exercise to get a representative value of scaling parameter ‘m’ that yields the best match to the original signal for that particular model
Chapter 4: Analysis of Wavelet Coefficients

The ultimate objective is to analyze the pressure data at wells using wavelets and subsequently obtain the wavelet coefficients at unsampled locations using a suitable spatial interpolation technique. The pressure profile at the unsampled location can then be obtained by applying the inverse wavelet transform. This chapter summarizes the application of wavelets for analyzing pressure fields simulated by a reservoir simulator (Eclipse 100) and analyzes the pattern of variability exhibited by the wavelet coefficients. Three fields have been modeled: i) Production with one injector ‘I’ and 3 producers ‘P’; ii) Production with one injector and one producer, and iii) Production with 10 injectors and 15 producers. Water was used for injection. All the cases in this study are two-phase oil/water models. The two dimensional grid is 100 x 100, with each grid block 100 ft x 100 ft. All wells are vertical. All the boundaries are closed. The reference reservoir pressure is assumed to be 1500 psi and the rock compressibility is $0.5 \times 10^{-5}$ psi$^{-1}$. and fluids are slightly compressible. The density of oil is 49.94 lb/ft$^3$ and that of water is 62.4 lb/ft$^3$. The porosity is maintained constant at 0.3 over the entire grid. The permeability field is heterogeneous with the direction of permeability anisotropy oriented along 135° azimuth direction.

4.1. 1x3 Synfield

Many cases were run for this field by varying the locations of the producers over the reservoir domain. The simulation was run with varying configuration of the wells with respect to the underlying heterogeneity. The resultant pressure profiles at the wells were analyzed using wavelets. The correlation between the wavelet coefficients at well
locations were analyzed with respect to the underlying reservoir heterogeneity. The procedure was repeated for two cases.

The permeability map of the entire reservoir and well locations is depicted in Fig. 4.1.1 Blue zones represent low-permeability barriers to flow. The algorithm for wavelet decomposition was rigorously implemented. Three scales ($2^{3+1} - 1$ coefficients) of the wavelet function were deemed sufficient to adequately represent the trends in the transient pressure signal. The linear correlation coefficient between the wavelet coefficients at two wells $X$ and $Y$ computed over the set of wavelet coefficients ($n=15$) was calculated using the formula

$$r_{XY} = \frac{n \sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{\sqrt{\left(n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i\right)^2\right)\left(n \sum_{i=1}^{n} Y_i^2 - \left(\sum_{i=1}^{n} Y_i\right)^2\right)}}$$

4.1.1 Case 1: The coefficients and the corresponding correlation coefficients between the well pairs are shown in Table 4.1.1 and Table 4.1.2, respectively. In this case, producers $P_2$ and $P_3$ are at similar positions with respect to the injector, while the producer $P_1$ is located farther away and the intervening geologic heterogeneity is different as compared to producers $P_2$ and $P_3$. 

28
As observed from the Table 4.1.1., the correlation coefficient between P2 and P3 is very high primarily because they are at almost the same distance from the injector and the intervening reservoir heterogeneity is also similar. Since the flow paths connecting producer P1 and the injector are oriented perpendicular to the direction of permeability anisotropy and quite different from that of P2 and P3, hence the correlation coefficient of P2 and P3 with P1 are lower.

<table>
<thead>
<tr>
<th>Well pairs</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 and P2</td>
<td>0.42</td>
</tr>
<tr>
<td>P1 and P3</td>
<td>0.30</td>
</tr>
<tr>
<td>P2 and P3</td>
<td>0.87</td>
</tr>
</tbody>
</table>
4.1.2 Case 2: In this case wells P1, P2 and P3 are located at similar distance from the injector I1. The intervening reservoir heterogeneity between the injector and P2/P3 are similar. The flow paths from I1 to P2 are in the direction of the reservoir heterogeneity.

![Image: Well locations in 1x3 synfield](image)

**Fig. 4.1.2: Well locations in 1x3 synfield**

**Results:**

<table>
<thead>
<tr>
<th>Well pairs</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 and P2</td>
<td>0.044566</td>
<td>0.008581</td>
<td>0.015192</td>
</tr>
<tr>
<td>P1 and P3</td>
<td>0.022846</td>
<td>0.00842</td>
<td>0.008607</td>
</tr>
<tr>
<td>P2 and P3</td>
<td>0.018231</td>
<td>0.000966</td>
<td>0.007652</td>
</tr>
<tr>
<td></td>
<td>0.016174</td>
<td>0.01305</td>
<td>0.008733</td>
</tr>
<tr>
<td></td>
<td>0.009372</td>
<td>5.86E-05</td>
<td>0.005018</td>
</tr>
<tr>
<td></td>
<td>0.00967</td>
<td>0.001903</td>
<td>-0.00504</td>
</tr>
<tr>
<td></td>
<td>0.012912</td>
<td>0.003646</td>
<td>0.007152</td>
</tr>
<tr>
<td></td>
<td>0.000871</td>
<td>-0.00251</td>
<td>-0.00206</td>
</tr>
<tr>
<td></td>
<td>0.010863</td>
<td>-0.00751</td>
<td>0.000933</td>
</tr>
<tr>
<td>P1 and P2</td>
<td>0.000982</td>
<td>0.00111</td>
<td>-0.00181</td>
</tr>
<tr>
<td></td>
<td>0.004249</td>
<td>0.00405</td>
<td>-0.0005</td>
</tr>
<tr>
<td></td>
<td>0.0010759</td>
<td>0.003513</td>
<td>0.001533</td>
</tr>
<tr>
<td></td>
<td>0.010133</td>
<td>0.007489</td>
<td>0.00282</td>
</tr>
<tr>
<td></td>
<td>0.005431</td>
<td>-0.00242</td>
<td>0.001729</td>
</tr>
<tr>
<td></td>
<td>-0.00133</td>
<td>-0.00242</td>
<td>0.002332</td>
</tr>
</tbody>
</table>

Table 4.1.4: Correlation coefficient

<table>
<thead>
<tr>
<th>Well pairs</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1 and P2</td>
<td>0.56</td>
</tr>
<tr>
<td>P1 and P3</td>
<td>0.83</td>
</tr>
<tr>
<td>P2 and P3</td>
<td>0.55</td>
</tr>
</tbody>
</table>
In this case, the pairs P1/P2 and P2/P3 exhibit a lower value of correlation between the wavelet parameters primarily because the flow paths from I1 to P1 and P3 are across the direction of reservoir continuity while that from I1 to P2 is in the direction of heterogeneity. In comparison, the wavelet coefficients for P1 and P3 are correlated because the underlying heterogeneity affects the flow paths from the injector to these wells similarly.

4.1.3 Observations

The following can be inferred from the above cases:

1. The correlation coefficient between the wavelet parameters of a pair of producers depends on the direction of orientation of the producer-injector pair relative to the direction of permeability anisotropy
2. The correlation coefficient also depends on the distance of producers from the injector
3. The nature of permeability heterogeneity, whether there is a barrier or any low permeability zone, which hinders the motion of fluid between the producer injector pair, also plays an important role.

