Stochastic Characterization of Carbonate Buildup Architectures, Using Two- and Multiple-Point Statistics, and Statistical Evaluation of These Methods

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Stochastic Characterization of Carbonate Buildup Architectures, Using Two- and Multiple-Point Statistics, and Statistical Evaluation of These Methods

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Dedication

To God, to whom I owe all my accomplishments.

To my beautiful wife and greatest gift, Lydia Madriz, for her unconditional support and love.

To my mom, Teresa Gonzalez, for her continuous guidance and encouragement.

In memory of my father, Freddy Madriz.
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Abstract

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The University of Texas at Austin, 2009

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Accurate reproduction of complex geological architectures is necessary to build realistic reservoir models, especially since these architectures are often important to flow behavior. Classical geostatistics presents severe modeling limitations because it only accounts for one- and two-point structural information. Complex geological structures cannot be captured based on these very restraining measures of spatial continuity (Caers, 2005).

A particular heterogeneous architecture, significant to the oil industry, is carbonate buildups. The objective of this thesis is to “better” characterize common geo-structures and architectural elements that are often present in carbonate buildups. A typical example
from the Sacramento Mountains in Southern New Mexico is analyzed, then modeled, using two main geostatistical algorithms: two-point statistics (TPS)-based SISIM, and multiple-point statistics (MPS)-based SNESIM. Both models are conditioned to facies pseudo-wells interpreted from high-resolution Light Detecting And Ranging (LIDAR) images of outcrops, themselves located in three canyons that intersect the buildups.

MPS-based algorithms extract structural information from conceptual 3D representations of reservoirs, called training images, that depict prior geological understanding. Algorithms are designed and coded to generate geologically realistic training images of carbonate buildups. Advanced MPS techniques, such as subgrid and multigrid simulations, targeted global proportions, and vertical proportion maps, are required to adequately model this complex carbonate stratigraphy.

The MPS-based model is compared to a TPS-based model, representative of traditional geostatistical models, by quantifying their similarities to the training image (true reference) using statistical and heterogeneity measures. Assuming that visual, statistical and flow behavior similarity to the training image indicates a “better” model, we conclude that the MPS-based model, as compared to its TPS analog, is the “better” alternative for the particular algal buildup studied. Despite this result, this thesis does not suggest that MPS techniques are, in general, the “superior” alternative. On the contrary, based on the parallel modeling performed of both simple and complex architectural elements, we observe that the more “appropriate” stochastic technique is not only application-specific, but also goal- and data-driven.
# Table of Contents

List of Tables .................................................................................................................. xii

List of Figures .................................................................................................................. xiv

Chapter 1: Introduction .................................................................................................. 1

Chapter 2: Literature Review ......................................................................................... 6
  2.1 Carbonate Mound Architecture and Geobodies in the Sacramento Mountains .......... 7
    2.1.1 Location ............................................................................................................. 7
    2.1.2 Schematic Diagram of Architecture and Geobodies ...................................... 10
  2.2 Previous Geocellular Models of the Carbonate Buildups in the Sacramento Mountains ......................................................... 13
    2.2.1 Pseudo-Deterministic Model .......................................................................... 14
    2.2.2 First Model Based on Multiple-Point Statistics ............................................. 17
  2.3 Geostatistical Simulation of Categorical Variables ............................................... 20
    2.3.1 Indicator Kriging .............................................................................................. 20
    2.3.2 Sequential Indicator Simulation (SISIM) ......................................................... 22
    2.3.3 Introduction to Multiple-Point Statistics ......................................................... 24
    2.3.4 Single Normal Equation Simulation (SNESIM) ............................................. 26

Chapter 3: Training Image Generation ...................................................................... 29
  3.1 Introduction ............................................................................................................. 30
  3.2 Modeling of Mound Core Surface ....................................................................... 32
    3.2.1 Mound Core Facies in Region 1 ..................................................................... 36
    3.2.2 Mound Core Facies in Region 2 ..................................................................... 37
    3.2.3 Mound Core Facies in Region 3 ..................................................................... 38
  3.3 Modeling of Debris Surface .................................................................................. 40
    3.3.1 Debris Facies in Region 2 ................................................................................ 41
    3.3.2 Debris Facies in Region 3 ................................................................................ 46
  3.4 Sensitivity Analysis on Mound Core and Debris Surfaces .................................. 49
3.4.1 Areal Density Parameter (V) .............................................53
3.4.2 Steepness Parameter (S)..................................................55
3.4.3 Scaling Parameter (H).....................................................58
3.4.4 Semivariogram Correlation Length (R)..............................60
3.4.5 Dome Parameters (Dome_top/Dome_bot) .......................63

Chapter 4: Application of Advanced Multiple-Point Statistics Tools to
Carbonate Buildup Modeling..................................................69

4.1 Introduction........................................................................69
4.2 Reproduction of Long Range Structures.............................73
  4.2.1 Multigrid Simulation....................................................74
  4.2.2 Subgrid Simulation.....................................................77
4.3 Reproduction of Curvilinear Features................................79
  4.3.1 Optimal Multiple-Point Template Selection.................80
  4.3.2 Resimulation of Nodes...............................................84
4.4 Honoring Facies Proportions and Distributions.....................85
  4.4.1 Target Global Distributions and Vertical Proportion Maps..86
  4.4.2 Multiregion Simulation..............................................89

Chapter 5: Geostatistical Modeling of Carbonate Mound Architectures and
Geobodies .............................................................................90

5.1 Introduction........................................................................90
5.2 Sequential Modeling Using Sequential Indicator Simulation
  (SISIM) ..............................................................................94
  5.2.1 Semivariogram Modeling ..............................................95
  5.2.2 Input Parameters .......................................................101
  5.2.3 Two-Point Statistics-Based Stochastic Realizations.......104
5.3 Sequential Modeling Using Single Normal Equation Simulation
  (SNESIM) ..........................................................................108
  5.3.1 Input Parameters .......................................................108
  5.3.2 Multiple-Point Statistics-Based Stochastic Realizations ..111
Chapter 6: Determining the Quality of the Two- and Multiple-Point Statistics Based Models

6.1 Introduction

6.2 Reproduction of Facies Proportions and Two-Point Spatial Structures

6.2.1 Facies Histograms

6.2.2 Indicator Semivariograms

6.3 Reproduction of Multiple-Point Spatial Structures

6.3.1 Truncated Multiple-Point Histograms

6.4 Reproduction of Spatial Heterogeneity

6.4.1 Entropy of Multiple-Point Histograms

6.4.2 Flow-Capacity-Storage-Capacity Curves and Lorenz Coefficients

Chapter 7: Application of Multiple-Point Statistics to Model a Carbonate Mound Complex in the Sacramento Mountains

7.1 Introduction

7.2 Geological Description of the Carbonate Mound Complex

7.3 Processing Conditioning Data

7.4 Modeling of Carbonate Mound Sequences

Chapter 8: Conclusions and Future Work

8.1 Research Hypothesis

8.2 Conclusions

8.3 Future Work
Appendix A: Geostatistical and Related Programs Developed ..................166
Appendix B: Application of Multiple-Point Statistics to Karst Modeling .......181
Appendix C: Vertical Semivariogram Modeling for Carbonate Buildups in the Sacramento Mountains ................................................................. 186
References ........................................................................................................ 189
Vita ...................................................................................................................... 193
List of Tables

Table 3-1: Important parameters used to generated the mound core surfaces for regions 1, 2, and 3 ......................................................... 39
Table 3-2: Parameters used in mound and debris transforms to generate the base surfaces ................................................................. 51
Table 3-3: Values assigned to parameters needed to perform sensitivity analysis of the areal density parameter ................................. 55
Table 3-4: Values assigned to parameters needed to perform sensitivity analysis of the steepness parameter ...................................... 58
Table 3-5: Values assigned to parameters needed to perform sensitivity analysis of the scaling parameter ........................................... 60
Table 3-6: Values assigned to parameters needed to perform sensitivity analysis of the correlation length ........................................... 63
Table 3-7: Values assigned to parameters needed to perform sensitivity analysis of the dome parameters .......................................... 65
Table 5-1: Main parameters of the 3D model used as training image for carbonate buildups in the Sacramento Mountains ......................... 92
Table 5-2: Indicator semivariograms used to model spatial correlations of facies in geological regions of the training image .................... 100
Table 5-3: Values assigned to additional parameters in SISIM .................. 103
Table 5-4: Values assigned to parameters used in SNESIM .................... 110
Table 6-1: Average porosity and permeability values ........................... 137
Table 6-2: Lorenz coefficients corresponding to the FC curves shown in figure 6-10 ................................................................. 141
Table 7-1: Length of geological regions within each mound sequence ........145
Table 7-2: Values of important parameters input into SNESIM to simulate mound sequences .................................................................152
Table B-1: Values of important parameters input into SNESIM to simulate karst architectures .................................................................181
List of Figures

Figure 2-1: Location of the Sacramento Mountains ................................................. 7
Figure 2-2: Digital elevation model of the Sacramento Mountains ......................... 8
Figure 2-3: Specific locations of the canyons that intersect the Sacramento
Mountains ............................................................................................................... 9
Figure 2-4: Extensive LIDAR coverage of the area of study ..................................... 10
Figure 2-5: Schematic diagram of the carbonate buildup under study .................... 11
Figure 2-6: Map view of SGSIM realizations used to model the spatial
correlation of the mound core facies within each region ................................. 14
Figure 2-7: Pseudo-deterministic model of mound plus debris surface ................. 16
Figure 2-8: Training image used to model an isolated carbonate platform .......... 18
Figure 2-9: Facies distribution model applied to calibrate the distribution of
the background facies in an isolated carbonate platform ................................. 18
Figure 2-10: Stochastic model of a carbonate platform developed using the
MPS/FDM modeling approach ............................................................................ 19
Figure 3-1: Schematic diagram of a carbonate mound growth in a dipping
depositional profile ............................................................................................. 31
Figure 3-2: An example of the extraction of physical dimensions from
LIDAR images .................................................................................................. 33
Figure 3-3: Unconditioned Gaussian field used to convey information about
areal variability ................................................................................................. 34
Figure 3-4: Mound core distribution for regions 1, 2, and 3 ................................. 36
Figure 3-5: Mound core surface modeling the geological structures of the
mound core facies in region 1 ........................................................................... 37
Figure 3-6: Mound core surface modeling the geological structures of the mound core facies in region 2 .................................................................38
Figure 3-7: Mound core surface modeling the geological structures of the mound core facies in region 3 ..............................39
Figure 3-8: Convolution matrix technique for image smoothing ..........41
Figure 3-9: Debris surface superimposed on the mound core surface in sub-region 2b .................................................................43
Figure 3-10: Kernel matrix used to enhance the debris profile ...............44
Figure 3-11: Final debris surface superimposed on the mound core surface in sub-region 2b .................................................................45
Figure 3-12: Debris surface modeling the geological structures exhibited by the debris facies in region 2 .................................................................45
Figure 3-13: Dome with its basic sections highlighted...............................47
Figure 3-14: Debris surface modeling the geological structures exhibited by the debris facies in region 3 .................................................................49
Figure 3-15: Base cases for sensitivity analysis on the mound core and debris surfaces .................................................................52
Figure 3-16: Shifting of the hyperbolic function used in mound transform caused by the 'v' parameter .................................................................54
Figure 3-17: Mound core surface generated by assigning a value of 2 to the areal density parameter .................................................................54
Figure 3-18: Mound core surface generated by assigning a values of -5 to the areal density parameter .................................................................55
Figure 3-19: Slope change of the hyperbolic function used in mound transform caused by the 's' parameter .................................................................56
Figure 3-20: Mound core surface generated by assigning a value of 4 to the sensitivity parameter .................................................................57

Figure 3-21: Mound core surface generated by assigning a value of 0.5 to the sensitivity parameter .................................................................57

Figure 3-22: Change of height of the hyperbolic function caused by the 'h' parameter.......................................................................................59

Figure 3-23: Mound core surface generated by assigning a value of 30 m to the scaling parameter ........................................................................59

Figure 3-24: Mound core surface generated by assigning a value of 8 m to the scaling parameter .........................................................................60

Figure 3-25: Gaussian semivariograms with different correlation lengths........61

Figure 3-26: Mound core surface generated based on a Gaussian semivariogram with 20 units of correlation length.............................................62

Figure 3-27: Mound core surface generated based on a Gaussian semivariogram with 5 units of correlation length.............................................62

Figure 3-28: Debris surface with a small transition zone as determined by the parameters 'dome_top' and 'dome_bot' ..............................................64

Figure 3-29: Debris surface with a large transition zone as determined by the parameters 'dome_top' and 'dome_bot' ..............................................64

Figure 3-30: Workflow used to generate 3D volumes from the mound core and debris surfaces ..............................................................................66

Figure 3-31: 3D volume used as training image for the carbonate buildup under study ........................................................................................67

Figure 3-32: Visual comparsion between a fence diagram of the training image and the discussed schematic diagram ..............................................68
Figure 4-1: Steps involved in simulating facies using an MPS algorithm........71
Figure 4-2: A section of region 2 used as the training image in this chapter .....73
Figure 4-3: Fine and coarse simulation grids........................................75
Figure 4-4: Sensitivity of MPS realizations to the number of grids ............76
Figure 4-5: Illustration of subgrid simulation tool..................................78
Figure 4-6: Sensitivity of MPS realizations to the application of the subgrid simulation method.................................................................79
Figure 4-7: Schematic diagram of the algorithm proposed to select the optimal multiple-point template.........................................................81
Figure 4-8: Multiple-point templates.....................................................83
Figure 4-9: Sensitivity of MPS realizations to the geometry of multiple-point templates .................................................................84
Figure 4-10: Sensitivity of MPS realizations to target global distributions .......88
Figure 5-1: Training image in different views.........................................93
Figure 5-2: Horizontal indicator semivariograms for the mound core facies in region 1 .................................................................97
Figure 5-3: Horizontal indicator semivariograms for mound core and debris facies in region 2 .................................................................98
Figure 5-4: Horizontal indicator semivariograms for mound core and debris facies in region 3 .................................................................99
Figure 5-5: Typical SISIM input parameter file .....................................102
Figure 5-6: Training image and SISIM realization of geological region 1 ......105
Figure 5-7: Training image and SISIM realization of geological region 2 ......106
Figure 5-8: Training image and SISIM realization of geological region 3 ......107
Figure 5-9: SNESIM graphical user interface ..........................................109
Figure 5-10: Training image and SNESIM realization of geological region 1..................113
Figure 5-11: 2D kernel matrix used to post-process SNESIM model of geological region 1..........................................................113
Figure 5-12: Training image and SNESIM realization of geological region 2....114
Figure 5-13: 2D kernel matrix used to post-process SNESIM model of geological region 2..........................................................115
Figure 5-14: Training image and SNESIM realization of geological region 3....116
Figure 5-15: Training image, SISIM realization, and SNESIM realization ....117
Figure 6-1: Training image, SISIM realization, and SNESIM realization ....120
Figure 6-2: Top, middle, and bottom images respectively show the facies distributions of geological regions 1, 2, and 3 ......................122
Figure 6-3: Horizontal indicator semivariograms capturing the spatial correlation of the mound core facies in region 1..............................124
Figure 6-4: Horizontal indicator semivariograms capturing the spatial correlation of the mound core and debris facies in region 2 ..................125
Figure 6-5: Horizontal indicator semivariograms capturing the spatial correlation of the mound core and debris facies in region 3 ..............126
Figure 6-6: Truncated multiple-point histograms.........................................129
Figure 6-7: Entropies of TMP histograms of region 1.................................132
Figure 6-8: Entropies of TMP histograms of region 2.................................133
Figure 6-9: Entropies of TMP histograms of region 3.................................134
Figure 6-10: FC curves corresponding to the geological regions..............139
Figure 6-11: Alternative FC curves corresponding to the geological regions ....140
Figure 7-1: Horizons of mound sequences ..............................................144
Figure 7-2: Training image in different views...........................................146
Figure 7-3: LIDAR image of carbonate buildups under study ..........................147
Figure 7-4: Facies pseudo-wells used as conditioning data of MPS simulations ........................................................................................................148
Figure 7-5: Conditioning facies pseudo-wells mapped into Cartesian grids ....151
Figure 7-6: MPS models of mound sequences developed using SNESIM.......155
Figure 7-7: Fence diagrams of MPS models from figure 7-6 ...........................156
Figure 7-8: MPS-based models mapped back into the stratigraphic grid .......156
Figure 7-9: Comparison between cross-sections from the MPS-based model and corresponding outcrop .................................................................157
Figure 7-10: Comparison between dome-like structures from the MPS-based model and its outcrop analog ..............................................................158
Figure 7-11: Comparison between mounded objects typical of geological region 2. ............................................................................................................159
Figure B-1: Survey of Wind Cave ..................................................................182
Figure B-2: Multiple-point templates used for karst modeling ....................182
Figure B-3: Pseudo-wells used to condition MPS-based model .................183
Figure B-4: MPS-based models of a karstic architecture ...............................184
Figure C-1: Vertical indicator semivariograms for mound core facies in region 1 ..............................................................................................................186
Figure C-2: Vertical indicator semivariograms for mound core and debris facies in region 2 ......................................................................................187
Figure C-3: Vertical indicator semivariograms for mound core and debris facies in region 3 ......................................................................................188
Chapter 1: Introduction

Reservoir characterization aims to provide numerical models that describe the subsurface as accurately as possible. Because there exists only limited information concerning the subsurface medium, which is often complex and heterogeneous, geostatistics has become a key element within reservoir characterization. Geostatistics combines various sources of information to develop realistic models of the subsurface, while describing our limited understanding, using measures of “uncertainties” (Journel, 1986).

Geostatistical estimation methods exploit the correlations between spatially close subsurface property values to estimate those at unsampled locations. Geostatistical simulation methods then use these estimated values to build stochastic reservoir models. Different analyses, such as estimation of original oil in place, well placements, design of enhanced oil recovery projects, can be subsequently carried out based on the stochastic reservoir models (Caers, 2005).

This thesis specifically addresses the challenge of developing “accurate” stochastic models of typical carbonate buildups. One such case is located within the Sacramento Mountains of New Mexico. Carbonate buildups are characterized by complex architectures and definite curvilinear geobodies (Lucia, 1999). This geological structure is particularly relevant to the oil industry because they have been found in highly productive reservoirs around the world.
In this thesis, two geostatistical estimation algorithms are used to stochastically model carbonate buildups: the two-point statistics (TPS)-based SISIM, and the multiple-point statistics (MPS)-based SNESIM. SISIM stands for Sequential Indicator SIMulation; while SNESIM stands for Single Normal Equation SIMulation.

TPS-based algorithms are limited to spatial correlations inferred from semivariograms; while multiple-point analogs extract spatial correlations from multiple-point patterns found in training images (Journel and Zhang, 2006). In view of the relatively limited spatial information considered by TPS-based algorithms, we hypothesize that the MPS-based model will “better” capture and reproduce the common curvilinear geo-structures and overall architecture present in carbonate buildups.

1.1 Problem Statement

Carbonate buildups are usually characterized by facies that form clear and intricate geo-structures. As a result of post-sedimentation biological and chemical processes, large contrasts in petrophysical properties are also typical of buildup facies (Lucia, 1999). In complex cases such as this, “inaccurate” facies distributions might lead to “inaccurate” flow responses (Caers, 2005).

Carbonate buildups have been extensively studied (Kerans et al., 1994; Lucia, 1999) and modeled (Janson et al., 2003; Janson, 2005; Bellian, 2008). However, existing models either fail to adequately reproduce the buildup structures or cannot be easily anchored to
conditioning information. Moreover, uncertainty that reflects the limited understanding of the buildups architecture is not quantified.

In this research, we propose using an MPS-based simulation algorithm (SNESIM) to build geocellular models of carbonate buildups. This technique is applied to a particular case study from the Sacramento Mountains. Scientists at the Bureau of Economic Geology (BEG) have compiled comprehensive information concerning its facies composition, structural patterns, and architecture (Janson et al., 2003; Janson, 2005). An alternative TPS-based model of the same carbonate buildup is also presented in this thesis. The TPS model, built using SISIM, is representative of classical or two-point geostatistics. Extensive comparisons between MPS- and TPS-based models are offered. Comparisons are based on different statistical measures, following the goal to identify the “better” model.

1.2 Research Objectives

This thesis focuses on presenting a workflow, based on geostatistical algorithms, that improve our ability to realistically model carbonate buildups, as compared to the classical approaches discussed in chapter 2. Along this line of work, we define five main objectives to accomplish in this research:

- Develop and code algorithms to build training images of carbonate buildups.
- Build a stochastic facies model of the carbonate buildup under study, using two-point statistics techniques.
• Build a stochastic facies model of the carbonate buildup under study, using multiple-point statistics techniques.
• Determine the “quality” of the TPS and MPS models according to a set of both standard and novel statistical measures.
• Propose and test a workflow, based on either TPS or MPS techniques, to model actual carbonate buildups.

1.3 Approach Overview

This research work is presented in eight chapters. An extensive literature review is given in chapter 2 on carbonate buildups and on geostatistical techniques. Chapter 3 introduces an algorithm that can be used to generate training images that reflect typical architectural elements of carbonate buildups, which is then applied to the carbonate buildup under study. Chapter 4 discusses the role of advanced MPS tools used to model reef buildups. Multigrid and subgrid simulations, resimulation of nodes, optimal template selection, vertical proportion maps, and target global distributions are discussed. In chapter 5, we build TPS and MPS models of the training image from chapter 3. The spatial structural information is retrieved from the training image, which is assumed to be the true reference. Chapter 6 draws a comparison between the TPS and MPS models using facies histograms, indicator semivariograms, multiple-point histograms, entropy coefficients, flow-capacity-storage-capacity curves, and Lorenz coefficients. They were selected to measure different geological and statistical characteristics of the models. Chapter 7
proposes a workflow to model carbonate mound complexes, which is then applied to the mound complex from the Sacramento Mountains. Finally, key conclusions and possible future areas of research are stated in chapter 8.
Chapter 2: Literature Review

Carbonate reservoirs are usually formed as a result of a series of complex depositional and diagenetic processes. The formation of carbonate rocks begins with the creation of calcium carbonate sediments, either by direct precipitation from seawater or by biological extraction. These sediments are then exposed to different chemical, biological, and mechanical degradations (diagenesis), resulting in formations characterized by complex geological structures and petrophysical properties (Lucia, 1999).

One particularly important carbonate structure to the oil industry is carbonate buildups, also known as reefs. Highly productive reservoirs sharing these structures are found around the world, including West Texas; New Mexico; Alberta, Canada; Bermejo, Ecuador (Lee and Castagna, 2007; Reading, 1996).

In section 2.1 of this chapter, we introduce a classic example of a carbonate algal buildup, found in the Sacramento Mountains of New Mexico. These particular reefs have been studied in Kerans et al. (1994), Janson et al. (2003), Janson (2005), and Bellian et al. (2008). Thus, there exists extensive information concerning their style and internal architecture. Scientists working at the Bureau of Economic Geology (BEG) have developed geological 3D models of this formation. These models integrate depositional environment considerations with physical measurements from outcrops (Janson et al., 2003). Two of these models are briefly covered in section 2.2, as they provide needed fundamental understanding. The basic theory of two-point and multiple-point statistics-
based algorithms for facies modeling is then presented in section 2.3. Two-point and multiple-point-based techniques are used to develop geostatistical models in this work.

2.1 Carbonate Mound Architecture and Geobodies in the Sacramento Mountains

2.1.1 Location

The Sacramento Mountains are found in the south-eastern region of New Mexico. They cover an area of approximately 4,700 mi². Their exact location is shown below in red on the map of the United States, in figure 2-1.

![Figure 2-1: Location of the Sacramento Mountains (from Janson et al., 2003)](image)

The eastern section of the Sacramento Mountains, near the Orogrande Basin, provides a typical example of a mixed siliciclastic-carbonate formation developed on a dipping shelf margin (Janson, 2005). This depositional profile, combined with water level dynamics,
resulted in complex facies architectures and geobodies. The details of this process are covered in section 2.1.2.

Figure 2-2 shows a digital elevation model (DEM) of the Sacramento Mountains-Orogrande Basin area. A red square, displayed in the DEM, encloses the carbonate formation under study.

Figure 2-2: Digital elevation model of Sacramento Mountains. The red square indicates the area where the algal buildups studied by Wilson (1967), Rankey (1999), and Janson et al. (2003), are located.
The stratigraphic architectures were inferred from outcrops observed in three canyons intersecting this carbonate buildup. The canyons are Yucca Canyon, Dry Canyon, and Beeman Canyon (figure 2-3). The rocks exposed in these canyons belong to the late carboniferous holder formation (Wilson, 1967; Rankey, 1999; Janson et al., 2003). These sediments were deposited on a narrow carbonate shelf attached to the pedernale uplifted land mass to the East. The shelf was itself uplifted to form the western escarpment of the Sacramento Mountains during the Ancestral Rocky Mountains Orogeny.

Figure 2-3: Specific locations of the canyons that intersect the Sacramento Mountains (from Janson et al., 2003).

The canyons and their surroundings were imaged using Light Detection and Ranging (LIDAR) technology to extract further information that could be used to condition the geological models (Bellian et al., 2008; Janson, 2005). Figure 2-4 shows the extent of the LIDAR coverage. This extensive image is the final outcome of the merging of more than 130 localized high-resolution 3D scenes (Janson, 2005).
Figure 2-4: Extensive LIDAR coverage of the area of study. The gray sections correspond to the regions imaged by LIDAR. Different shadings are used to show the data resolution. The grayer a section is, the higher its resolution is (from Janson, 2005).

2.1.2 Schematic Diagram of Architecture and Geobodies

Reefs are usually characterized by highly-curvilinear structures, with intricate facies distributions (Lucia, 1999). The one found in the Sacramento Mountains is no exception. A schematic diagram was necessary to describe the overall architecture and most important geobodies of this carbonate formation. A comprehensive study, containing the needed schematic diagram, was put together by Janson (2005).

This reef buildup strictly consists of 18 lithological facies, identified from outcrops. However, based on similar petrophysical properties and similar depositional environment, they can be grouped into 4 simplified facies categories: shallow subtidal marine carbonate facies, deeper subtidal marine carbonate facies, mound core facies, and phylloid algae floatstone facies. By relaxing the constraint of similar depositional
environment, we can further simplify the previous classification and introduce the following 3 composite facies: mound core facies, debris facies (same as phylloid algae floatstone facies), and background facies (merging of the shallow and the deeper subtidal marine carbonate facies) (Janson et al., 2003)

Figure 2-5 displays a schematic diagram that show the main structures in terms of the three composite facies: mound core, debris, and background facies. In this figure, the mound core facies is colored green, the debris facies is cross-hatched, and the background facies is left blank (zone above the debris and mound core facies).

Figure 2-5: Schematic diagram of the carbonate buildup under study (from Janson et al., 2003).

The schematic diagram in figure 2-5 consists of 3 main regions chosen based on the characteristics and dimensions of the mound core and debris facies. The particular architecture depicted there can be explained by hydrodynamic and biochemical principles.
The reef buildup under study is the result of algae production and its accumulation as either mound core facies or debris facies. The mounds in region 1 could not grow tall because of reduced algae production. This is a result of being below the photic zone (a zone in a body of water that has enough light exposure for photosynthesis to occur). In other words, the restricted light exposure led to limited algae production that was only enough to accumulate in the small mounded structures of region 1.

The mounds in region 2 grew until they reached the high-energy zone. This zone is somewhere between the fair-weather wave base (FWWB) and storm-weather wave base (SWWB) thresholds shown in figure 2-5. In the high-energy zone, the algae in the mound core facies were reworked into debris sediments. The debris sediments were then deposited over the mounds and inter-mound areas, forming a thin overlaying bed. The thickness of this debris bed depended on localized erosion rates, which, in turn, depended on relative elevation.

The mounds in region 3 reached the high-energy zone faster, because of less accommodation space. Therefore, they could not grow as tall as the mounds in region 2. Moreover, the algae productivity was large because it was within the photic zone and had extended exposure to high-energy waters, resulting in large amounts of debris material. This debris material was initially deposited similarly to the one from region 2, that is, uniformly over the reef buildup. However, the excess debris that was created did not follow this profile; instead, it formed dome-like structures in sub-region 3a and tabular beds of massive thickness in sub-region 3b (Janson et al., 2003).
The schematic diagram in figure 2-5 efficiently summarizes the relevant geological features of this reef buildup, which were derived from advanced energy principles and expert geological interpretation.

2.2 Previous Geocellular Models of the Carbonate Buildups in the Sacramento Mountains

In this section, we briefly cover two geological models of the reef buildup under study. Both of them were developed by the BEG. The first model was created by combining geostatistical and deterministic techniques, hence, the model was named ‘pseudo-deterministic model.’ The second model was created by using a multiple-point statistics based algorithm developed at Chevron Energy Technology Company.

The workflows used to develop the two models are covered thoroughly in this report. The details of the implementations have been purposively left out for the following reasons:

- In the case of the ‘pseudo-deterministic model,’ the determination of the ‘right values’ for the parameters used to generate the model was a highly iterative process. Therefore, stating the specific parameter values will not give any additional insights.

- In the case of the MPS model, the available information concerning the details of the implementation is very limited.
2.2.1 Pseudo-Deterministic Model

Dr. Janson designed and implemented a novel modeling approach, based on stochastic and deterministic principles, to model the curvilinear geometry of the mounds. The first step, which is the stochastic portion of the algorithm, consists of using SGSIM realizations conditioned to variograms with different correlation lengths to model the mound shapes and variability (Janson, 2005).

Figure 2-6 shows the set of SGSIM realizations in map view used to model the mound core facies for the three different regions. A color-coded coded scheme is used to represent the mound heights in the actual simulation grid.

![Figure 2-6: Map view of SGSIM realizations used to model the spatial correlation of the mound core facies within each region. The SGSIM realizations are shown in the actual simulation grid using a color-coded coded scheme (from Janson, 2005).](image)

The low amplitude and high frequency mounds observed in region 1 are modeled using a variogram with short range and high nugget. The variogram used to model the mounds and debris in region 2 has larger correlation length and smaller nugget. Finally, the
The variogram used to model the mounds and debris in region 3 has the largest correlation length and smallest nugget.