4.2 1 x 1 Synfield

4.2.1 Case Description

The reservoir properties are similar to the one in the previous cases. There is one injector whose position is fixed at the centre and the location of the producer is varied all over the reservoir domain and the simulator is run for each producer-injector configuration. A
map of each of the 15 wavelet coefficients is then plotted. Figure 4.2 shows the spatial
distribution of some of the wavelet coefficients.

As depicted in Fig. 4.2, the permeability heterogeneity field is almost exactly
reproduced by the map of magnitudes of wavelet coefficients, albeit at a coarser
resolution. This indicates that the well bottom-hole pressure is directly correlated to the
permeability heterogeneity and hence reflects itself in the wavelet parameter map. Fig.
4.2 (a) is the permeability field while the following figures are the maps for some wavelet
parameters. The absolute value of the coefficients may be different but the general pattern
of reservoir heterogeneity is reproduced.

4.2.2 Inference

Should the wavelet parameters calibrated at the producer locations be extrapolated to
unsampled locations, a spatial correlation measure or semivariogram (Issaks and
Srivastava, 1989) is necessary. Ideally, one semivariogram is essential for every
coefficient. However, since the wavelet parameters reproduce the permeability field, it is
hypothesized that a single semivariogram model developed on the basis of the underlying
geologic model will suffice to generate the parameter map. That semivariogram can be
implemented within a stochastic simulation approach such as sequential Gaussian
simulation in order to obtain the wavelet coefficients at unsampled locations.
4.3 10 x 15 Synfield

4.3.1 Case description

The field has 10 injectors (●) and 15 producers (○) arranged in a staggered fashion as shown in Fig. 4.3.

Fig.4.2: Map of wavelet coefficient at different scales ‘m’ and integer translates, ‘n’

The objective in this case is to fit the wavelet coefficients to reflect the pressure data at the producers and subsequently use those coefficients together with the semivariogram model representative of the underlying heterogeneity to obtain an interpolated map of
wavelet coefficients. The simulator was run using the above well configuration and the pressure response at each well location was fit using wavelet functions. The semivariogram (Issacs and Srivastava, 1989) used for the spatial interpolation is shown in Fig. 4.4 in two directions (red – 0° and green 90°). Sequential Gaussian Simulation (Goovaerts, 1997) was used to generate a spatially correlated map of wavelet parameters using the specific semivariogram model. Maps were generated for all the wavelet coefficients.

The coefficient maps are displayed in Fig. 4.5 for a few of the coefficients. The interpolated spatial distribution of wavelet coefficients differs somewhat from the permeability heterogeneity field, primarily because of the sparse well data available to condition the simulated models. Based on the interpolated maps of wavelet parameters, a particular grid block (●) as shown in Fig 4.3 was selected and the pressure profile was regenerated at that location using the corresponding interpolated set of parameters. To verify the accuracy of the interpolation procedure, a simulator was run placing a producer
at the selected location. The pressure profile obtained by flow simulation was compared with the one developed from the map of parameters. The two profiles are shown in Fig. 4.6

4.3.2 Inference:
As seen from Fig. 4.6 the Haar wavelet manages to capture the trend in the pressure profiles effectively. However, there are certain limitations. As seen in Fig. 4.6, there is a sharp drop in pressure towards the end of profile generated using the Haar wavelets,

![Coefficient maps at scales m=2 and 3 generated by Sequential Gaussian Simulation](image)

Fig. 4.5: Coefficient maps at scales m=2 and 3 generated by Sequential Gaussian Simulation

indicating that it might be difficult to reproduce pressure profile that either exhibit edge effects or are flat. Moreover, accurate reproduction of the signal depends on the sampling
in the time domain and the spacing between two subsequent time instants. This is because the coefficients evaluated as an integral using the condition for orthonormality are approximated by a sum. This requires a necessary condition of the spacing between two time instants ‘$\Delta t$’ to be infinitesimally small. However, this drawback is overcome using the Lifting Scheme that is discussed in the next chapter.

![Pressure profile graph](image)

**Fig. 4.6: Comparison between geostatistical (wavelet) and flow simulated (actual) pressure profiles**
Chapter 5: Lifting Scheme and Application to Synthetic Fields

5.1 Wavelet Implementation with a Lifting Scheme

The primary advantage of the Lifting Scheme (Sweldens, 1995) is that it uses the sample values for decomposition of the transient signal without being affected by the spacing in the time domain. The decomposition results in averages and details at various levels of resolution. Consider a sample transient signal $s_j$ with $2^j$ values where ‘$j$’ is the resolution level. The signal may be transformed into a coarser signal $s_{j-1}$ and a detail signal $d_{j-1}$ using the following scheme.

![Diagram of the lifting scheme](image)

Fig. 5.1: The lifting scheme

**Step 1:** Split the signal into even $s_{2l}$ and odd parts $s_{2l+1}$ where ‘$l$’ is the sample number at that particular scale.

**Step 2:** In the Haar wavelet case used here, the odd sample uses its left neighboring sample as its predictor. The detail is then defined as the difference between the odd sample and its prediction.

$$d_{j-1} = s_{j,2l+1} - P(s_{j,2l})$$  \hspace{1cm} (5.1)

Thus an operator $P$ is defined as
\[ d_{j,1} = odd_{j,1} - P(\text{even}_{j,1}) \]  \hspace{1cm} 5.2

**Step 3:** The update stage evaluates the signal at a coarser resolution ‘\( j-1 \)’ as

\[ s_{j-1,l} = s_{j,2l} + U \left( \frac{d_{j-1,l}}{2} \right) \]  \hspace{1cm} 5.3

In the Haar case, the operator \( U \) is

\[ s_{j,1} = even_{j,1} + U(d_{j,1}) \]  \hspace{1cm} 5.4

The original signal may be reconstructed using the inverse of the above scheme.

![Fig. 5.2: Inverse Transform of the Lifting Scheme](image)

**Step 1:** The even samples can be regenerated by subtracting the update information.

\[ even_{j,1} = s_{j,1} - U(d_{j,1}) \]  \hspace{1cm} 5.5

This may also be written as

\[ s_{j,2l} = s_{j-1,l} - U \left( \frac{d_{j-1,l}}{2} \right) \]  \hspace{1cm} 5.6

**Step 2:** The odd samples may be recovered by adding the prediction information.

\[ odd_{j,1} = d_{j,1} + P(\text{even}_{j,1}) \]  \hspace{1cm} 5.7

This may be written as
\[ s_{j,2l+1} = d_{j,2l} + s_{j,2l} \]

**Step 3:** Merge the odd and the even samples to retrieve the signal. This implies the calculated odd and even samples are alternately placed one after another to reconstruct the original signal.