The second step, a rather ambitious one, consists of imposing intricate trend maps on the SGSIM surfaces to honor overall height trends of the mound plus debris surface along with some conditioning data from the LIDAR images (Janson, 2005).

Figure 2-7 summarizes the general workflow used in this modeling approach. The 3 images at the top left corner show the SGSIM realizations used to model the heterogeneity of regions 1, 2, and 3, respectively. They display the height variability of each region using color-coded schemes in map view. The image at the top center shows the trend map used to honor the overall height trend and match some conditioning data. The surface modeling the mound core plus debris shape is displayed in map and perspective views in the top right and bottom left corners. Janson (2005) contains the details of this implementation.
Figure 2-7: Pseudo-deterministic model of mound plus debris surface (from Janson, 2005).
2.2.2 First Model Based on Multiple-Point Statistics

Bellian et al. (2008) pioneered the application of multiple-point statistics (MPS) algorithms to model the carbonate buildup under consideration.

The first step in his modeling approach was to build a 3D model defining important geological structures and facies interactions. A total of 4 facies were used: mound core facies, debris facies, background facies, and siliciclastic facies. The first three facies are based on Dr. Janson’s definitions, presented in the previous section. The siliciclastic facies refers to the sediments filling a channel in the northeastern section of the model. The artificial channel was created to illustrate additional features of this modeling approach.

Figure 2-8 displays a conceptual 3D model of a carbonate formation. This non-location-specific model is also known as a ‘training image’ in MPS terminology (Levy et al., 2008).

The second element of this workflow is the facies distribution model (FDM). The main purpose of this tool is to define facies proportions locally by using an additional set of probabilities. This information is then used as soft data to further condition the multiple-point simulations. Figure 2-9 shows the corresponding FDM corresponding to the carbonate from figure 2-8 (Levy et al., 2008).
Figure 2-8: Training image used to model an isolated carbonate platform. The blue- and green-colored structures represent, respectively, the debris, and mound core facies. The white and purple shaded areas stand for regions where the background facies is present (modified from Levy et al., 2008).

Figure 2-9: Facies distribution model applied to calibrate the distribution of the background facies in an isolated carbonate platform. Coloring scheme was not available (modified from Levy et al., 2008).

The third and final step consists of performing multiple-point sequential simulations, conditioned to both the multiple-point probabilities extracted from the training image,
and the soft probability cubes from the FDM models. The tau model (Journel, 2002) is used to merge these two probabilities. Figure 2-10 displays the final MPS model. The color scheme used in the training image has also been applied here.

![Stochastic model of a carbonate platform](image)

Figure 2-10: Stochastic model of a carbonate platform developed using the MPS/FDM modeling approach. The blue- and green-colored structures represent, respectively, the debris, and mound core facies. The purple shaded areas stand for regions where the background facies is present (modified from Levy et al., 2008).

In regards to the modeling of carbonate buildups, Bellian et al. (2008) arrived at the following conclusions:

- Multiple-point statistics-based algorithms are promising alternatives to model highly-structured reef buildups. “Additional modeling finesse is required” for this particular implementation (Bellian et al., 2008).
- A more complex training image than the one currently being used is necessary to generate a realistic MPS model in this case study.
• LIDAR data can be directly integrated into an MPS model, though some rethinking is necessary.

2.3 Geostatistical Simulation of Categorical Variables

Geostatistics is the branch of statistics that uses the non-randomness of geological phenomena to estimate the values of unobserved variables based on available data (Caers, 2005).

There are essentially two main statistical approaches to estimate unknown variables given some conditioning information:

• The traditional two-point statistics approach
• The recently developed multiple-point statistics (MPS) approach

In this section we present the fundamental theories behind both. We also cover their advantages and disadvantages.

2.3.1 Indicator Kriging

Most of the two-point statistics-based algorithms use some form of kriging to establish the dependence between unknown variables and available data. Indicator kriging is a particular type of kriging that applies to binary variables reflecting the occurrence of an event. These binary variables are also known as ‘indicator variables’ (Deutsch and Journel, 1992). An indicator variable can be defined as follows:
\[ I_k(u) = \begin{cases} 1 & \text{if the event } k \text{ occurs} \\ 0 & \text{if the event } k \text{ fails to occur} \end{cases} \] (2-1)

where ‘u’ is a vector representing a particular reservoir location, and ‘k’ is any definite event. In this thesis, the ‘k’ event is the presence of a particular facies.

A unique feature of indicator kriging is that its kriging estimate is an actual approximation of the probability of the event ‘k’ occurring at a particular location, given the available conditioning information (Goovaerts, 1997). The more general expression to calculate the indicator kriging estimate is given below:

\[
I_{sk}^*(u) = \text{Prob} \{ I(u) = 1 \mid (n) \} = \sum_{a=1}^{n} \lambda_a(u_a) I_k(u_a) + \left[ 1 - \sum_{a=1}^{n} \lambda_a(u) \right] \cdot p_0(u) \] (2-2)

where ‘p_0(u)’ represents the prior probability assigned to the occurrence of event ‘k’ at location u, and ‘\lambda_a(u)’s’ are the kriging weights associated to the indicator conditioning data. In simple indicator kriging, the sum of the weights (\(\Sigma \lambda_a\)) does not necessarily add up to 1. The prior ‘p_0(u)’ is obtained from global categories proportions.

Indicator kriging uses kriging weights to update the prior probabilities of the occurrence of event ‘k’ based on indicator conditioning data. Kriging weights account for two-point spatial relationships between conditioning data and estimated location, and they also account for redundant information caused by clustered conditioning data (Deutsch and Journel, 1992).
An extension of this technique that allows integration of additional information into the simulation stage is indicator cokriging. This technique uses additional kriging weights to account for spatial information derived from secondary data.

2.3.2 Sequential Indicator Simulation (SISIM)

This technique was originally introduced to simulate categorical variables (e.g. lithological facies). It was later extended to simulation of continuous variables, made discrete by using interval classes (Journel and Huijbregts, 1978). In this work, only the former application is considered.

Sequential indicator simulation (Sisim) uses indicator kriging to estimate the probability of occurrence of each category, building categorical distributions. Sisim then simulates the unobserved nodes by randomly sampling from these distributions, following the sequential paradigm (Isaaks and Srivastava, 1989).

The following algorithm, modified from Jensen et al. (1997), clearly presents the steps used to generate a realization, using this technique:

1. Define a random path visiting all reservoir locations that are to be simulated. The locations should be visited once and only once.
2. For each location visited \( u \) do the following
   a. Find neighboring facies conditioning data up to a pre-specified number \( N \):
      \[
      z(u_\alpha), \alpha = 1, 2, \ldots, N.
      \]
b. Write each condition datum $z(u_a)$ in terms of indicators corresponding to the occurrence of a particular facies $k$:

$$i(u_a) = [i(u_a, \text{facies 1}), i(u_a, \text{facies 2}), \ldots, i(u_a, \text{facies k})]$$

c. For each facies, estimate the indicator random variable $I(u, k)$ by solving the kriging system in equation 2-2 based on individual facies semivariograms.

d. The estimates of the indicator random variables are actually estimates of the conditional probabilities for the occurrence of each facies. Normalize these conditional probabilities by the sum of all the conditional probabilities for each location $u$. Use the normalized conditional probabilities to define the cumulative density function (CDF) for the facies.

e. Randomly sample a facies from this CDF and assign it to location $u$. The recently simulated node becomes conditioning data.

3. Repeat step 2 until all nodes in the grid are simulated.

Sisim is a variogram-based technique. As a result, it performs better when it is used to reproduce the high-entropy distributions from unstructured reservoirs. In this context, entropy is a measure of heterogeneity. In other words, high-entropy corresponds to ‘homogeneously heterogeneous’ features (Caers, 2005).
2.3.3 Introduction to Multiple-Point Statistics

Equation 2-2 defines the method used to calculate the indicator kriging estimate of an unknown variable. This approach roughly consists of relating the unobserved value to each conditioning datum one at a time. An extension of this idea proposed by Journel and Alabert (1989) consists of grouping the conditioning data “in pairs, in triplets, […] and in one group altogether,” establishing their joint relationships among themselves and to the unobserved value. This approach results in the extended indicator kriging expression shown below:

\[
I^*_k(u) = \text{Prob}\{I(u) = 1 \mid (n)\} = p_0 + \sum_{\alpha=1}^{n(u)} \lambda^{(1)}_{\alpha}(u)[I(u_{\alpha}) - p_0]
\]

\[
+ \sum_{\alpha=1}^{(n(u),2)} \lambda^{(2)}_{\alpha}(u)[I(u_{a_1})I(u_{a_2}) - E\{I(u_{a_1})I(u_{a_2})\}]
\]

\[
+ \sum_{\alpha=1}^{(n(u),3)} \lambda^{(3)}_{\alpha}(u)[I(u_{a_1})I(u_{a_2})I(u_{a_3}) - E\{I(u_{a_1})I(u_{a_2})I(u_{a_3})\}]
\]

... 

\[
+ \lambda^{(n(u))}_{\alpha}(u)[I(u_{a_1})I(u_{a_2})I(u_{a_3})...I(u_{a_n}) - E\{I(u_{a_1})I(u_{a_2})I(u_{a_3})...I(u_{a_n})\}]
\]

Equation 2-3 is an indicator cokriging estimator, where the cokriging weights convey information from the conditioning data taken in pairs, in triplets, up to all at a time.

This equation is known as ‘the extended system of normal equations’. Its solution would provide the exact conditional probability for the occurrence of the event I(u)=1; however, it would require solving for \(2^{n(u)}\) kriging weights from \(2^{n(u)}\) equations (Journel and Alabert, 1989).
The information carried by the conditioning data taken jointly in pairs, triplets, up to n-tuples is not redundant. Therefore, by restricting the priors to the two-point relationships conveyed in the semivariogram, valuable information is lost.

Following the kriging paradigm, the new multiple-point information might be conveyed in the form of additional semivariograms or covariances. For example, if information contained by pairs of data was to be considered, we would additionally need a 3-point covariance, linking the 2-point data to the unknown, and a 4-point covariance, linking any two pairs of data. Moreover, the dimension of the new kriging matrix to be solved would be \((n + \binom{n}{2})\), where \(n\) is the number of conditioning data. This approach is not practical, even for small \(n\), especially if data is to be considered more than two at a time (Strebelle, 2000).

A more practical approach to integrate the joint data information is to take all conditioning data as a single multiple-point event. This is the task of the last term in equation 2-3. The new indicator kriging becomes:

\[
I^*_sk(u) = \lambda(u) \cdot \prod_{\alpha=1}^{n(u)} I(u_\alpha)
\]  

(2-4)

where \(\lambda(u)\) is a single multiple-point kriging estimate.

Strebelle showed that this new estimator can be used to approximate the following conditional probability (2000):
\[
I_{st}^*(\mathbf{u}) = \frac{Prob\{I(\mathbf{u}) = 1|n(\mathbf{u})\}}{Prob\{n(\mathbf{u})\}} = \frac{Prob\{I(\mathbf{u}) = 1, n(\mathbf{u})\}}{Prob\{n(\mathbf{u})\}}
\]

\[
= \frac{Prob\{I(\mathbf{u}) = 1, I(\mathbf{u}_a) = i_a, \alpha = 1,...,n(\mathbf{u})\}}{Prob\{I(\mathbf{u}_a) = i_a, \alpha = 1,...,n(\mathbf{u})\}}
\]

Equation 2-5 contains information about non-centered multiple-point covariances: the numerator is a non-centered, \((n + 1)\)-point covariance, and the denominator is a non-centered \(n\)-point covariance. This fact can be clearly seen when rewriting the numerator and denominator as follows:

\[
Prob\{I(\mathbf{u}) = 1, I(\mathbf{u}_a) = i_a, \alpha = 1,...,n(\mathbf{u})\} = E[I(\mathbf{u}) \cdot I(\mathbf{u}_1) \cdot I(\mathbf{u}_2) \cdot ... \cdot I(\mathbf{u}_n)]
\]

\[
Prob\{I(\mathbf{u}_a) = i_a, \alpha = 1,...,n(\mathbf{u})\} = E[I(\mathbf{u}_1) \cdot I(\mathbf{u}_2) \cdot ... \cdot I(\mathbf{u}_n)]
\]

Equation 2-5 has also been written in Bayes form, suggesting an alternative way to compute the \(I^*(\mathbf{u})\) estimate based strictly on conditional probabilities. This approach avoids the need to compute and model multiple-point covariances (Strebelle, 2000).

2.3.4 Single Normal Equation Simulation (SNESIM)

The advantage of multiple-point statistics over two-point statistics is that it allows the reproduction of complex patterns and curvilinear structures. We have found this claim to hold for carbonate buildups. Supporting results are shown in chapters 5 and 7.

Two-point statistics relies on the semivariogram to capture spatial relationships. The semivariogram conveys very limited information by only considering two points at a time. The local conditional probabilities are then estimated from semivariogram-based
kriging. This approach turned out to be insufficient to reproduce complex architectures and geobodies (Strebelle, 2000).

MPS algorithms avoid the variogram limitation by directly calculating the conditional probabilities from ‘training images’. Training images are conceptual models that display the essential spatial patterns. Training images are then scanned only to retrieve patterns that match the current conditioning data event. Based only on these patterns, the needed conditional probabilities can then be estimated from pattern proportions (Caers, 2003). This approach was initially proposed by Guardiano and Srivastava (1993) and implemented (using the search tree structure) by Strebelle (2000) as the “Single Normal Equation SIMulation” (SNESIM) algorithm.

There are two main parts to the SNESIM algorithm:

- The construction of a search tree to store pattern proportions from training images.
- The sequential simulation section where simulated values are drawn based on these proportions.

The algorithm corresponding to the simplest SNESIM implementation is given below (Strebelle, 2000):

1. Define a multiple-point template $T_j$ to scan the training image.
2. Scan the training image using the multiple-point template $T_j$ and store the pattern proportions in a search tree object.
3. Assign available conditioning data to their nearest simulation node.

4. Define a random path visiting all locations to be simulated once and only once.

5. For each location \( u \) do the following
   a. Determine the current conditioning data event \( \text{con}_J(u) \) within the template \( T_J \).
   b. Calculate the following conditional probability distribution
      \[ Prob(I(u) = k \mid \text{con}_J(u)) \]
      based on the pattern proportions from the search tree.
   c. Randomly sample from this conditional probability distribution and assign this simulated value to location \( u \). Treat the simulated value as conditioning data.

6. Repeat step 5 until all nodes in the grid are simulated.

The additional multiple-point information provided by the SNESIM algorithm comes at the expense of increase RAM usage and more CPU time.
Chapter 3: Training Image Generation

Current implementations of multiple-point statistics (MPS) based algorithms rely on training images for extracting high-order statistics. High-order statistics are measures containing information about spatial relationships of data considered jointly. These statistics are later used to derive the conditional probabilities needed to generate MPS stochastic models following the sequential simulation approach (Caers, 2005). Based on these facts, training images can be considered one of the most important components (if not the most important) of a multiple-point based algorithm.