**5.2 Application of Lifting Scheme (Synthetic case 1)**

A two dimensional permeability field with 100 producers (○) and 100 injectors (●) was used to demonstrate the application of the lifting scheme to analyze well data. Only 100 of the total 200 wells are shown in Fig. 5.3.

![2-D Permeability field](image)

Fig.5.3: Well model with the underlying permeability field

The grid dimensions are 1000 x 1000 x 50 ft with 100 grids each in the x and y direction and one grid block in the z direction. Each grid block dimension was thus 10 x 10 ft. The rock and fluid properties were maintained similar to the cases discussed in Chapter 4. The flow rates of the producers and injectors were maintained constant in each layer. The
producer flow rate is 100 STB/day at the top row of producers and 1000 STB/day at the bottom row of wells. The production rate increments by 100 STB/day for each row from top to bottom. The injector flow rates vary in the reverse order from top to bottom row of injectors. This order was selected at random just to test the robustness of geostatistical interpolation to variable flow conditions. The scale on the side of Fig. 5.3 shows permeability in millidarcies.

5.2.1 Lifting Scheme Implementation Approach

**Step 1:** Run a flow simulation and record well bottomhole pressure at each of the 100 producer locations.

**Step 2:** Decompose each of these pressure transients into averages \( s_j \) and details \( d_j \) using the Lifting scheme, at different levels of resolution. Thus there are \( 2^j \) integer translates as shown in Fig. 5.6 at the \( j^{th} \) resolution level.

**Step 3:** In totality, 32 maps of these coefficients resulted corresponding to 4 scales. Plot an experimental semivariogram for each of these coefficients.

**Step 4:** Model the experimental semivariograms to identify the spatial correlation structure and carry out Sequential Gaussian Simulation of the coefficients.

**Step 5:** Generate pressure transient data at the desired location using the inverse wavelet transform. Apply the inverse lifting scheme using the spatially interpolated coefficients.

**Step 6:** Place a well at a desired location and generate pressure variations at that location by running a flow simulation. Compare this profile with the pressure profile generated by geostatistical interpolation.
5.2.2 Observations

Pressure transient signals at three locations in the reservoir are shown in Fig. 5.4. The dramatic change in the nature of the well bore hole pressure is indicative of the heterogeneity in the underlying permeability field.

![Fig. 5.4 Pressure transients at three locations in the reservoir](image)

The maps of the wavelet coefficients at different resolution levels are shown in Fig. 5.5 (a) and (b). At each resolution, the pressure values over a time window are averaged and that constitutes an integer translate. For scale 0, all pressure values are averaged to yield one value. Thus at scale ‘0’ there is only $2^0$ i.e. ‘1’ integer translate and consequently one map. For scale ‘1’ the time domain is divided into $2^1$ i.e. 2 windows and thus there result
two pressure averaged values and 2 maps. Generalizing, at scale ‘m’ there result $2^j$ maps representing integer translates or time windows from $0^{th}$ to $(2^j-1)^{th}$ time window. A maximum of 4 integer translates are shown, as others exhibit similar kind of variability. The maps exhibit maximum continuity in the x-direction with a very high range which is indicative of zonal anisotropy in the corresponding semivariogram (Issaks and Srivastava, 1989). Some of the red regions are indicative of positive pressure values color scales of which haven’t been shown in the figure; as they are differences between original signal and the average pressures.

Fig.5.5(a): Maps of details at scales ‘j’ and integer translates ‘n’

An interesting observation is that the first integer translate at scales ‘3’ and ‘4’ exhibit dramatically different spatial variability as compared to the other maps. This is primarily because the first integer translate ($n=1$) corresponds to the early time characteristics of the pressure transient signal and that is influenced strongly by the fluid compressibility.
In other words, fluid compressibility effects mask the influence of underlying geology and also the smooth variations in the pressure field. This is illustrated for a particular well in Fig. 5.6.

**Fig. 5.5(b): Maps of averages at scales ‘j’ and integer translates ‘n’**

### 5.2.3 Inference

The map of subsequent integer translates exhibit smooth variability and are consistent with the underlying geology. The similarity of maps at various scales brings out the self-similar characteristics of wavelets.

If there are sharp noisy spikes in the transient signal at some time instants, the spatial variability exhibited by averages and details at different scales may be different and this multiscale variability will reflect itself in the coefficient maps. In that case, it is expected that specific semivariogram models would be necessary to model spatial variability at each scale.
5.3 Modeling Spatial Variability of the Coefficients

As observed from the maps all maps except the first integer translate at scales 3 and 4 exhibit similar kind of variability with some subtle differences. It will thus be a good approximation to model 3 semivariograms one for the first integer translates at scales 3 and 4 and a third one for all other coefficients. The experimental semivariograms of the first integer translates of the averages in the 0 and 90° directions have been displayed in Fig. 5.7. These semivariograms are plots of the semivariance of stationary increments of wavelet coefficients at two locations separated by a lag \( h \). These semivariograms were modeled using nested Gaussian structures and are displayed in Fig.5.8. Figure 5.8(b)
and (c) exhibit slight difference in the semivariogram because of the influence of compressibility as explained earlier. Fig.5.8 (a) represents the variability corresponding to all other scales and integer translates.

Fig.5.7: Experimental semivariograms in 0 and 90° azimuth directions

The experimental semivariograms have been shown in yellow and red while the modeled semivariograms have been displayed in green and black in the 0 and the 90° azimuth directions respectively. As the phenomena is continuous in the x-direction (90° azimuth angle) and exhibits zonal anisotropy, it is modeled with an infinite correlation range in the x-direction. An infinite range also implies that the structure does not contribute to the variability in the perpendicular direction (y-direction). The sill (Deutsch and Journel, 1992) indicates the standardized prior variance of the coefficients.
(a) $m = 0, 1, 2$ and $n = 1, 2, 3, 4$ and $m = 3, 4$ and $n = 2$ to 16

(b) $m = 3$ and $n = 1$

(c) $m = 4$, $n = 1$

Fig. 5.8: Experimental semivariograms & models of coefficients in 0 & 90°
As appears from Table 5.1, the above is a case of zonal anisotropy. For scales 0, 1, 2 and translates from 1 to 4, structure 1 contributes to x-direction only as its y-range is infinite. Thus, the nugget and sill of structure 1 add up to a total of 0.081 in the x-direction. On the other hand, structure 2 doesn’t contribute anything to the x-direction as its x-range is infinite. Thus the nugget added to the sill of 0.999 yields a total sill of 1. On similar lines, the contributions of sills from individual structures may be explained.

5.4 Sequential Gaussian Simulation

Sequential Gaussian Simulation (Goovaerts, 1997) was carried out using the above semivariogram models and maps of coefficients representing values over all grid blocks were generated. These maps are displayed in Fig. 5.10. 32 samples have been split into 1 coarsest average at scale ‘0’ and 31 fine details at different scales between ‘0’ and ‘4’ as
indicated in Fig. 5.9(a). Thus, there will be 32 maps required to reconstruct pressure profile at any location as shown in Fig. 5.9 (b).

(a) Splitting of sample signal into averages and details at multiple scales;

1 average and 1+2+4+8+16 = 31 details

(b) Reconstruction of sample signal from averages and details at multiple scales

Fig. 5.9: Scheme illustrating splitting and reconstruction of pressure signal

As observed from Fig. 5.10, the maps at different scales exhibit subtle differences in spatial variability though their overall nature is similar. Maps of first integer translate at
scales 3 and 4 in Fig.5.10 exhibit dramatically different characteristics as compared to the other maps on account of the compressibility effect of fluids as explained earlier.

Fig.5.10: Interpolated maps of coefficients at scales ‘m’ and integer translates ‘n’
5.5 Reconstruction of Pressure Signal at an Unsampled Location

Fig. 5.11 shows the locations of the wells in the reservoir where the pressure profiles were reconstructed using the scheme illustrated in Fig. 5.9(b). The coefficients for the reconstruction were collected from the extrapolated maps.

Fig. 5.11: Reconstructed pressure profiles at the indicated locations

Fig. 5.11 shows the relative comparison between the flow simulated (actual) profiles and the geostatistically simulated profiles (predicted). The match between the profiles is reasonably good. The similarities of the profiles validate the robustness of the geostatistical simulation exercise. The locations of the wells were deliberately selected to
highlight the robustness of the procedure at locations close to the reservoir boundary where there might be non-stationarity in pressure response due to the boundary influence.

5.6 Synthetic Case 2

In reality, field models may not have enough data locations to develop representative semivariograms. Hence a synthetic case with the same underlying permeability field as case 1 but only 36 producer wells was tested. The underlying permeability field is unknown in the field case. However, pressure data were generated from a simulator. The location of the wells is shown in Figure 5.12.

![Well model with the underlying permeability field](image)

Fig.5.12: Well model with the underlying permeability field

5.6.1 Wavelet Decomposition

The grid dimensions are 1000 x 1000 x 50 ft with 100 grids each in the x and y direction. Each grid block dimension was thus 10 ft x 10 ft. The rock and fluid properties were maintained similar to the previous case. The production flow rates were allowed to vary
from one well to the next ranging from 300 to 1800 STB/day. However, each well was produced at a set rate to evaluate changes in the pressure response with time. The pressure response recorded at each well was subjected to decomposition using the lifting scheme and maps were plotted at different scales. These maps are shown in Fig. 5.13.

The decomposition begins at level 5 as there are $2^5$ i.e. 32 pressure samples recorded at each well location. It then progresses stepwise from scale 4 to scale 0 decomposing the average at each scale into a subsequent average at the next scale and its corresponding detail as depicted in Fig. 5.9 (a). At scale ‘0’ there is just one average and this cannot be decomposed further as it is the coarsest scale. All other details are retained.

### 5.6.2 Kriging

Field pressure data are noisy. However, for this case of simulator generated pressure data, kriging (Goovaerts, 1997) was deemed appropriate as opposed to sequential Gaussian simulation because kriging yields smooth pressure profiles that are expected to comply with simulated smooth pressure profiles. If a specific semivariogram model was used to model spatial variability of each average and detail then the process would become computationally intensive. In order to test the sensitivity of estimation to the semivariogram model and scaling, a single semivariogram was used for modeling the average at scale ‘0’ and another for all details. These semivariograms are shown in Fig. 5.14. The points represent the experimental semivariogram values in the $0^\circ$ and $90^\circ$ azimuth directions that have been inferred from the data maps i.e. average at scale ‘0’ and details at scale ‘0’. The solid lines represent models fitted to the experimental semivariograms.
Fig. 5.13: Maps of coefficient at well locations at scales ‘m’ and integer translates ‘n’
Kriged maps at different scales were developed using the above two semivariogram models and are displayed in Fig. 5.15. Two integer translates shouldn’t be compared with each other as they refer to different time windows. However, the first integer translate at different scales may be compared to observe the reservoir response characteristics corresponding to the same time window at different scales. Similarly, one may compare the last integer translate at each scale i.e. the first integer translate at scale 0, the second integer translate at scale ‘1’, the fourth integer translate at scale 2, the 8th integer translate at scale and so on to observe the multi-scale characteristics of the pressure response.
5.6.3 Reconstruction Using Inverse Lifting Scheme

Inverse lifting scheme wavelet transform as shown in Fig. 5.9(b) has to be used to reconstruct the pressure profile at unsampled location. One such pressure profile is compared with an actual profile generated using a flow simulator by placing a well at that location is shown in Fig. 5.16.
Fig. 5.16: Reconstructed pressure profiles at location (990 ft, 990 ft)

5.6.4 Inference

The predicted profile differs significantly from the actual one because only one semivariogram was used to model all details rather than modeling each detail individually using a separate semivariogram. Also the inferred semivariogram models may not account for non-stationarity effects resulting from influence of the boundary. The whole idea of wavelet analysis coupled to geostatistics suffers from a few drawbacks. Considering a realistic scenario with a limited number of wells, it is difficult to infer a representative semivariogram for each of the coefficients. One might want to club all coefficients at multiple scales together and develop some kind of a representative mean semivariogram for all of them. However, this is not justified because it will distort our predictions as the reconstructed pressure profile developed on the basis of a single semivariogram will not comply with multiscale reservoir operation.
Chapter 6:
Joint Space-time Analysis Using Spatiotemporal Covariances

An alternative to analyzing the pressure transient data at wells as a time series and subsequently extrapolating the parameters of the time series to all locations in the reservoir is to directly analyze the joint space-time variability of the pressure field using appropriate covariance measures. Several researchers have looked at tools for characterizing space-time variability, however, the following two approaches appear promising for the purposes of this research:

1. Simple extension of spatial extrapolation techniques in geostatistics to space-time systems by including time as another pseudo-spatial dimension. In that case, spatial measures such as semivariograms can be expressed in terms of effective lags that are a composite of spatial and temporal vectors. Traditional geostatistical techniques can then be applied for the task of spatiotemporal extrapolation. However, such an extension may not be justified for all circumstances of space-time data analysis (Rouhani and Wackernagel, 1990). Qualitative differences between spatial and temporal dimensions, temporal periodicities, spatial non-stationarity, fundamental physical differences between spatial and temporal scales and ordering of temporal data are some factors that need to be considered before proceeding with such an analysis.

2. A second more complete approach is to infer and model the spatiotemporal covariances ensuring positive definiteness and legitimacy of the proposed models. This is necessary for ensuring that the estimation variances remain positive and
the kriging estimate is unique. A simple manifestation of the linear model of co-regionalization is the sum-product model that requires fitting the model for the spatial and temporal models separately and subsequently combining them using coefficients that are adjusted to ensure positive definiteness. The fitted spatiotemporal covariance models can be used in a robust space-time kriging procedure.

Both these procedures are implemented were attempted in this thesis and the results are summarized in this chapter. The essential motivation stems from the need to analyze cross-correlations between pressure samples that are not only separated in space but also located at different instants in time.