In this chapter, we present a complete workflow for generating “accurate” training images for a carbonate buildup. An “accurate” training image is one that combines different sources of information (geological interpretation, physical measurements) to reflect relevant architectural features and geobodies. To accomplish this task, we start by reviewing a conceptual model that conveys important geological information about a particular carbonate buildup formation. We then propose a set of transformations and techniques used to generate surfaces, honoring the suggested sedimentary unit shapes. Next, we perform a comprehensive sensitivity study of the important parameters in these transformations. We conclude by briefly comparing the generated training image and the original ideal model to verify the reproduction of important geological features.
3.1 Introduction

Multiple-point statistics (MPS) based algorithms borrow high-order moments from conceptual 3D reservoir representations called “training images.” Training images are used to convey prior geological understanding of both major geological structures and heterogeneity nature to MPS algorithms. These algorithms scan the training images to extract high-order statistics, which are subsequently merged with hard data, to develop specific stochastic models for a target reservoir (Strebelle, 2000). The high-order statistics capture multiple-point spatial correlations jointly which, in turn, allow for the reproduction of complex geo-structures and curvilinear features. As a result of the fundamental role played by training images in MPS algorithms, “accurate” training images capturing the relevant geological features of a reservoir are needed to achieve a satisfactory MPS characterization. A significant part of the work of this research consists of developing algorithms to generate geologically realistic training images for carbonate mounds and related geobodies.

A comprehensive analysis of the depositional environment and facies composition of the mixed siliciclastic/carbonate formation under study is presented by Janson (2005). Figure 3-1 shows a schematic diagram describing the overall architecture and the most important geobodies of this carbonate formation.
Figure 3-1: Schematic diagram of a carbonate mound growth in a dipping depositional profile.

The above illustration consists of 3 main regions that have been chosen based on the mounds’ architectural characteristics. Regions 2 and 3 have been further divided into 4 sub-regions, based on the debris facies’ architecture. The region and sub-region partitioning was necessary to resolve non-stationarity issues caused by different heterogeneity types.

The particular architecture depicted in figure 3.1 can be explained based on hydrodynamic and biochemical principles, as noted in section 2.1.2. The considered carbonate mound complex is a result of phyloid algae production. The algal mounds in region 1 did not grow tall because of reduced algae material production. This reduction is a result of lack of light exposure from being beneath the photic zone (a zone in a body of water that has enough light exposure for photosynthesis to occur).

The mounds in region 2 grew until they reached the high-hydrodynamic energy zone. In the high-hydrodynamic energy zone, the mound core was reworked into debris and was
deposited uniformly over the mound core facies. Finally, in region 3, because of smaller initial accommodation space, the mounds reached the high-energy zone faster, and were not able to grow as tall as the mounds in region 2. Region 3 is also characterized by massive debris deposition over the mound core facies. This large debris accumulation is a direct result of extended exposure to the high-energy zone.

This conceptual model developed by Janson (2003) reflects that this carbonate buildup presents very different architectures and geobodies in each region, which must be modeled separately to avoid non-stationarity issues. An accurate training image should honor style, size, and internal architecture in all regions. We found that a model with an approximate area of 3 km$^2$ provided the range of variability required to honor all relevant geobodies without exceeding current computer limitations. Such a model also had a size that allowed it to be repetitive enough in geo-structures, warranting the production of robust multiple-point statistics. The details of the workflow followed to generate the training image for this carbonate buildup are described below:

### 3.2 Modeling of Mound Core Surface

Training images need not have physical scales attached to them. It suffices for them to contain relative dimensions and, most importantly, relevant geological features (Strebelle, 2000). Some geostatisticians prefer to work with dimensionless training images because they provide more general representations. Physical dimensions are introduced into the models during the simulation stage using dimensional multiple-point templates. The other standard approach is to apply scaling factors to the training images, so that they also
contain the physical dimensions. Then, the multiple-point template is used only to capture and reproduce geo-structures during the simulation stage. In this thesis, we selected to follow the second alternative as it reduces abstraction by allowing work on geological structures having real dimensions attached to them.

The first step of the mound-core modeling was to extract specific physical dimensions for the different geobodies contained in the schematic diagram. The mound characteristic dimensions were obtained by averaging several measurements acquired from LIDAR images of outcrops found in this formation. Figure 3-2 shows one such measurement.

![Figure 3-2: An example of the extraction of physical dimensions from LIDAR images.](image)

Using this approach, we found the following dimensions for the mound core geobodies: the width of the mounds is approximately 5 m in region 1, and approximately 100 m in regions 2 and 3. Region 1 mounds have heights ranging from 2 to 4 m, and region 2
mounds have a uniform height of 30 m. Finally, region 3 mounds have heights ranging from 6 to 8 m.

The next step was to capture and reproduce the areal variability of the mounds in each region. Area variability should be correlated to the number of mounds per dimensionless area in this report. To accomplish this task, a series of 2D unconditional Gaussian fields were generated using the sequential Gaussian simulation algorithm (Sgsim). The Sgsim realizations were constrained to Gaussian semivariograms with different correlation lengths. The correlation lengths were selected to reproduce the desired areal variability of each region. Therefore, we used three different correlation lengths corresponding to the three different regions. This step follows the workflow presented in Jennings (2003). Figure 3-3 shows one of these Gaussian fields in 3D for easier visualization (the third dimension consisting of the Sgsim simulated nodes).

Figure 3-3: Unconditioned Gaussian field used to convey information about areal variability.
The Gaussian fields by themselves provide a poor representation of the mounds, and there is no direct way to control their areal density and shapes. To better model this carbonate buildup, a modified version of a transformation proposed by Janson et al. (2003) is used. The transformation ‘mound_surf’ is defined below:

\[
mound\_surf(x, y, r, v, s, h) = \frac{h \times (1 + \tanh(s \times (v + r(x, y))))}{2}
\]  

(3-1)

where ‘r’ is the previously generated Gaussian field in meters. The parameter ‘s’ controls the steepness of the mound flanks, while the parameter ‘v’ controls the mound spacing, that is, the number of mounds per area. Both of these parameters are dimensionless. The parameter ‘h’ is a scaling factor that controls the average height of the mound core facies in meters. A comprehensive sensitivity analysis on these parameters is in section 3.4 of this report. This section clearly illustrates the influence of each parameter on the mound core profile.

This transformation was selected because it preserves the stochastic variability of the Gaussian fields, which mimics the area variability of the mounds. It also creates more realistic-looking profiles for carbonate buildups compared to the models described in section 2.2. Finally, this transformation provides control over important characteristics of the mounds, such as steepness of mound flanks, mound spacing, and average mound core height. Figure 3-4 shows a mound core surface generated using this technique. All dimensions are given in meters.
In the next sections we will illustrate the application of this technique to generate the mound core surface used in our model.

3.2.1 Mound Core Facies in Region 1

Region 1 is in the lowest portion of the dipping depositional profile in this buildup. This region is below the high-energy zone, resulting in preservation of the mound core facies and lack of production of the debris facies. Region 1 is also below the photic zone, leading to poor algae production, which in turn resulted in a relatively low number of small mounds.

The mounds in this region have an average height of 3 m and an average width of 5 m. Figure 3-5 shows the surface modeling the architecture of the mound core facies in region 1. The dimensions of the bounding 3D volume are (432 x 864 x 5.4) m.
3.2.2 Mound Core Facies in Region 2

Region 2 is in the mid-section of the dipping depositional profile. The mound structures are larger in this region as a result of healthy algae production (from good light exposure) and large available accommodation space. In the high-energy zone, water agitation limited the growth of the mounds by preventing preservation of the algae. Instead, the algae material was reworked into debris and shed laterally.

The mounds in region 2 have an average height of 30 m and an average width of 100 m. Figure 3-6 shows the surface modeling the architecture of the mound core facies in region 2. The dimensions of the 3D bounding volume are (432 x 864 x 30.8) m.
3.2.3 Mound Core Facies in Region 3

Region 3 is in the top-section of the dipping depositional profile. The mound structures are medium in size because of reduced accommodation space. Moreover, there are some zones where the high-energy currents fully prevented the preservation of algae material as mound core. This phenomenon led to patches of mounds spread out across region 3.

The mounds in region 3 have an average height of 6 m and an average width of 100 m. Figure 3-7 shows the surface modeling the architecture of the mound core facies in region 3. Two mound patches modeled independently are displayed. The dimensions of the bounding 3D volume are (432 x 740 x 25.6) m. Table 3-1 contains the values of the important parameters used to generate the mound core surfaces to model the architectures and geobodies in region 1, region 2, and region 3.
Figure 3-7: Mound core surface modeling the geological structures of the mound core facies in region 3.

Table 3-1: Important parameters used to generate the mound core surfaces for regions 1, 2, and 3.

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<th>Parameters</th>
<th>Value/Type</th>
<th>Parameters</th>
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</tr>
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</table>
3.3 Modeling of Debris Surface

The debris facies is a result of high-energy currents reworking the mound core facies. As a result, the mound core and debris facies are genetically related. However, because of different amounts of compaction and degradation, their petrophysical properties are significantly different. This fact makes necessary precise modeling of particular facies architectures, as these might be low-resistance flow paths. In this carbonate buildup, the debris facies presents a highly curvilinear architecture and a set of petrophysical properties typical of low-resistance flow facies. A high-resolution model is therefore important to capture the particular flow response of the debris facies. More about this topic using flow-capacity-storage-capacity curves is discussed in section 6.5.

As mentioned before, the debris facie is not present in region 1. However, it is present in regions 2 and 3 with large heterogeneity, even within each region. To better model the heterogeneity of the debris facies, regions 2 and 3 have been further divided into sub-regions 2a, 2b, 3a, and 3b. We used a different strategy for modeling the debris profile in each of these sub-regions because of their very different geometries. The explanations and details are given in their corresponding sections below.

A very important characteristic common to the debris profile throughout this formation was its smoothness. This smoothness is a result of erosion. We wanted this particular feature to be preserved in our model to make it more realistic. The novel idea of using the convolution matrix technique to mimic the effects erosion is introduced in this work.
The use of convolution matrix to reduce noise (smooth images) is a well-established technique used in image processing (Foltz and Welsh, 1998). We inferred that the same principles could work to smooth the debris surface. Figure 3-8 uses a simple application to summarize the main components involved in this technique. The left image shows a picture affected by white noise (noise at all frequencies). The middle image corresponds to a kernel matrix; in this case, a noise-filtering matrix. This kernel matrix spreads out the random noise distortions to cancel them out. This process results in the enhanced picture shown on the right.

Figure 3-8: Convolution matrix technique for image smoothing.

The kernel matrix is the key element of the convolution matrix technique. It contains detailed information on how to enhance the ‘quality’ of an image. Quality should be understood in this context as having more desired characteristics.

3.3.1 Debris Facies in Region 2
This debris facies contains large-scale and small-scale heterogeneities that are addressed separately by this modeling workflow. Large-scale heterogeneities refer to drastic
differences in architecture, while small-scale heterogeneities consist of varying physical dimensions within an architectural element. Large- and small-scale heterogeneities are respectively modeled by using regions and subregions. This two-step partitioning, that is, the assignment of specific locations to regions and sub-regions, is based on the conceptual geological model, built from outcrop observations, shown in figure 3-1.

Region 2 was further divided into sub-regions 2a and 2b. In both sub-regions, the debris profile closely follows the mound core surface. However, the thickness of the superimposed debris bed differs. In sub-region 2a, the thickness of the debris layer varies from 0 to 3 m; while in sub-region 2b, the thickness of the debris layer varies from 0 to 5 m.

The variable thickness feature observed in the debris facies is the combined result of sediment accumulation, erosion, and a dipping depositional profile. The thickness of the debris layer can be modeled as inversely proportional to the relative elevation of the mounds. The larger the relative elevation of a particular section, the more sediment reworking and erosion there are, caused by high-hydrodynamic energy environment above the storm wave base. The more erosion, the thinner the debris layer is. Considering this phenomenon, we proposed the following mathematical function to create a debris surface reflecting this signature of erosion:

\[
Debris\_Surf = Mound\_Surf + \frac{\text{max}\_\text{thick}}{\text{mound}\_\text{top}} \times (\text{mound}\_\text{top} - Mound\_\text{Surf}) \quad (3-2)
\]
where ‘max_thick’ is a scalar containing the maximum thickness of the debris bed, ‘mound_top’ is a scalar containing the largest elevation in the mound core surface, and ‘Debris_Surf’ and ‘Mound_Surf’ are matrices containing respectively the elevations for the debris and mound core profiles in every node of a sub-region. ‘Debris_Surf’ is a function of ‘mound_top’, ‘max_thick’, and ‘Mound_Surf’. The last parameter, being a function depending on 6 variables itself (see eq. 3-1), causes ‘Debris_Surf’ to depend on 8 variables.

It sufficed to modify the parameter ‘max_thick’ to generate the debris distributions in sub-regions 2a and 2b. In sub-region 2a, this parameter takes on the value of 3 m; while in sub-region 2b, it takes on the value of 5 m. The ‘mound_top’ parameter assumes the value of 30 m for both sub-regions. Figure 3-9 shows a debris surface generated using this algorithm for sub-region 2b.

Figure 3-9: Debris surface superimposed on the mound core surface in sub-region 2b. Brown and yellow-colored structures represent respectively the mound core and debris surfaces.
The dipping depositional profile combined with erosion in this carbonate buildup resulted in a smooth debris profile with larger debris accumulation towards the plunging direction. The convolution matrix technique was used to model this feature by convoluting the debris surface, shown in figure 3-9, and a 5x5 kernel matrix. Figure 3-10 shows the specific node values selected for the kernel matrix. The matrix ‘Debris_Surf’ contains the elevations of every node in the debris surface. The particular values in the kernel matrix were determined by numerical experiment. This matrix performs a weighted average of the altitudes of the debris surfaces that favors higher altitudes in the plunging direction.

<table>
<thead>
<tr>
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<th>0.04</th>
<th>0.08</th>
<th>0.02</th>
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<td>0.08</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figure 3-10: Kernel matrix used to enhance the debris profile.

Figure 3-11 shows the debris surface generated after applying the matrix convolution technique to create a smooth debris surface in sub-region 2b.
Figure 3-11: Final debris surface superimposed on the mound core surface in sub-region 2b. Brown and yellow-colored structures represent the mound core and debris surfaces.

Figure 3-12 shows the debris surface modeling the geological features of the debris facies in sub-regions 2a and 2b. The dimensions of the bounding 3D volume are (432 x 864 x 30.8) m.

Figure 3-12: Debris surface modeling the geological structures exhibited by the debris facies in region 2.
3.3.2 Debris Facies in Region 3

Region 3 is in the top section of the depositional profile. It is characterized by a small initial accommodation space and extended exposure to high-energy waters. This resulted in large debris production and accumulation. Region 3 is partitioned into sub-regions 3a and 3b. In sub-region 3a, the debris architecture consists of domes with flat tops having an average diameter of 250 m and an average height of 25 m overlaying patches of mounds. In sub-region 3b, the debris profile resembles a sinusoidal function with average amplitude of 6 m with, and no mounds are present.

The dome-like structures found in region 3a are modeled using the following set of functions:

\[
\text{dist}(x, y) = \sqrt{(x - x_c)^2 + (y - y_c)^2}
\]

\[
\text{Dome} \_ \text{Surf} = h \times (1 + \tanh\left(\frac{6}{(\text{dome} \_ \text{top} - \text{dome} \_ \text{bot})}\right)) \times \text{dist} + (3 - \frac{\text{dome} \_ \text{top} \times 6}{(\text{dome} \_ \text{top} - \text{dome} \_ \text{bot})})
\]

where \((x_c, y_c)\) are the (x, y) coordinates of the center of a dome in meters, ‘dome_top’ is a scalar containing the horizontal distance in meters at which the flattened top section of the dome ends, ‘dome_bot’ is a scalar containing the horizontal distance in meters at which the flattened bottom section of the dome starts, ‘h’ is a scalar containing the height of the top surface of the dome, and ‘Dome_Surf’ is a matrix containing the elevations of the debris surface at every node. ‘Dome_Surf’ is a function of ‘h’, ‘dome_top’,...
‘dome_bot’, and ‘dist.’ The parameter ‘dist’ is a function itself of ‘x’, ‘y’, ‘x_c’, and ‘y_c’.

Hence, ‘Dome_Surf’ is a function of 8 parameters.

Figure 3-13 displays a flattened dome with its important sections highlighted.

![Dome with its basic sections highlighted.](image)

Figure 3-13: Dome with its basic sections highlighted.

Equation 3-4 calculates the physical distance of a particular node relative to the center of a dome. Using the values of ‘dome_top’ and ‘dome_bot’, the algorithm determines a linear transformation from the physical space into the argument of the hyperbolic tangent function. This transformation guarantees that every node falls into its corresponding section (flat top, transition, or flat bottom).

In sub-region 3b, the architecture of the debris facies is modeled based on the following sinusoidal function:

\[
Debris\_Surf(y, h, y_i) = h \times \sin\left(\frac{\pi}{30}(y - y_i)\right)
\]  

(3-5)
where \( y_i \) is the y-coordinate in meters at the start of region 3b, ‘h’ is a scalar that contains the altitude of the highest point of the debris surface, and ‘Debris_Surf’ is a matrix containing the elevations of the debris surface at every node in sub-region 3b.

Figure 3-14 shows the debris surface modeling the architecture of the debris facies in sub-regions 3a and 3b. The dimensions of the bounding 3D volume are (432 x 740 x 25.6) m.
3.4 Sensitivity Analysis on Mound Core and Debris Surfaces

A series of parameters were introduced by equations (3-1), (3-2), and (3-4). In this section, we refer to equations (3-1) and (3-4) as the ‘mound transform’ and the ‘debris transform’, respectively. We also refer to the surfaces modeling the architecture of the
mound core facies in region 2 and debris facies in region 3 as ‘mound core surface’ and ‘debris surface’, respectively.

The architectures of the mound core facies are very similar in all 3 regions with variations only in physical dimensions and areal density. Hence, a sensitivity analysis on the mound geometries in region 2 suffices to cover the structures in all regions. The debris facies has 3 types of architectures. The architecture of the debris facies in region 2, being a thin layer overlaying the mound core facies, closely resembles the mound geometries in this region. As a result, the sensitivity analysis on the mound geometries applies to this debris architecture. The architecture of the debris facies in sub-region 3a has unique characteristics requiring a separate sensitivity analysis. Finally, we believe a sensitivity analysis of the debris architecture in sub-region 3b is not necessary because of its simplicity.

The goal of this section is to perform a brief sensitivity analysis of some of the parameters in the mound transform and debris transform to clearly identify their specific roles in terms of characteristics in the mound core surface and debris surface. The parameters considered in this section are:

- Areal density parameter (v)
- Flank steepness parameter (s)
- Scaling parameter (h)
- Semivariogram correlation length (r)
- Dome parameters (dome_top/dome_bot)

We start by generating base cases for the mound core surface and the debris surface. Table 3-2 shows the values assigned to all parameters in the mound transform and debris transform to generate the base surfaces. Figure 3-15 shows the base cases for the mound core surface and debris surface.

Table 3-2: Parameters used in mound and debris transforms to generate the base surfaces.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value/Type</th>
<th>Parameters</th>
<th>Value/Type</th>
</tr>
</thead>
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<td><strong>Mound Transform</strong></td>
<td></td>
<td><strong>Debris Transform</strong></td>
<td></td>
</tr>
<tr>
<td>Sgsim Parameters</td>
<td></td>
<td>Debris Transform Parameters</td>
<td></td>
</tr>
<tr>
<td>Semivariogram Structure</td>
<td>Gaussian with a sill of 0.9</td>
<td>Top Surface Parameter (dome_top)</td>
<td>50 meters</td>
</tr>
<tr>
<td>Nugget Effect</td>
<td>0.1</td>
<td>Bottom Surface Parameter (dome_bot)</td>
<td>90 meters</td>
</tr>
<tr>
<td>Correlation Length</td>
<td>10 meters (isotropic)</td>
<td>Scaling Factor (h)</td>
<td>12.5 meters</td>
</tr>
<tr>
<td>Mound Transform Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flank Steepness Parameter (s)</td>
<td>1.0 unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Areal Density Parameter (v)</td>
<td>-1.50 units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scaling Parameter (h)</td>
<td>15 meters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3-15: Base cases for sensitivity analysis on the mound core and debris surfaces.

After generating the base cases for the mound and debris surfaces, we are ready to start the sensitivity analysis. The analysis is performed by fixing all parameters except the one under consideration to isolate the influence of a particular parameter on the resulting surfaces. This workflow allows the identification of a parameter’s specific role. The parameter under consideration is assigned both larger and smaller values relative to the base case to explore different effects.
### 3.4.1 Areal Density Parameter (v)

The areal density parameter controls the number of mounds per dimensionless area. This is accomplished by shifting the sinusoidal function used in the mound transform. Positive ‘v’ values lead to higher numbers of mounds per area; while negative ‘v’ values result in smaller numbers of mounds per area.

Figure 3-16 illustrates the effect of the areal density parameter on the hyperbolic tangent function used in the mound transform. Assume the yellow region represents the values taken on by the argument of this function. The plots are actual profiles generated by the hyperbolic function used to create the mound-like structures. In this plot, we can see that a ‘v’ value of 2 causes a shift to the left, leading to a profile with larger y values; that is, more mounded areas. On the other hand, a ‘v’ value of -2 causes a shift to the right, leading to a profile with smaller y values; that is, more inter-mound areas. These findings are confirmed by studying figures 3-17 (v = 2) and figures 3-18 (v = -5). Table 3-3 summarizes the values assigned to parameters for the sensitivity analysis of the areal density parameter.
Figure 3-16: Shifting of the hyperbolic function used in mound transform caused by the ‘v’ parameter.

Figure 3-17: Mound core surface generated by assigning a value of 2 to the areal density parameter (v).
Figure 3-18: Mound core surface generated by assigning a value of -5 to the areal density parameter (v).

Table 3-3: Values assigned to parameters needed to perform sensitivity analysis of the areal density parameter. This parameter and its values are in italics.

<table>
<thead>
<tr>
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<td>Semivariogram Structure</td>
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<td>Correlation Length</td>
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</tr>
<tr>
<td>Scaling Parameter (h)</td>
<td>15 meters</td>
</tr>
</tbody>
</table>

3.4.2 Steepness Parameter (s)

The steepness parameter controls the slope of the mound flanks. In other words, it controls the steepness in the structures modeling the mounds. This is accomplished by increasing or decreasing the slope of the hyperbolic function used within the mound
transform. ‘s’ values larger than 1 produce steeper flanks; while ‘s’ values smaller than 1 lead to more gentle flanks.

Figure 3-19 illustrates the effect of the steepness parameter on the hyperbolic tangent function used in the mound transform. The plot below shows that an ‘s’ value of 2 causes a steeper profile, while an ‘s’ value of 1/2 causes a more gentle profile. These observations are confirmed by studying figures 3-20 (s = 4) and figures 3-21 (s = 0.5). Table 3-4 summarizes the values assigned to parameters for the sensitivity analysis of the steepness parameter.

Figure 3-19: Slope change of the hyperbolic function used in mound transform caused by the ‘s’ parameter.
Figure 3-20: Mound core surface generated by assigning a value of 4 to the sensitivity parameter (s).

Figure 3-21: Mound core surface generated by assigning a value of 0.5 to the sensitivity parameter (s).
Table 3-4: Values assigned to parameters needed to perform sensitivity analysis of the steepness parameter. This parameter and its values are in italics.

<table>
<thead>
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<th>Parameters</th>
<th>Value/Type</th>
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<tr>
<td>Nugget Effect</td>
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</tr>
<tr>
<td>Correlation Length</td>
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</tr>
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<td><strong>Mound Transform Parameters</strong></td>
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</tr>
<tr>
<td>Flank Steepness Parameter (s)</td>
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<tr>
<td>Areal Density Parameter (v)</td>
<td>-1.5 units</td>
</tr>
<tr>
<td>Scaling Parameter (h)</td>
<td>15 meters</td>
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</tbody>
</table>

3.4.3 Scaling Parameter (h)

The scaling parameter introduces physical dimensions into the models, controlling the average height of the mound core facies. Figure 3-22 shows the role played by the ‘h’ parameter in terms of the hyperbolic function used in the mound transform. This plot shows how the scaling parameter controls the height of the hyperbolic function profile, which, in turns, corresponds to the average mound height. Figures 3-23 (h = 30) and 3-24 (h = 8) confirm this observation. Table 3-5 summarizes the values assigned to parameters for the sensitivity analysis of the scaling parameter.
Figure 3-22: Change of height of the hyperbolic function caused by the ‘h’ parameter.

Figure 3-23: Mound core surface generated by assigning a value of 30 m to the scaling parameter (h).
Figure 3-24: Mound core surface generated by assigning a value of 8 m to the scaling parameter (h).

Table 3-5: Values assigned to parameters needed to perform sensitivity analysis of the scaling parameter. This parameter and its values are in italics.

<table>
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</tr>
<tr>
<td>Scaling Parameter (h)</td>
<td>30, 8 meters</td>
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3.4.4 Semivariogram Correlation Length (r)

The semivariogram directly manifests its influence in the Gaussian random field (r) which is used as an argument in the mound transform. Roughly speaking, the correlation length regulates the mound spacing by establishing spatial correlation between the elevation values of the structures. In other words, the correlation length determines the
‘variability’ of this mounded morphology. A semivariogram with a large correlation length results in continuous geobodies with smooth profiles (small variability); a semivariogram with a small correlation length leads to uncorrelated structures with drastic elevation changes (large variability).

Figure 3-25 is a plot of three Gaussian semivariograms with different correlation lengths. Correlation length should be understood as the lag distance where 95% of the sill is reached in this context. These semivariograms, from left to right, conditioned the Gaussian random fields that were ultimately used to generate the mound surfaces in figures 3-26, the base case, and figure 3-27. Study of these mound surfaces confirm that the longer the semivariogram correlation length is, the less ‘variable’ the structures are. Table 3-6 summarizes the values assigned to parameters for the sensitivity analysis of the semivariogram correlation length.

Figure 3-25: Gaussian semivariograms with different correlation lengths.
Figure 3-26: Mound core surface generated based on a Gaussian semivariogram with 20 units of correlation length.

Figure 3-27: Mound core surface generated based on a Gaussian semivariogram with 5 units of correlation length.
Table 3-6: Values assigned to parameters needed to perform sensitivity analysis of the correlation length. This parameter and its values are in italics.

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<tr>
<td>Scaling Factor (h)</td>
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### 3.4.5 Dome Parameters (dome_top/dome_bot)

The debris facies is distributed in geo-structures resembling flattened domes in region 3. Figure 3-13 shows the 3 main zones of this structure: flat top, transition, and flat bottom. In this section, we analyze the influence of the ‘dome_top’ and ‘dome_bot’ parameters from the debris transform in terms of the interactions of these 3 main divisions. We previously stated that the ‘dome_top’ parameter corresponds to the horizontal distance, relative to the center of the dome, where the flat top division ends. We also claimed that the ‘dome_bot’ parameter correlates with the horizontal distance, with respect to the center of the dome, where the flat bottom starts. In other words, these two parameters defined the boundaries of the divisions. The goal of the current analysis is to verify these two claims.

Figure 3-28 shows a case where the difference between ‘dome_bot’ and ‘dome_top’ is 5 m, leading to a narrow transition zone. Figure 3-29 shows a case where the difference
between these parameters is 60 m, leading to a large transition zone. Table 3-7 shows the particular values assigned to both parameters.

Figure 3-28: Debris surface with a small transition zone as determined by the parameters ‘dome_top’ and ‘dome_bot’.

Figure 3-29: Debris surface with a large transition zone as determined by the parameters ‘dome_top’ and ‘dome_bot’.
Table 3-7: Values assigned to parameters needed to perform sensitivity analysis of the dome parameters. These parameters and their values are in italics.

<table>
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<td>Scaling Factor (h)</td>
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3.5 Generation of 3D Models based on Mound and Debris Surfaces

A training image for the current case study should be a 3D representation of how the mound core and debris facies are jointly distributed in space. In this chapter, we have described how to generate surfaces accurately that accurately model the topography of the tops of the mound core and debris facies. Although these surfaces show appropriate facies distribution, they cannot be used as training images, as they are not 3D volumes. In this section we present the workflow used to generate the desired 3D model from the mound core and debris surfaces.

The debris and mound surfaces were initially generated as matrices in MatLab®. They were then written in XYZ file format and exported as ASCII files. Using Gocad®, these ASCII files were imported as point clouds. From the point cloud objects, mound core and debris surfaces were generated within Gocad®. A 3D grid (voxet object) was defined with corresponding physical dimensions. Finally, using the mound core and debris surfaces as geological boundaries, we defined mound core and debris regions within the 3D grid. Figure 3-30 summarizes this workflow using the mound core profile as an example.
Figure 3-30: Workflow used to generate 3D volumes from the mound core and debris surfaces.
Figure 3-31 shows the full 3D model generated following this approach. This 3D model is used as the training image of the carbonate buildup under study in chapters 5 and 7.

Figure 3-31: 3D volume used as training image for the carbonate buildup under study.

3.6 Training Image vs. Ideal Model

We conclude this chapter by briefly comparing the training image and the schematic diagram of this carbonate buildup. Figure 3-32 graphically compares these two. The top image shows a fence diagram of the final 3D model (training image). The bottom figure shows the ideal model (schematic diagram). The training image honors the overall architecture and facies distribution reflected in the ideal model in the following aspects:

- Small mounds in region 1.
- Large mounds covered by a thin debris layer in region 2.
- Medium mounds covered by a massive debris bed in region 3.
Figure 3-32: Visual comparison between a fence diagram of the training image and the discussed schematic diagram. Brown- and yellow-colored structures respectively represent the mound core and debris facies.
Chapter 4: Application of Advanced Multiple-Point Statistics Tools to Carbonate Buildup Modeling

4.1 Introduction

The multiple-point statistics (MPS) based approach is the result of the combination of two well-established modeling techniques: semivariogram-based geostatistics and object-based algorithms (Journel and Zhang, 2006).

Semivariogram-based geostatistics are stochastic pixel-based methods. Semivariogram-based algorithms create geological models by simulating the desired properties one node at a time while honoring some input statistics. Being pixel-based methods, they can easily honor even the densest conditioning data. The conditioning data can be anchored to a grid before simulation starts. Being stochastic, these algorithms also provide the means to quantify uncertainty. A number of ‘feasible models’ corresponding to possible scenarios can be generated. Based on these models, an uncertainty measure can be developed (e.g., E-type). However, semivariogram-based geostatistics cannot reproduce complex architectural elements, being conditioned to only first and second order-moments (Journel and Zhang, 2006).