6.1 Spatiotemporal covariances

Let \( P(u, t) \) denote a spatiotemporal random process observed at \( N \) space-time coordinates \((s_1, t_1), \ldots, (s_n, t_n)\). The ultimate objective is the optimal prediction of ‘P’ at unsampled locations in space and time subject to observations

\[
P = (P(s_1, t_1), \ldots, P(s_n, t_n))
\]

To ensure optimal prediction one has to develop a model to characterize how pressure samples \( P \) jointly vary in space and time. The observed spatiotemporal variations between recorded samples can be characterized using a spatiotemporal semivariogram function which is defined as:
The whole idea of spatiotemporal modeling depends on the fitting of a deterministic function to the above developed observed spatiotemporal semivariogram function.

6.2 **Spatiotemporal covariance models**

Some common classes of spatiotemporal models have been proposed in the literature. They can be summarized as:

1. **The Metric model**: Dimitrakopoulos and Luo (1994) expressed the covariance function in terms of a linear combination of spatial and temporal lags

   \[ C_{st}(h, \tau) = C(a^2|h|^2 + b^2|\tau|^2) \]

2. **The product model**: De Cesare et al. (2001) proposed space-time covariances as a product of individual spatial and temporal covariances.

   \[ C_{st}(h, \tau) = C_s(h)C_t(\tau) \]

   Cs and Ct are admissible positive definite covariance models in space and time like an exponential, spherical, or Gaussian model and hence they are permitted to be combined in product form.

3. **The Linear model**: Another idea coined by Rouhani and Hall (1989) clubs the covariances directly as a sum.

   \[ C_{st}(h, \tau) = C_s(h) + C_t(\tau) \]

4. **The non-separable model**: Certain classes of nonseparable spatiotemporal stationary covariance functions have been derived by Cressie and Huang (1999). The functions contain the spatial and temporal implicitly coupled with each other.
6.3 Synthetic Case 1

Pressure variations were generated at 50 time instants each with a time spacing of 0.2 days using Eclipse reservoir simulator for a one dimensional reservoir domain split into 51 grid blocks each with a size of 100 ft. The porosity was maintained uniform at 0.3. The rock compressibility was $0.5 \times 10^{-5}$ psi$^{-1}$ and oil compressibility was $3.33 \times 10^{-7}$ psi$^{-1}$. Data were measured at 7 equally spaced intervals starting at grid block 1. An effective spatiotemporal lag vector ($m$) (Armstrong et al., 1993) is defined as a resultant of the two coordinates, spatial and temporal. If two wells are a spatial lag ‘$h$’ apart and pressure values are measured at $t_1$ days and $(t_1 + \tau)$ days at well 1 and 2 respectively, then the effective lag corresponding to these two pressure samples would be defined as

$$m = \sqrt{\left(\frac{h}{L}\right)^2 + \left(\frac{\tau}{T}\right)^2}$$  \hspace{1cm} (6.6)

Note that effective lag is in dimensionless form with $L$ and $T$ representing respective semivariogram ranges in space and time. Analogous to spatial geostatistics, the semivariance of stationary lag increments between a pair of pressure samples corresponding to an effective lag $m$ defines a spatiotemporal statistic called a spatiotemporal semivariogram.

6.3.1 Inferring spatiotemporal semivariogram

A spatiotemporal semivariogram was generated using the ‘gamv’ program in GSLIB and is shown in Fig. 6.1. Various combinations of azimuth angles were tested to identify the direction of maximum and minimum continuity and the two directions and the corresponding semivariograms have been indicated in the Fig. 6.1.
Fig. 6.1: Experimental semivariograms and corresponding fitted models

The parameters of the semivariogram model are shown in Table 6.1.

<table>
<thead>
<tr>
<th>Model type</th>
<th>Nugget</th>
<th>Temporal range</th>
<th>Spatial range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.01</td>
<td>95</td>
<td>65</td>
</tr>
</tbody>
</table>

The semivariogram model equation now defined in terms of effective lag appears as

\[
\gamma(m) = 1 - \exp\left( - \frac{3m^2}{(95^2 + 65^2)} \right)
\]

6.7

The model ranges are approximately 3 times the experimental ranges as observed from Fig. 6.1 primarily because a Gaussian model is used. The experimental ranges spanning the whole gamut of spatiotemporal correlations are shown in the form of black lines in
Fig. 6.2. Based on the model parameters, the fitted ellipse of space-time ranges is shown in pink.

6.3.2 Sequential Gaussian Simulation

The above semivariogram model parameters were input to the SGSIM program in GSLIB to extrapolate pressure samples over the entire reservoir domain. One such pressure profile was generated at the fourth grid block and compared with an actual pressure profile simulated using a flow simulator. This comparison is shown in Fig. 6.3. The predicted profile seems to match the simulated profile closely. The noise exhibited by the maps could be eliminated by resorting to kriging, however at the expense of poor reproduction of the spatio-temporal covariance.

Fig. 6.2: Anisotropy exhibited by spatiotemporal variogram
6.4 Synthetic Case 2

After the successful implementation of the effective lag concept for the spatiotemporal covariance in the one dimensional case, the same concept was extended to two dimensional reservoir to validate the robustness of spatiotemporal extrapolation. Reservoir dimensions were 1500 ft x 1500 ft and 36 wells were assumed at equal intervals as shown in Fig. 5.12. Other properties were maintained similar to the synfield case 1. Different flow rates were assumed for the wells and ranged between 30-180 STB/day.

6.4.1 Inferring Spatiotemporal Semivariogram

In this case the effective spatiotemporal lag vector is a resultant of two spatial co-ordinates and one temporal co-ordinate. Define the expression for the effective lag in this case. Similar to the 1-D case, the ‘gamv’ program was used to generate the
spatiotemporal semivariogram as shown in Fig. 6.4. Parameters of this semivariogram model are tabulated in Table 6.2. The ranges are dimensionless because the effective lag was evaluated as a resultant of the spatial and temporal co-ordinates standardized by their respective ultimate time (last measurement time instant) and spatial co-ordinates(dimensions of reservoir).

Table 6.2: Parameters of spatiotemporal semivariogram

<table>
<thead>
<tr>
<th>Model type</th>
<th>Nugget</th>
<th>Temporal range</th>
<th>X-range</th>
<th>Y-range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.01</td>
<td>165</td>
<td>65</td>
<td>145</td>
</tr>
</tbody>
</table>

Fig.6.4: Three dimensional Spatiotemporal semivariogram for two dimensional reservoir
6.4.2 Spatiotemporal Kriging

Analogous to Simple Kriging in traditional geostatistics, we can develop similar system of equations for spatiotemporal kriging. The estimator for pressure \( P^*(h_o, t_o) \) can be written as

\[
P^*(u_o, t_o) = \sum_{\alpha=1}^{n(u,t)} \lambda_\alpha \left[ P(u_\alpha, t_\alpha) - P_m \right] + P_m
\]

Where \( P_m \) is the mean of the data values \( P(u_o, t_o) \). The \( n(u,t) \) weights \( \lambda_\alpha \) are determined with an attempt to minimize the error variance \( \sigma^2 E(u,t) = \text{Var}\{P^*(u,t) - P(u,t)\} \) under the unbiasedness constraint 6.9.

\[
E\left\{ P^*(u,t) - P(u,t) \right\} = P_m - P_m = 0
\]

The error variance can be expressed as a double linear combination as

\[
\sigma^2 E(u,t) = \text{Var}\left\{ P^*(u,t) \right\} + \text{Var}\left\{ P(u,t) \right\} - 2 \text{Cov}\left\{ P^*(u,t), P(u,t) \right\}
\]

\[
= \sum_{\alpha=1}^{n(u,t)} \sum_{\beta=1}^{n(u,t)} \lambda_\alpha \lambda_\beta \text{C}(u_\alpha - u_\beta, t_\alpha - t_\beta) + \text{C}(0,0) - 2 \sum_{\alpha=1}^{n(u,t)} \lambda_\alpha \text{C}(h_{ao}, \tau_{ao})
\]

\[
= \text{OF}(\lambda_\alpha)
\]

\( h_{ao} \) and \( \tau_{ao} \) are respectively, lags associated with \( u_{ao} - u_o \) and \( t_{ao} - t_o \)

OF represents an objective function which on minimization will yield the optimal combination of weights \( \lambda_\alpha \). Minimization is achieved by setting first order partial derivatives of the objective function to zero.
\[
\frac{1}{2} \sum_{\beta=1}^{n(u,t)} \lambda_{\beta} (C(u_{\alpha} - u_{\beta}, t_{\alpha} - t_{\beta}) - C(u_{\alpha} - u_{\beta}, t_{\alpha} - t_{\beta})) = 0
\]
\[
\alpha = 1, 2, \ldots n(u,t)
\]

In matrix form the above system of equations may be written as

\[
K\lambda = k
\]

Where \(K\) is the \(n(u,t) \times n(u,t)\) matrix of data to data covariances

\[
K = \begin{bmatrix}
C(u_1 - u_1, t_1 - t_1) & \ldots & C(u_1 - u_n, t_1 - t_n) \\
\vdots & \ddots & \vdots \\
C(u_n - u_1, t_n - t_1) & \ldots & C(u_n - u_n, t_n - t_n)
\end{bmatrix}, \quad \lambda = \begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_n
\end{bmatrix}
\]

And ‘\(k\)’ is matrix of data to unknown covariances.

\[
k = \begin{bmatrix}
C(u_1 - u_0, t_1 - t_0) \\
\vdots \\
C(u_n - u_0, t_n - t_0)
\end{bmatrix}
\]

Thus, the weights can be determined by inverting the matrix relation 6.12

\[
\lambda_{\alpha} = K^{-1}k
\]

The covariance matrix values may be evaluated by utilizing the spatiotemporal semivariogram model under second order stationarity

\[
C(h, \tau) = C(0, 0) - \gamma(h, \tau)
\]

In the above case of effective lag vectors, an analogous relationship may be written as

\[
C\left(m = \sqrt{h^2 + \tau^2}\right) = C(0) - \gamma\left(m = \sqrt{h^2 + \tau^2}\right)
\]
$C(0)$ is the standardized sill value in the spatiotemporal semivariogram model which is 1.

Fig. 6.5: Pressure response generated using space-time kriging
The parameters of the semivariogram were input to the ‘kt3D’ code in GSLIB (Deutsch and Journel, 1991) to generate pressure profiles as shown in Fig. 6.5. Interpolated pressure maps were also plotted at different time intervals from the kriged output. These have been shown in Fig. 6.6.

![Pressure maps at various time instants](image)

**Fig.6.6: Pressure maps at various time instants**

### 6.4.3 Inference

The pressure profiles shown in Fig. 6.5 show a good match between the actual and predicted values validating the spatiotemporal interpolation. However, as seen from Fig. 6.5(f), boundary influences tend to introduce distinct non-stationarity in the simulated pressure response and in order to reflect such a non-stationary response, the kriging procedure would have to be modified. Spatiotemporal kriging with a spatiotemporal trend would have to be performed.

The spatial pattern exhibited by the underlying permeability field is as such unknown before estimation. The interpolated pressure maps at times $t = 0.8, 1, 2$ and $2.5$
days however show a distinct similarity to the underlying permeability field in terms of patterns. High bottomhole pressure locations indicated in red in pressure maps comply with corresponding high permeability zones at the same locations on the permeability map. This is consistent with the understanding that if the permeability is high in a region, very little drawdown is required to produce at the specified flowrate, that in turn allows the bottom hole pressure to remain high for longer time.

6.5 Linear Model of Coregionalization

One of the common problems in spatiotemporal geostatistics is inference of a valid spatiotemporal semivariogram model. Expressing time and space dimensions as effective lag vector is not always justified because of the fundamental difference between the two dimensions. Thus a requirement is to model the spatiotemporal semivariogram surface maintaining the two dimensions as separate co-ordinates. Several researchers have developed techniques to express the spatiotemporal semivariogram as a linear model of coregionalization (De Iaco et al., 2003). A subset of the linear models is the product-sum models in which the spatiotemporal semivariogram is expressed as a product and sum of the marginal spatial and temporal semivariograms.

6.5.1 Product-sum models

If $Z = \{Z(u,t), (u,t) \in D \times T\}$ is a second order stationary spatio-temporal random field, where D represents n-dimensional space and T represents another additional dimension. The expected value, covariance and semivariogram of the RF $Z$ is represented by:
\[ E(Z(u,t)) = 0, \]
\[ C(h, \tau) = \text{Cov}(Z(u+h, t+\tau), Z(u, t)), \]
\[ \gamma(h, \tau) = \frac{\text{Var}(Z(u+h, t+\tau) - Z(u, t))}{2}, \]

The function \( C(h, \tau) \) in 6.16 must be positive definite in order to be an permissible covariance model (De Iaco et al, 2001).

This entails that for any set of co-ordinates within the dimensions of the system \((D \times T)\), the covariance function should be such that the following condition should be satisfied for all positive integers \( n_t \) and \( n_t \)

\[
\sum_{i=1}^{n_t} \sum_{j=1}^{n_t} \sum_{k=1}^{n_s} \sum_{l=1}^{n_s} a_{ij}a_{kl} C(u_i - u_k, t_j - t_l) \geq 0 \quad 6.17
\]

De Cesare et al. (2001) introduced the following class of valid product-sum covariance models.

\[ C(h, \tau) = k_1 C_x(h) C_t(\tau) + k_2 C_x(h) + k_3 C_t(\tau) \quad 6.18 \]

where \( C_x \) and \( C_t \) are valid spatial and temporal covariance models respectively. Thus, the conditions for positive definiteness are now transformed to \( k_1 > 0, k_2 \geq 0 \) and \( k_3 \geq 0 \). i.e. positive product and sums of licit covariances gives rise to a licit spatio-temporal covariance model.