Object-based techniques employ basic objects to reproduce geological structures and model heterogeneity. The fundamental objects are then distributed according to depositional and physical laws. Many basic objects are currently available, allowing these techniques to reproduce geobodies and architecture of highly-complex reservoirs.
However, honoring conditioning data, done through iteration, can be a very time-consuming and CPU-demanding task. There is also a lack of widely accepted methods that integrate soft data, such as production data and seismic surveys (Zhang et al., 2006).

Multiple-point statistics (MPS) is a stochastic pixel-based technique that can capture and reproduce curvilinear structures and complex architectures by honoring high-order moment statistics. MPS represents a powerful technique, borrowing only the strengths of semivariogram-based and object-based algorithms. The basic theory behind MPS is in section 2.3.3 of this thesis. Figure 4-1 shows the steps involved in simulating the sand/shale property using an MPS algorithm. The top right image shows a simulation grid and a conditioning data event. The left image shows a training image, which has been scanned for patterns that satisfy the conditioning data event. The conditional probabilities of the event sand/shale are calculated in the node to be simulated based on pattern proportions. According to these probabilities, a facies is randomly sampled. The bottom right image shows the grid after simulation.

One of the most successful MPS implementations is the ‘Single Normal Equation SIMulation’ (SNESIM) program by Strebelle (2000). Details of its implementation are in section 2.3.4 of this thesis.
Figure 4-1: Steps involved in simulating facies using an MPS algorithm. A conditioning data event is determined. The training image is scanned for replicates of the conditioning data event. Based on replicate proportions, conditional probabilities are calculated. According to these probabilities, a facies is randomly sampled and added to the simulation grid (modified from Strebelle, 2000).

SNESIM integrates a set of advanced tools into the MPS modeling approach to accurately honor facies proportions, and to better reproduce long range structures and curvilinear features. All of these tools are necessary to successfully model the architectural elements and heterogeneity of carbonate buildups.

This section explores the role of the following advanced MPS tools when used to model reef buildups:
- Multigrid simulation
- Subgrid simulation
- Optimal template selection
- Resimulation of nodes
- Vertical proportion map
- Target global distribution

A section from region 2, characterized by large mounds covered by a thin debris layer, is used as the training image in the following analysis. Figure 4-2 shows this facies configuration.
4.2 Reproduction of Long Range Structures

In region 2 of this carbonate buildup, there are architectural elements and heterogeneity of very different scales. The overall architecture is characterized by relatively large mounds with a thin superimposed debris layer. The mound core facies forms highly-curvilinear mounded shapes, while the debris facies is distributed following the mounded
shapes, in a narrow layer of varying thickness. We would like to capture and reproduce all these geo-structures in a high-resolution geocellular model.

Multiple-point (MP) templates are used to extract high-order moments from training images. In other words, the training images are scanned, using MP templates, to retrieve MP patterns. The MP patterns are the only means to convey information from the training images to the MPS algorithms. Therefore, if the MP patterns do not capture all types of heterogeneity and structures, these features will be missing in the MP realizations, resulting in inaccurate models. The obvious solution is to increase the size of the MP templates until all important features are included; however, this technique is not generally accepted as it might lead to prohibitively large demands of CPU time and RAM memory.

In the next section, we explore tools that allow the capture of heterogeneities and structures of different scales while working within limited CPU time and RAM memory.

4.2.1 Multigrid Simulation

The most common technique used to capture structural information at different scales is the multigrid simulation tool. Multigrid simulation consists of performing sequential simulation using multiple grids that feature an equal number of nodes but a different spatial extent.

The original MP template used is always the finest grid. Coarser grids, those with larger spatial extent, are then constructed from the finest grid by having their size increased by a factor, and then being down-sampled by the same factor. Figure 4-3 shows a coarse grid (right) obtained from re-scaling the original, fine, MP template (left).
Nodes are initially simulated on the grid that has the greatest spatial coverage, allowing the capture of large-scale features. Nodes are then simulated on the second coarsest grid, using the previously simulated nodes as conditioning data. This process is continued until nodes are simulated on the finest grid, capturing small-scale features. By simulating in progressively finer templates, we can capture features observed at all scales, which leads to realistic representations (Strebelle, 2002).

To test the effect of using multiple grids when modeling carbonate buildups, we generated three unconditioned MP realizations by simulating with one, two, and finally three grids. The full MP realizations and their corresponding fence diagrams are shown in figure 4-4.

The mound core and debris facies are represented by the brown and yellow-colored structures. Case 1 was simulated using only the finest grid. Case 2 was simulated using
two grids; a fine and a medium grid. Case 3 was simulated using three grids; a fine, a medium, and a coarse grid. Based on visual comparison against the training image, we conclude the following: case 1 completely fails to capture the mounded shapes and overall architecture; case 2 reproduces the mound shapes but not the overall facies interactions; case 3 reproduces the overall architecture and geo-structures the most accurately.

Figure 4-4: Sensitivity of MPS realizations to the number of grids. The brown and yellow-colored structures represent, respectively, the mound core and debris facies. Cases 1, 2, and 3 were simulated, respectively, using one, two, and three grids.
4.2.2 Subgrid Simulation

Subgrid simulation enhances the continuity of the geo-structures by using highly conditioned data events. Figure 4-5 illustrates the basic steps of the subgrid simulation method.

Subgrid simulation works in conjunction with the multigrid simulation technique described in section 4.2.1. The subgrid technique divides the original grids into smaller grids (subgrids). A subgrid-based simulation starts by populating the coarsest grid following the multigrid approach. The nodes in the coarsest grid have been labeled ‘A’ in figure 4-5. The ensemble of nodes labeled ‘A’ constitutes the first subgrid. The idea behind the subgrid implementation is to primarily use nodes ‘A’ as conditioning data, as these are all informed, to simulate the nodes labeled ‘B’. One can optionally include some of the nodes ‘B’ that were previously simulated. Following this approach, good and reliable conditioning is warranted to simulate all nodes. The collection of nodes ‘B’ then constitutes the second subgrid. Finally, nodes ‘C’ are simulated based on conditioning data events formed by mainly including nodes ‘A’ and ‘B’, that is, subgrids 1 and 2.
Figure 4-5: Illustration of subgrid simulation tool (modified from Strebelle, 2002).

Figure 4-6 shows two unconditioned MPS realizations modeling the features of the training image shown in figure 4-2. Case 1 was generated using two simulation grids and the subgrid simulation technique (displayed on the left). Case 2 was generated using two simulation grids without the subgrid simulation technique (displayed on the right).

Visual comparison of the MP realizations and their fence diagrams is not sufficient to determine if, in this case, subgrid simulation enhanced the reproduction of the geological features (the structures look similar in both realizations). However, by considering the horizontal mound core indicator semivariograms of the realizations, we can conclude that this is indeed the case. Case 1 has a spatial structure, described by an exponential semivariogram, with a practical correlation length of 80 m. Case 2 has a spatial structure, also described by an exponential semivariogram, with a practical correlation length of only 50 m. We can conclude, based on these findings, that the geological elements in case 1 are more continuous structures than those present in case 2.
Figure 4-6: Sensitivity of MPS realizations to the application of the subgrid simulation method. The brown and yellow-colored structures represent, respectively, the mound core and debris facies. The subgrid simulation method was only used in case 1.

### 4.3 Reproduction of Curvilinear Features

The two-point spatial correlations used in semivariogram-based geostatistics are severely limited in their ability to reproduce curvilinear features. In contrast, the multiple-point spatial correlations (MP correlations) used in MPS algorithms allow the capture of these complex features.
These important MP correlations are extracted from training images using MP templates. Both training images and MP templates play important roles within an MPS algorithm. The training image should contain the relevant geological structures, while the template should have the shape needed to capture them. Wang (1996) found that the MP correlations, and thus the success of the MP simulations, depend greatly on the geometry of the MP template, proposing the existence of the ‘optimal template.’

The MP spatial correlations are encoded as conditional probabilities during the simulation stage. These probabilities, based on the conditioning data events, guide the sequential simulation process. Strebelle (2002) emphasizes the importance of ‘highly informed’ conditioning data events in reproducing curvilinear features.

In this section we present tools that can be used to efficiently determine good MP templates and ensure proper conditioning data events.

### 4.3.1 Optimal Multiple-point Template Selection

Strictly speaking, multiple-point (MP) templates can only extract statistical patterns from training images. These patterns are then stored in the form of high-order statistics used to direct the simulation that models actual geological patterns.

This modeling approach presents one major difficulty. Different templates generate very different sets of statistical patterns from the same training image. These different sets lead to the simulation of very diverse architectural elements. Most of these elements do not reproduce the geological features from the training image (Eskandari, 2008).
Eskandari (2008) proposes a simple yet satisfactory algorithm based on two-point correlations. This algorithm finds a subset of highly-spatially correlated nodes within a template frame. These nodes are then retained to form the ‘optimal multiple-point template’ (2008). Optimal multiple-point templates are templates that extract statistical patterns consistent with the geological patterns to be simulated. Figure 4-7 illustrates the main steps of this algorithm.

![Figure 4-7: Schematic diagram of the algorithm proposed to find the optimal multiple-point template. The top left image shows a training image. The other top images correspond to template frames illustrating the steps of the algorithm. The bottom left image shows the spatial correlations of the template nodes. The bottom right image displays the optimal template found (modified from Eskandari, 2008).](image)

The user defines a rectangular template frame used as the algorithm working domain and specifies the number of nodes to be retained in the final template. The template frame
should be large enough to provide enough space to clearly define the optimal template geometry. The training image is initially scanned using the template frame, and all the patterns found are recorded. Based on these patterns, the algorithm then calculates two-point correlations between the central node and all remaining nodes in the template frame. The user-specified nodes, with the highest spatial correlations, are retained.

This algorithm was used to find an MP template based on the following inputs:

- Template frame dimensions are (9 by 9 by 9) m
- Number of nodes to be retained is 250

Figure 4-8 shows a template determined by the above algorithm and an arbitrary elliptical template created as a comparison. Both of these templates are composed of 250 nodes. The dimensions for the elliptical template are (15 by 7 by 5) m.

Figure 4-9 shows two unconditioned MPS realizations generated by use of the three simulation grids, as well as the templates in figure 4-8. Case 1 shows the MP realization generated by use of the ‘optimal template’. Case 2 shows the MP realization generated by use of the arbitrary elliptical template. Case 1 simulation more accurately reproduces the geological features than does case 2 simulation, even though they are both based on the same training image.
Figure 4-8: Multiple-point templates. Template 1 was determined based on two-point spatial correlations. Template 2 was arbitrarily defined. 250 nodes were retained in both templates.
Figure 4-9: Sensitivity of MPS realizations to the geometry of multiple-point templates. The brown and yellow-colored structures represent, respectively, the mound core and debris facies. A template based on spatial correlations was used in case 1. An arbitrary elliptical template was used in case 2.

4.3.2 Resimulation of nodes

This is another tool used to enhance the reproduction of complex geological features. Rich conditioning data events provide more structural information than do poorly informed conditioning events. Hence, all nodes should ideally be simulated based on rich conditioning events (Remy, 2001).

The random path, followed when performing sequential simulation in stochastic algorithms, might lead to inadequately informed simulations. These types of simulations are characterized by large uncertainty and limited geological pattern reproduction (Remy,
To prevent these unsatisfactory simulations, the idea of resimulating nodes was introduced into SNESIM.

A threshold corresponding to the minimum number of conditioning data accepted is specified by the user. This threshold is used to divide simulated nodes, based on their conditioning data events, into two groups: well-informed and poorly-informed nodes. The SNESIM program records the poorly-informed nodes. After the typical sequential simulation process has been completed, a new random path is defined that visits only the poorly-informed nodes. Following this path, a second sequential simulation process is implemented to resimulate these nodes based on better conditioning.

Based on visual comparison against the training image, this technique did not produce significant improvement in modeling the carbonate buildup under consideration.

### 4.4 Honoring Facies Proportions and Distributions

Training images should ideally contain large-scale architectural elements, small-scale geological features, and overall facies proportions that characterize a particular formation. Chapter 3 introduced a workflow to generate TIs of carbonate buildups. As can be inferred from this chapter, the generation of training images can be a challenging and time-consuming task. In some cases, the exact information about facies proportions may be omitted from the TIs to ease modeling.

Multiple-point algorithms retrieve statistical patterns from TIs. This retrieving requires stationary TIs. In geological terms, all described geo-structures should be repetitive and equally likely to occur throughout the TIs. The architecture of most, if not all carbonate
deposits, is not stationary making its reproduction using geostatistical techniques challenging. In this section, we introduce tools that allow MPS algorithms to use TIs lacking information about facies proportions to model complex reservoirs.

4.4.1 Target Global Distributions and Vertical Proportion Maps

There is a current interest in developing libraries of training images for different depositional environments (Journel and Zhang, 2006). These training images focus on capturing only main stratigraphic architecture and general facies proportion associated with geological deposits. It does not, however, contain specific lateral and vertical facies proportion. We might soon be able to find geologically acceptable training images that do not have correct facies proportions.

A tool built within the SNESIM program allows the use of a target global proportion to correct inaccurate facies ratios. This tool saves the work of building a completely new training image. When the target global proportion tool is used, there are essentially two sets of statistics to honor: the first set is retrieved from the training image, and the second set comes from the target proportion. During the simulation stage, the TI statistics are encoded as conditional probabilities as follows:

\[
Pr(A|B) = \frac{Pr(A,B)}{Pr(B)}
\]

(4-1)

whereby A and B represent the node to be simulated and the conditioning data event.

Using Bayes theorem, equation 4-1 can be rewritten as follows:

\[
Pr(A|B) = \frac{Pr(B|A)Pr(A)}{Pr(B|A)Pr(A) + Pr(B|\bar{A})Pr(\bar{A})}
\]

(4-2)
where $\overline{A}$ is the complement of the event $A$. In equation 4-2, all probabilities are the actual TI facies proportion. Strebelle proposed replacing the facies proportion $\Pr(A)$ from the TI by the facies proportion $\Pr(A')$ from the target global proportion to merge these two set of statistics (2002). The author argued that this change allows updating the facies proportion while maintaining the likelihood $\Pr(B|A)$. Equation 4-3 is the equation used to perform a simulation that honors a global target proportion.

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A')}{\Pr(B|A)\Pr(A') + \Pr(B|\overline{A})\Pr(\overline{A}')}$$ \hspace{1cm} (4-3)

Figure 4-10 shows two unconditioned MPS realizations honoring different target global proportions. This figure also shows the target proportions used in both cases. The MPS realization in case 1 was conditioned to a target proportion with relatively large presence of mound core and debris facies. The MPS realization in case 2 shows the opposite effect. Based on visual comparison against the training image, we conclude that both realizations reproduce the architectural elements from the TI.