In terms of semivariograms using stationarity, the above relation 6.18 may be expressed as

\[ \gamma(h, \tau) = (k_2 + k_1 C_t(0)) \gamma_x(h) + (k_3 + k_1 C_x(0)) \gamma_t(\tau) - k_1 \gamma_x(h) \gamma_t(\tau) \quad 6.19 \]

Where \( \gamma_x \) and \( \gamma_t \) are the spatial and temporal semivariogram models while \( C_x(0) \) and \( C_t(0) \) are sill values. Ideally both the spatial and temporal sills should be equal to the
population variance as per the assumption of stationarity, but in real field cases the sills may be different. De Cesare et al. (2001) defined relationships between the spatiotemporal and marginal semivariograms as:

$$\gamma(h,0) = (k_2 + k_1 t_0)\gamma_x(h) = k_1 \gamma_x(h)$$  \hspace{1cm} 6.20

$$\gamma(0,t) = (k_3 + k_1 t_0)\gamma_t(t) = k_1 \gamma_t(t)$$  \hspace{1cm} 6.21

Combining the above set of equations 6.19, 6.20 and 6.21 with

$$C(0,0) = k_1 C_x(0) t_0(0) + k_2 C_x(0) + k_3 C_t(0)$$  \hspace{1cm} 6.22

Solving for $k_1$, $k_2$ and $k_3$

$$k_1 = \frac{k_1 C_x(0) + k_1 t_0(0) - C(0,0)}{C_x(0) t_0(0)}$$  \hspace{1cm} 6.23

$$k_2 = \frac{C(0,0) - k_1 t_0(0)}{C_x(0)}$$  \hspace{1cm} 6.24

$$k_3 = \frac{C(0,0) - k_1 C_x(0)}{C_t(0)}$$  \hspace{1cm} 6.25

$$k = \frac{k_1}{k_1 k_t} = \frac{k_1 C_x(0) + k_1 t_0(0) - C(0,0)}{k_1 C_x(0) + k_1 t_0(0)}$$  \hspace{1cm} 6.26

where $k_1 C_x(0)$, $k_1 t_0(0)$ and $C(0,0)$ are sill values of $\gamma(h,0)$, $\gamma(0,t)$ and $\gamma(h,t)$

According to the paper by De Iaco et al.(2001), the condition of positive definiteness is ensured by

$$0 < k \leq \frac{1}{\max \{sill(\gamma(h,0))sill(\gamma(0,t))\}}$$  \hspace{1cm} 6.27
6.5.2 Implementation Approach

Say there are data recorded at \( n_x \) locations in space and \( n_t \) locations in time. Then the following steps are followed to compute and model the spatiotemporal semivariogram surface:

1. Compute the spatial and temporal semivariograms using the following relations

\[
\gamma(h, 0) = \frac{1}{2} \sum_{(n(h))} \left( \frac{Z(u + h, t) - Z(u, t)}{M(0)} \right)^2
\]

\[
\gamma(0, \tau) = \frac{1}{2} \sum_{(M(\tau))} \left( \frac{Z(u, t + \tau) - Z(u, t)}{M(\tau)} \right)^2
\]

Where \( N(h) \) and \( M(\tau) \) represent the total number of pairs of samples separated by a spatial lag distance ‘\( h \)’ and temporal lag distance ‘\( \tau \)’ respectively.

2. Evaluate the best fit model for each of the above estimated experimental sample semivariograms. Also estimate the spatial and temporal sills i.e. \( \lim_{h \to \infty} \gamma(h, 0) \) and \( \lim_{\tau \to \infty} \gamma(0, \tau) \).

3. Compute the sample spatiotemporal semivariogram using the following relation

\[
\gamma(h, \tau) = \frac{1}{2} \sum_{(N(h, \tau))} \left( \frac{Z(u + h, t + \tau) - Z(u, t)}{N(h, \tau)} \right)^2
\]

\( N(h, \tau) \) represents the total number of pairs of sample points separated by a spatial lag ‘\( h \)’ and temporal lag ‘\( \tau \)’

4. From the experimental semivariogram surface, estimate the global sill \( C(0,0) \)

The value of ‘\( k \)’ can be estimated using 6.26 and the condition for positive definiteness is verified from equation 6.27.
6.6 Synthetic One Dimensional Case

The above algorithm was tested on a one dimensional reservoir of length 5100 ft split into 51 grid blocks. An injector was placed at the centre and two producer wells were placed each one grid block away from the producer on either side. Bottom hole pressure profiles were generated at both the producers using ECLIPSE simulator for 55 time steps each with a spacing of 0.1 day. In the next run of the simulator the lag distance between the two producers was increased and pressure data were again recorded for the same duration of production. On a similar basis, pressure data were recorded for different combinations of lag distances. Thus, utilizing these different sets of pressure data, a sample spatiotemporal semivariogram surface was generated as shown in Fig. 6.7.

![Variogram surface](image)

Fig. 6.7: Spatiotemporal semivariogram surface for one dimensional case
6.6.1 Observations

The semivariogram reaches a peak in the far right corner which is where the spatial lag =0 and temporal lag is maximum. This peak indicates temporal non-stationarity. The temporal non-stationarity results because the simulated pressure profiles are partly in the infinite acting or transient regime and partly in the pseudo steady state regime. The semivariogram surface was re-plotted after reducing the maximum temporal lag and the resultant truncated semivariogram surface is depicted in Fig. 6.8. This truncation was done in accordance with the assumption of stationarity, which is necessary to promote any further geostatistical analysis. The new semivariogram surface stabilizes at a theoretical sill that is attained at a time lag of 13 units as seen from the x-axis of Fig. 6.8. The residual non-stationarity due to the boundary effects would have to be accounted for by explicitly incorporating a trend model in the kriging procedure.

![Truncated Variogram](image)

Fig.6.8: Truncated semivariogram surface
6.6.2 Modeling spatiotemporal semivariogram surface

The sill values of the above truncated semivariogram are tabulated in Table 6.3. The experimental semivariogram values were standardized by the spatiotemporal sill value \( (C(0,0)) \) and are plotted in Fig.6.10(a). A model for the semivariogram surface was developed on the basis of spherical and Gaussian models respectively as shown in Fig. 6.9.