A servosystem correction factor to include target global proportions is also built into SNESIM (Strebelle, 2002). However, the geological features of the carbonate buildup under consideration are not preserved in the MPS simulation when the servosystem factor is used.
Figure 4-10: Sensitivity of MPS realizations to target global distributions. The brown and yellow-colored structures represent, respectively, the mound core and debris facies. The abbreviations ‘bg’, ‘db’, and ‘mc’ stand for background, debris, and mound core facies in the histograms. The target proportions used are displayed as histograms.
Some heterogeneous reservoirs have very different facies proportions in each horizontal layer. This information cannot be conveyed by one target global proportion. To handle these heterogeneously stratified cases, the technique of vertical proportion maps was introduced. A vertical proportion map allows the definition of a set of target global proportions to condition the MP simulation on a layer basis. The previously introduced Bayesian technique is used to update the conditional probabilities during the simulation, one layer at a time. This technique is used extensively in chapter 5 of this report.

4.4.2 Multiregion Simulation

Oil and gas reservoirs, largely affected by faulting and folding, usually present compartmentalized geological architectures, characterized by fundamentally different elements. We explained in chapter 3 that training images must be stationary so that they can provide robust MPS statistics. A training image should therefore not be used to capture the geological features of these complex reservoir. The multiregion simulation tool was introduced by Remy (2001) and is intended to handle these situations within an integrated approach.

In this technique, different regions are defined as corresponding to zones with particular architectures. Training images, representing the regional geology, are then built for each region. The MPS sequential simulation follows a random path within a region and populates the nodes based on conditional probabilities retrieved from its regional training image. Simulated nodes from different regions are used as conditioning to allow gradual transitions between reservoir compartments, mimicking the subsurface.

The multiregion simulation tool is used extensively in chapters 5 and 7 of this thesis.
Chapter 5: Geostatistical Modeling of Carbonate Mound Architectures and Geobodies

5.1 Introduction

Geostatistical algorithms combine geological and statistical information to develop information-consistent models. Geostatistical models must conform to the statistics inferred from the available conditioning data as well as to the interpreted geological constraints.

In geostatistics, unknown variables to be estimated are treated as random variables. Random variables are those with an attached probability distribution that describes their outcomes. Given the fact that, in most modeling cases, many unknown variables are to be simulated; the goal of geostatistical modeling can be summarized as that of building multivariate conditional probability distributions of the unobserved variables, conformed to the conditioning data, and randomly sampling from them (Goovaerts, 1997). Equation 5-1, written in the form of a conditional cumulative probability distribution (CCPD), summarizes this approach.

\[
\Pr\{ Z(u) \leq z, u \in S | n(u) \} = \frac{\Pr\{ Z(u) \leq z, u \in S, n(u) \}}{\Pr\{ n(u) \}}
\]  

(5-1)

where ‘Z(u)’ is the unknown variable at location ‘u’, ‘n(u)’ is the available conditioning data, and ‘S’ is the set of locations of unknown variables to be simulated.

Modeling the multivariate CCPDs that jointly describe the unobserved variables is the main challenge of geostatistics. This difficult task is traded for the easier problem of
simulating a series of univariate CCPDs, while accounting for spatial dependence through sequential simulation (Deutsch and Journel, 1992). Multivariate CCPDs can be decomposed as follows:

\[
\text{Pr}\{Z(\mathbf{u}_1) \leq z_1, Z(\mathbf{u}_2) \leq z_2, Z(\mathbf{u}_3) \leq z_3, \ldots, Z(\mathbf{u}_N) \leq z_N \mid n\} = \text{Pr}\{Z(\mathbf{u}_1) \leq z_1 \mid n\} \\
\cdot \text{Pr}\{Z(\mathbf{u}_2) \leq z_2 \mid n, Z(\mathbf{u}_1) \leq z_1\} \cdot \text{Pr}\{Z(\mathbf{u}_3) \leq z_3 \mid n, Z(\mathbf{u}_1) \leq z_1, Z(\mathbf{u}_2) \leq z_2\} \cdot \ldots \\
\cdot \text{Pr}\{Z(\mathbf{u}_N) \leq z_N \mid n, Z(\mathbf{u}_1) \leq z_1, \ldots, Z(\mathbf{u}_{N-1})\} \leq z_{N-1}\}
\]

where ‘\(Z(\mathbf{u}_i)\)’ are the ‘\(N\)’ unknown variables to be estimated, and ‘\(n\)’ is the available conditioning data.

Equation 5-2 rewrites a multivariate joint CCPD in terms of elementary CCPDs involving the simulation of only one random variable at a time. The spatial dependence of the random variables \(Z(\mathbf{u}_i)\) is accounted for by using previously simulated variables to condition the new ones.

This approach assumes that we know how to calculate the conditional univariate CCPDs. In geostatistics, there are two main avenues used to solve this problem: two-point statistics-based algorithms, or multiple-point statistics-based algorithms.

In this chapter, we use these two geostatistical approaches to develop models of carbonate buildups in the Sacramento Mountains. In section 5.2, we use the sequential indicator simulation (SISIM) technique, which is a two-point statistics-based algorithm, to model the structures of these carbonate buildups. In section 5.3, we use the Single Normal Equation Simulation (SNESIM) algorithm, which is a multiple-point statistics-based algorithm, to perform the same modeling task.
The two-point and multiple-point statistics are extracted from the 3D model developed in chapter 3. This model, known as the training image, only contains the non-location-specific geological information of these buildups.

The training image is divided into 3 regions based on their specific geological structures. Table 5-1 summarizes the principal characteristics of this model. Figure 5-1 reproduces the training image and its corresponding fence diagram.

Table 5-1: Main parameters of the 3D model used as training image for carbonate buildups in the Sacramento Mountains.

<table>
<thead>
<tr>
<th>Region 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Cells</strong></td>
<td>108 x 216 x 3 = 69,984</td>
</tr>
<tr>
<td><strong>Cell Dimensions</strong></td>
<td>(4 x 4 x 0.9) meters</td>
</tr>
<tr>
<td><strong>Facies Present</strong></td>
<td>Mound Core Facies, Background Facies</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Cells</strong></td>
<td>108 x 216 x 14 = 326,592</td>
</tr>
<tr>
<td><strong>Cell Dimensions</strong></td>
<td>(4 x 4 x 2.2) meters</td>
</tr>
<tr>
<td><strong>Facies Present</strong></td>
<td>Mound Core Facies, Debris Facies, Background Facies</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Cells</strong></td>
<td>108 x 242 x 16 = 418,176</td>
</tr>
<tr>
<td><strong>Cell Dimensions</strong></td>
<td>(4 x 4 x 1.6) meters</td>
</tr>
<tr>
<td><strong>Facies Present</strong></td>
<td>Mound Core Facies, Debris Facies, Background Facies</td>
</tr>
</tbody>
</table>
Figure 5-1: Training image in different views. Top image shows the training image in perspective view. Bottom image shows a fence diagram of the training image. In both images, brown- and yellow-colored structures represent the mound core and debris facies, respectively. In the fence diagram, the background is shaded blue.
5.2 Sequential Modeling Using Sequential Indicator Simulation (SISIM)

One of the most important steps in sequential simulation is the calculation of the single variable conditional probability distributions required by equation 5-2. A set of algorithms, collectively known as two-point statistics algorithms, has been successful in accomplishing this endeavor (Caers, 2005).

The sequential indicator simulation (SISIM) program has been widely used to simulate discrete variables (such as facies). The algorithm behind this technique is outlined in section 2.3.2 of this thesis.

The two-point statistics approach to performing simulations consists of conveying the spatial dependence between the single unknown \((Z(\mathbf{u}))\) and the conditioning data \((n)\), taken one at a time. These two-point statistics are represented in the form of a semivariogram (or a covariance). Some form of kriging is then used to perform the sequential simulation. The final model obtained is explicitly conditioned by first and second order moments, while it is implicitly conditioned to high order moments hidden in the chosen algorithm (Journel and Zhang, 2006).

In the case of the SISIM algorithm, simulation performance is accomplished by approximating the exact conditional probability of an indicator variable (eq. 2-3) by an equation that only includes the first two terms. Equation 5-3 shows this approximation.

\[
I^*_x(\mathbf{u}) = \text{Prob}^* \{ I(\mathbf{u}) = 1 \mid (n) \} = p_0 + \sum_{\alpha=1}^{n(u)} \lambda^{(1)}_{x\alpha}(\mathbf{u})[I(\mathbf{u}_\alpha) - p_0] \tag{5-3}
\]
The high-order moments implicitly built into most two-point statistics algorithms, including SISIM, lead to reproduction of high-entropy structures. High-entropy characteristics make it difficult to reproduce specific lower-entropy geological elements (entropy is a measure of statistical variability, which is in turn a measure of heterogeneity).

5.2.1 Semivariogram Modeling

The semivariogram is a statistical tool used to describe spatial relationships between unknown variables and conditioning data sets. The semivariogram captures the correlations between random variables ‘\( Z(\mathbf{u}) \)’ and ‘\( Z(\mathbf{u}+\mathbf{h}) \),’ separated by a distance ‘\( h \)’ within a region, deemed to have similar properties. The covariance is the plot of these relationships versus \( h \) (the lag distance). The semivariogram is then defined as follows:

\[
\gamma(h) = C(h) - C(0)
\]

where: \( C(h) = \text{E}[(Z(\mathbf{u}) - m)(Z(\mathbf{u}+h) - m)] \), \( C(0) = \text{Var}[Z(\mathbf{u})] \), and \( m = E[Z(\mathbf{u})] \)

In practice, an experimental semivariogram \( \gamma^*(h) \) is built based on available conditioning data or geological interpretation. The following equation is commonly used:

\[
\gamma^*(h) = \frac{1}{2n(h)} \sum_{n(h)} [z(\mathbf{u}_\alpha + h) - z(\mathbf{u}_\alpha)]^2
\]

where ‘\( z(\mathbf{u}_\alpha) \)’ are specific outcomes of the random variable ‘\( Z(\mathbf{u}_\alpha) \),’ and ‘\( n(h) \)’ are the number of pairs available at approximate lag distance ‘\( h \).’
A semivariogram model $\gamma(h)$ is then created by interpolating and extrapolating the experimental semivariogram.

In this section, the above approach is used to model indicator semivariograms from the training image. The indicator semivariograms correspond to the previously defined composite facies. Following the SISIM algorithm in section 2.3.2, we indicator-code the mound core, debris, and background facies, as follows:

$$i(u) = [i(u, \text{mound core}), i(u, \text{debris}), i(u, \text{background})].$$

We then establish the spatial relationships for each facies by building individual facies semivariograms for each region.

Figure 5-2 shows horizontal indicator semivariograms characterizing the mound core facies in region 1 (the debris facies is not present). Figure 5-3 shows horizontal indicator semivariograms individually characterizing the mound core and debris facies in region 2. Figure 5-4 shows horizontal indicator semivariograms individually characterizing the mound core and debris facies in region 3. The geological structures in the training image are better correlated in the horizontal plane; hence, the horizontal semivariograms are the most important ones. The vertical indicator semivariograms for the three facies in each region, as well as the horizontal indicator semivariogram for the background facies in each region, are shown in appendix C of this thesis.

Table 5-2 gives the specific 3D indicator semivariogram models used to individually characterize the mound core and debris facies in each region.
Figure 5-2: Horizontal indicator semivariogram for the mound core facies in region 1. The red marks are points from the experimental semivariogram. Lag distances are in meters. Indicator semivariogram values are dimensionless.
Figure 5-3: Horizontal indicator semivariograms for mound core (top image) and debris (bottom image) facies in region 2. The red marks are points from the experimental semivariograms. Lag distances are in meters. Indicator semivariogram values are dimensionless.
Figure 5-4: Horizontal indicator semivariograms for mound core (top image) and debris (bottom image) facies in region 3. The red marks are points from the experimental semivariograms. Lag distances are in meters. Indicator semivariogram values are dimensionless.
Table 5-2: Indicator semivariograms used to model spatial correlations of facies in geological regions of the training image. The correlation lengths are in meters.

<table>
<thead>
<tr>
<th>Facies</th>
<th>Semivariogram Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Region 1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Mound Core</strong></td>
<td>( \gamma(h_x, h_y, h_z) = 0.001 + 0.007 \cdot Sph\left(\sqrt{\left(\frac{h_x}{8}\right)^2 + \left(\frac{h_y}{8}\right)^2 + \left(\frac{h_z}{2}\right)^2}\right) )</td>
</tr>
</tbody>
</table>

| **Region 2** | |
| **Mound Core** | \( \gamma(h_x, h_y, h_z) = 0.001 + 0.08 \cdot \text{Exp}\left(\sqrt{\left(\frac{h_x}{18}\right)^2 + \left(\frac{h_y}{18}\right)^2 + \left(\frac{h_z}{10}\right)^2}\right) \) 

+ 0.04 \cdot \text{Gau}\left(\sqrt{\left(\frac{h_x}{1000}\right)^2 + \left(\frac{h_y}{1000}\right)^2 + \left(\frac{h_z}{20}\right)^2}\right) |

| **Debris** | \( \gamma(h_x, h_y, h_z) = 0.001 + 0.05 \cdot \text{Exp}\left(\sqrt{\left(\frac{h_x}{18}\right)^2 + \left(\frac{h_y}{18}\right)^2 + \left(\frac{h_z}{10}\right)^2}\right) \) 

+ 0.01 \cdot \text{Exp}\left(\sqrt{\left(\frac{h_x}{200}\right)^2 + \left(\frac{h_y}{200}\right)^2 + \left(\frac{h_z}{10}\right)^2}\right) |

| **Region 3** | |
| **Mound Core** | \( \gamma(h_x, h_y, h_z) = 0.038 \cdot \text{Exp}\left(\sqrt{\left(\frac{h_x}{20}\right)^2 + \left(\frac{h_y}{20}\right)^2 + \left(\frac{h_z}{15}\right)^2}\right) \) 

+ 0.062 \cdot \text{Gau}\left(\sqrt{\left(\frac{h_x}{1000}\right)^2 + \left(\frac{h_y}{1000}\right)^2 + \left(\frac{h_z}{10}\right)^2}\right) |

| **Debris** | \( \gamma(h_x, h_y, h_z) = 0.056 \cdot \text{Exp}\left(\sqrt{\left(\frac{h_x}{20}\right)^2 + \left(\frac{h_y}{20}\right)^2 + \left(\frac{h_z}{8}\right)^2}\right) \) 

+ 0.35 \cdot \text{Sph}\left(\sqrt{\left(\frac{h_x}{10000}\right)^2 + \left(\frac{h_y}{10000}\right)^2 + \left(\frac{h_z}{8}\right)^2}\right) |
where, \( Sph(h) = \begin{cases} \frac{3}{2} \frac{\|h\|}{a} - \frac{1}{2} \left( \frac{\|h\|}{a} \right)^3 & \text{if } \|h\| \leq a, \\ 1 & \text{otherwise} \end{cases} \)

\[
Exp(h) = 1 - e^{-\frac{3\|h\|}{a}}, \quad \text{and} \quad Gau(h) = 1 - e^{-\frac{3\|h\|^2}{a^2}}.
\]

where, in turn, ‘a’ is the correlation length in ‘Sph(h),’ and the practical correlation length in ‘Exp(h)’ and ‘Gau(h).’ Exponential semivariograms are used to characterize poorly correlated subsurface medium. On the other hand, spherical and Gaussian semivariograms indicate high-correlation between geo-structures. To model the architecture and heterogeneity exhibited in the training image, all semivariogram types were necessary.

5.2.2 Input Parameters

Besides the indicator facies semivariograms, a set of parameters needs to be selected to perform the sequential simulations using SISIM. Figure 5-5 shows a parameter file for the SISIM program coded in the GSLIB Software (Deutsch and Journel, 1992).