Table 6.3: Parameters of spatiotemporal semivariogram surface

<table>
<thead>
<tr>
<th>Spatial sill</th>
<th>Temporal sill</th>
<th>Spatiotemporal sill</th>
</tr>
</thead>
<tbody>
<tr>
<td>9158</td>
<td>10308</td>
<td>13858</td>
</tr>
</tbody>
</table>

The sum of least squares error between the semivariogram values obtained from the model and experimental semivariogram values, was used as a criterion to set a value for ‘\( k \)’. The selected value also satisfies the legitimacy condition for \( k \) i.e eqn. 6.27. This marginal error was found to be 0.574 and thus indicates a good fit. The parameters of the standardized semivariogram are tabulated in Table 6.4.
Fig. 6.9: Sample semivariograms and fitted models

(a) Temporal semivariogram and model fit

(b) Spatial semivariogram and model fit

Fig.6.9: Sample semivariograms and fitted models
(a) Standardized semivariogram surface

(b) Product-sum model

Fig. 6.10: Experimental and product-sum model
Table 6.4: Variance of the spatiotemporal semivariogram surface

<table>
<thead>
<tr>
<th>Spatial sill</th>
<th>Temporal sill</th>
<th>Spatiotemporal sill</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.672</td>
<td>0.7562</td>
<td>1</td>
</tr>
</tbody>
</table>

As appears from the Table 6.4, we see a difference in the spatial and temporal sills which could be interpreted as a case of zonal anisotropy. As the temporal sill is higher, one would expect the phenomenon to be relatively smoother in space than in time. Using equation 6.26 and the sill values tabulated in Table 6.4 the value of parameter ‘k’ was estimated to be 0.84. This value of ‘k’ satisfies the legitimacy condition, 6.27. From equation 6.18, and using the marginal semivariogram models, the product-sum semivariogram model was estimated and is depicted in Fig. 6.10(b). The ridges that appear on the experimental semivariogram surface are as a result of boundary influence that is smoothened out in the product-sum model.

6.6.3 Space-time kriging

The developed spatiotemporal semivariogram model can be used in space-time kriging (Rouhani and Hall, 1989) to estimate the pressure at an unsampled location and at an arbitrary time. The development of a space-time kriging code however is left as a future course of work. The above semivariogram surface modeling exercise was carried out only in 1-D to explore the amount of wells essential to develop a representative semivariogram surface. In the above cases with well locations corresponding to 25 lag distances were run to generate pressure samples at 50 time instants. In real field case scenario, one may not have substantial number of wells to characterize the entire spatial
domain even though there could be sufficient number of data points in time. Further analysis needs to be carried out to determine number of wells required to characterize a spatiotemporal semivariogram surface in one dimensional and subsequently extended to two dimensional and three dimensional.

6.7 Discussion

An attempt was made to determine how phenomena co-vary in space and time. The traditional geostatistical analysis was extended by assuming the time as another spatial dimension. In joint space-time analysis, issues concerning fitting spatiotemporal semivariogram models to experimental semivariograms were addressed. The stability conditions associated with the proposed semivariogram model were analyzed with respect to few synthetic cases. Mere observations of the semivariogram surface and space-time anisotropic structures enabled us determine the relative degree of smoothness in each domain, space or time, influence of boundary conditions and non-stationarity resulting from different operating reservoir conditions. Reconstructed pressure maps from space-time kriging showed some correlation to the underlying permeability field in accordance with Darcy’s law thus enabling location of new wells based on the pressure response.
Chapter 7: Conclusions and Future Work

Wavelets are useful mathematical tools that effectively decompose transient signals into trends and residuals thereby represent the signals with a minimal set of coefficients. In this research, the correlation between wavelet coefficients at different producer locations was analyzed and related to the underlying reservoir heterogeneity. The map of wavelet coefficients generated for a synthetic reservoir case by repeating the flow simulation corresponding to varying producer locations in the reservoir, reflect the spatial characteristics of the underlying permeability field. This suggested that the semivariogram characteristics of the underlying permeability field could be used to represent the spatial variations of the pressure field. The semivariogram model together with the pressure data was used to extrapolate the pressure field to unsampled locations in the reservoir. Subsequently, the pressure profile at an unsampled location was generated by inverting the traditional wavelet scheme with a Haar wavelet. The implemented approach has the following advantages:

− In all reservoir management applications, it is the spatial variation in response quantities such as pressure or saturation that is of primary economic interest since they are directly related to the production rate or water breakthrough time, etc. The proposed approach performs a direct extrapolation of these quantities of interest.

− Since the production record at the wells are used as conditioning information in the extrapolation procedure, the resultant models are, by construction, history matched.

The Haar wavelet has limitations on number of samples that can be used for fitting the wavelet coefficients and the spacing between the samples. These drawbacks were
overcome using the lifting scheme. As the lifting scheme is independent of the spacing in the time domain ‘\(\Delta t\)’, there is no approximation of an integral by a sum as was the case with the traditional scheme and thus, the signal is exactly reproduced. Profiles at a few locations were reconstructed by collecting coefficients from the extrapolated maps and compared with the pressure profiles generated using a simulator. A reasonable match between the two profiles validates the procedure for direct spatial interpolation of the time series data, at least for the stationary case where the pressure profile is unaffected by boundary influences and variations in operating conditions.

An alternate approach of spatiotemporal interpolation is explored to directly develop pressure response at an unsampled node and at a future time. The spatial and temporal dimensions are combined together as an effective lag vector to develop a spatiotemporal semivariogram that can be subsequently used for interpolation of time series coefficients. Spatiotemporal kriging establishes a pressure response at an unsampled node that reflects a good match with the corresponding flow-simulated response; thus, validating the robustness of the geostatistical approach.

However, this approach ignores certain important factors like temporal periodicities and the difference in the scaling characteristics of the phenomenon in space and time dimensions. To counter this drawback, the spatiotemporal semivariogram surface is developed using a linear model of coregionalization that uses a sum-product combination of the spatial and temporal semivariogram. The linear model of corregionalization ensures that the resulting model is legitimate and satisfies the condition of positive definiteness. The licit spatiotemporal covariance model can be
subsequently used in space-time kriging to predict the pressure characteristics at unsampled locations and at future time instants without running a flow simulator.

The results presented in this thesis indicate that spatiotemporal extrapolation of pressure using traditional geostatistical tools is an interesting and efficient alternative to the traditional method of first generating maps of reservoir attributes and subsequently processing these maps through a flow simulator.

**Future Course of Work**

1) Development of a space-time kriging procedure:

   The spatiotemporal semivariogram surface for one dimensional case has been developed in this thesis. Hereafter, the traditional kriging techniques need to be extended to time domain. Some of the traditional geostatistical techniques would fail when directly extended because temporal phenomena may be non-stationary as was observed from the one dimensional synthetic case in section 6.6. Thus, to account for non-stationarity it is essential that one resorts to kriging with a trend model.

2) Extension of procedure to other response variables:

   In realistic field case scenario, one would expect the production to be BHP controlled with the production flow rates changing as a function of time. It would thus be intriguing to perform similar spatiotemporal analysis using flow rate or water cut as the dynamic response variable.

3) Application of the procedure to a realistic field example with noisy response data:

   The true potential of all the above interpolation exercises can be realized only when tested on noisy pressure data available from realistic field case scenario. The
technique needs to be validated against actual pressure and rate data obtained in a field setting.
References


35. Schlumberger Geoquest: ECLIPSE Technical Description Manual, 2003A,


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