Table 5-3 summarizes the values assigned to the most important parameters that were used to perform the simulations in each of the three regions. A total of 12 parameters were specified to perform each simulation. Deutsch and Journel carefully explained the effects of each parameter on the simulations (1992).
### Parameters for SISIM\_GS

**START OF PARAMETERS:**

1  \( l=\text{continuous(cdf)}, 0=\text{categorical(pdf)} \)
5  \( \text{number thresholds/categories} \)
0.5 1.0 2.5 5.0 10.0 \( \text{thresholds / categories} \)
0.12 0.29 0.50 0.74 0.88 \( \text{global cdf / pdf} \)
cluster.dat \( \text{file with data} \)
1 2 0 3 \( \text{columns for X,Y,Z, and variable} \)
ydata.dat \( \text{file with gridded soft indicator input} \)
4 \( \text{columns for secondary variable} \)
bicalib.cal \( \text{file with calibration table} \)
-0.5 1.0e21 \( \text{trimming limits} \)
0.0 30.0 \( \text{minimum and maximum data value} \)
1 1.0 \( \text{lower tail option and parameter} \)
1 1.0 \( \text{middle option and parameter} \)
1 1.0 \( \text{upper tail option and parameter} \)
cluster.dat \( \text{file with tabulated values} \)
3 5 \( \text{columns for variable, weight} \)
3 \( \text{debugging level: 0,1,2,3} \)
SISIM\_gs0.dbg \( \text{file for debugging output} \)
SISIM\_gs.out \( \text{file for simulation output} \)
1 \( \text{number of realizations} \)
50 0.5 1.0 \( \text{nx,xmn,xsiz} \)
50 0.5 1.0 \( \text{ny,ymn,ysiz} \)
1 0.5 1.0 \( \text{nz,zmn,zsiz} \)
69069 \( \text{random number seed} \)
12 \( \text{maximum original data for each kriging} \)
12 \( \text{maximum previous nodes for each kriging} \)
12 \( \text{maximum soft indicator nodes for kriging} \)
0 \( \text{assign data to nodes? (0=no,1=yes)} \)
1 3 \( \text{multiple grid search? (0=no,1=yes),num} \)
0 \( \text{maximum per octant (0=not used)} \)
10.0 10.0 10.0 \( \text{maximum search radii} \)
0.0 0.0 0.0 \( \text{angles for search ellipsoid} \)
1 0.15 \( \text{One nst, nugget effect} \)
1 0.85 0.0 0.0 0.0 \( \text{it,cc,ang1,ang2,ang3} \)
10.0 10.0 10.0 \( \text{a_hmax, a_hmin, a_vert} \)
1 0.10 \( \text{Two nst, nugget effect} \)
1 0.90 0.0 0.0 0.0 \( \text{it,cc,ang1,ang2,ang3} \)
10.0 10.0 10.0 \( \text{a_hmax, a_hmin, a_vert} \)

Figure 5-5: Typical SISIM input parameter file.
Table 5-3: Values assigned to additional parameters in SISIM. Categories 1, 2, and 3 correspond to the background, debris, and mound core facies.

<table>
<thead>
<tr>
<th>Region 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
<td>20 x 20 x 3 meters</td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
<td>20 values</td>
</tr>
<tr>
<td><strong>Lower Tail Extrapolation</strong></td>
<td>Power law model with omega equals 3</td>
</tr>
<tr>
<td><strong>Upper Tail Extrapolation</strong></td>
<td>Power law model with omega equals 0.333</td>
</tr>
<tr>
<td><strong>Number of Categories and Category Proportions</strong></td>
<td>2 categories. The proportions of categories 1 and 3 are 98 and 2%, respectively.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
<td>80 x 80 x 10 meters</td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
<td>60 values</td>
</tr>
<tr>
<td><strong>Lower Tail Extrapolation</strong></td>
<td>Power law model with omega equals 3</td>
</tr>
<tr>
<td><strong>Upper Tail Extrapolation</strong></td>
<td>Power law model with omega equals 0.333</td>
</tr>
<tr>
<td><strong>Number of Categories and Category Proportions</strong></td>
<td>3 categories. The proportions of categories 1, 2, and 3 are 86, 6, and 8%, respectively.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
<td>60 x 60 x 8 meters</td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
<td>80 values</td>
</tr>
<tr>
<td><strong>Lower Tail Extrapolation</strong></td>
<td>Power law model with omega equals 3</td>
</tr>
<tr>
<td><strong>Upper Tail Extrapolation</strong></td>
<td>Power law model with omega equals 0.333</td>
</tr>
<tr>
<td><strong>Number of Categories and Category Proportions</strong></td>
<td>3 categories. The proportions of categories 1, 2, and 3 are 44, 48, and 8%, respectively.</td>
</tr>
</tbody>
</table>
5.2.3 Two-Point Statistics-Based Stochastic Realizations

Based on the indicator facies semivariograms and the corresponding set of additional parameters, two-point statistic-based realizations that model the facies interactions within each region were generated. In this section, we present these stochastic models and briefly compare them against the training image. The training image is assumed to be the true reference.

Figure 5-6 shows region 1 of the training image, which is the true reference. This figure also shows a corresponding unconditioned SISIM realization. By visually comparing these two images, we can conclude that the SISIM realization successfully reproduces the simple structural elements of region 1.

Figure 5-7 shows region 2 of the training image, which is the true reference. This figure also shows a corresponding unconditioned SISIM realization. By visually comparing these two images, we can conclude that the SISIM realization completely fails to capture the curvilinear features and architectures of region 2. Only the facies proportions are honored, this is further discussed in chapter 6.
Figure 5-6: Training image and SISIM realization of geological region 1. Top image shows region 1 of the training image (true reference). Bottom image shows a corresponding SISIM realization. In both images, brown-colored structures represent the mound core facies.

Figure 5-8 shows region 3 of the training image, which is the true reference. It also shows a corresponding unconditioned SISIM realization. By visually comparing these two images, we can conclude that the SISIM realization does not reproduce the important features of the geology in region 3. Only the facies proportions are honored; this is discussed further in chapter 6.
Figure 5-7: Training image and SISIM realization of geological region 2. Top image shows region 2 of the training image (true reference). Bottom image shows a corresponding SISIM realization. In both images, brown- and yellow-colored structures represent the mound core and debris facies.
Figure 5-8: Training image and SISIM realization of geological region 3. Top image shows region 3 of the training image (true reference). Bottom image shows a corresponding SISIM realization. In both images, brown- and yellow-colored structures represent the mound core and debris facies.
5.3 Sequential Modeling Using Single Normal Equation Simulation (SNESIM)

An alternative to computing the univariate CCPDs, required by equation 5-2, is to use multiple-point statistics-based algorithms.

The single normal equation simulation (SNESIM) program is the most widely used multiple-point statistics (MPS) algorithm to simulate discrete variables (such as facies) (Caers, 2005). The algorithm behind this technique is given in section 2.3.4 of this report.

The main idea of MPS algorithms consists of conforming models to spatial relationships conveyed by sets of ‘N’ data considered jointly; where ‘N’ is the number of nodes in the multiple-point template (chapter 2). This information is retrieved from training images (chapter 3). These patterns can then optionally be combined with conditioning data to produce the final stochastic models.

5.3.1 Input Parameters

In this work, the SNESIM program, coded in the Stanford geostatistical modeling software (SGEMS), is used to generate the MPS geostatistical models. Figure 5-9 shows the user interface for the SNESIM program. It contains four main sections: general, conditioning, rotation/affinity, and advanced.
Figure 5-9: SNESIM graphical user interface. The 4 main sections (general, conditioning, affinity/rotation, and advanced) are displayed.
Besides the geological information provided by the training image, a set of parameters needs to be selected to carry out unconditioned simulations within SNESIM. Table 5-4 summarizes the values assigned to these parameters. Strebelle explained the roles of all parameters in the SNESIM algorithm (2000).

Table 5-4: Values assigned to parameters used in SNESIM. Categories 1, 2, and 3 correspond, respectively, to the background, debris, and mound core facies.

<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
<td>20 x 20 x 3 meters</td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
<td>40 values</td>
</tr>
<tr>
<td><strong>Multigrid Simulation and Number of Grids</strong></td>
<td>Yes, 2 grids.</td>
</tr>
<tr>
<td><strong>Subgrid Simulation and Number of Subgrids</strong></td>
<td>Yes, 4 grids.</td>
</tr>
<tr>
<td><strong>Servosystem Factor</strong></td>
<td>0</td>
</tr>
<tr>
<td><strong>Affinity/Rotation</strong></td>
<td>none</td>
</tr>
<tr>
<td><strong>Resimulation of Nodes and Resimulation Threshold</strong></td>
<td>no</td>
</tr>
<tr>
<td><strong>Number of Categories and Category Proportions</strong></td>
<td>2 categories. The proportions of categories 1 and 3 are 98 and 2%, respectively.</td>
</tr>
<tr>
<td>Resimulation of Nodes and Resimulation Threshold</td>
<td>Yes, if simulated with less than 5 conditioning nodes (threshold).</td>
</tr>
<tr>
<td>Number of Categories and Category Proportions</td>
<td>3 categories. The proportions of categories 1, 2, and 3 are 86, 6, and 8%, respectively.</td>
</tr>
</tbody>
</table>

**Region 3**

| Search Ellipsoid | 100 x 100 x 8 meters |
| Maximum Conditioning Data | 100 values |
| Multigrid Simulation and Number of Grids | Yes, 4 grids. |
| Subgrid Simulation and Number of Subgrids | Yes, 4 grids. |
| Servosystem factor | 0 |
| Affinity/Rotation | None |
| Resimulation of Nodes and Resimulation Threshold | Yes, if simulated with less than 10 conditioning nodes (threshold). |
| Number of Categories and Category Proportions | 3 categories. The proportions of categories 1, 2, and 3 are 44, 48 and 8%, respectively. |

5.3.2 Multiple-Point Statistics-Based Stochastic Realizations

Based on the geological information from the training image and the corresponding set of selected parameters, MPS models were independently generated for each region. The training image is again assumed to be the true reference.

Figure 5-10 shows region 1 of the training image (true reference) at the top and a corresponding MPS realization at the bottom. By visually comparing these two images, we can conclude that the MPS-based geostatistical model reproduces the geological
architecture of region 1. Moreover, the facies proportions are honored. Facies histograms that display this information are presented in section 6.2.

This MPS realization was post-processed by convoluting it against the 2D kernel matrix in figure 5-11. This technique is proposed as an alternative to the moving window average tool, typically used in MPS algorithms to improve facies continuity. During post-processing, only small modifications of the geological patterns of the MPS model were allowed. This restriction was accomplished by storing the structural patterns in a multiple-point histogram and allowing overall changes of less than 1%. This process is further discussed in chapter 6.
Figure 5-10: Training image and SNESIM realization of geological region 1. Top image shows region 1 of the training image (true reference). Bottom image shows a corresponding SNESIM realization. In both images, brown-colored structures represent the mound core facies.

Figure 5-11: 2D kernel matrix used to post-process SNESIM model of region 1.

Figure 5-12 shows region 2 of the training image (true reference) at the top and a corresponding MPS realization at the bottom. By visually comparing these two images, we can conclude that they have similar geological architecture and geobodies. Therefore,
based on this criterion, the MPS model performs better than its two-point statistics counterpart.

This MPS realization was post-processed by convoluting it against the kernel matrix in figure 5-13. Only small modifications of the geological patterns in the MPS model were allowed.

Figure 5-12: Training image and SNESIM realization of geological region 2. Top image shows region 2 of the training image (true reference). Bottom image shows a corresponding SNESIM realization. In both images, brown- and yellow-colored structures represent the mound core and debris facies.
The MPS realizations could not simultaneously reproduce the dome-like structures and the large-scale architecture. Based on the geological information of region 3 of the training image, the domes were simulated one at a time. They were then placed randomly with the constraint that they do not overlap. Figure 5-14 shows region 3 of the training image (true reference) on top, and a corresponding MPS realization below. The structures formed by the debris facies are displayed on the left; those formed by the mound core facies are shown on the right. In region 3, the mound core facies is overlaid by the debris facies. Based on visual comparison between the training image and the MPS model, we conclude that the MPS model better reproduces the geology of region 3 than does its two-point statistics counterpart. The kernel matrix in figure 5-13 was also used to post-process the SNESIM model.
Figure 5-14: Training image and SNESIM realization of geological region 3. Top left and right images respectively show the distribution of the debris and mound core facies in region 3 of the training image (true reference). Bottom left and right images show the same information of a corresponding SNESIM realization. In both images, brown- and yellow-colored structures represent the mound core and debris facies.

Figure 5-15 reproduces the full training image, along with the full two-point statistics and MPS-based models. Strictly based on visual comparison against the true reference, the MPS model appears to better reproduce the curvilinear geobodies and complex architectures of this carbonate buildup. In chapter 6, we will further elaborate on the comparison of these two geostatistical models, using different statistical measures.
Figure 5-15: Training image, SISIM realization, and SNESIM realization. Top image shows the training image (true reference). The middle and bottom images respectively show the two-point and multiple-point statistics based models. Brown- and yellow-colored structures represent the mound core and debris facies.
Chapter 6: Determining the Quality of the Two- and Multiple-Point Statistics Based Models

6.1 Introduction

The main objective of geostatistics is to develop geologically realistic models of reservoirs that reflect their corresponding “uncertainties”. This task is performed by reproducing either geological patterns extracted from training images, or spatial correlations inferred from semivariograms. The final decision made concerning which method to use is reservoir-specific and, to some extent, subjective (Journel and Zhang, 2006).

This thesis specifically addresses the challenge of developing “accurate” stochastic models of carbonate buildups. In chapter 5, two-point and multiple-point statistics techniques were used to capture and reproduce the complex structures and geological patterns of a reef buildup. Basing our decision on visual comparison against the true reference, we concluded that the MPS model “better” reproduces the geology of the reefs.

Visual evaluation of 3D models can be very subjective, largely dependent on the judgments of geoscientists. Therefore, visual evaluation should not be used alone to determine the quality of 3D models (Caers, 2005). Nevertheless, it still remains the main way to assess the geological realism of geocellular models and should be a prerequisite to more quantified evaluations.
In this chapter, we propose a set of additional statistical measures that quantify heterogeneity, spatial correlation, and structural information, so that the quality of geostatistical models can be accurately determined.

Figure 6-1, a reproduction of figure 5-15 included here for convenience, shows the training image (TI), along with the two-point statistics (TPS) and multiple-point statistics (MPS)-based realizations.

Section 6.2 compares the TPS and MPS models against the training image (true reference), based on their spatial correlations and facies proportions. Spatial correlations are measured through semivariograms, and facies proportions are measured through facies histograms. Section 6.3 uses multiple-point (MP) histograms to compare structural information of the geostatistical models against that of the TI. Section 6.4 uses spatial entropy to quantify geological pattern heterogeneity of the models. Finally, section 6.5 uses flow-capacity-storage-capacity curves and Lorenz coefficients to assess the heterogeneity of the models in the context of flow performance.
Figure 6-1: Training image, SISIM realization, and SNESIM realization. Top image shows the training image (true reference). The middle and bottom images, respectively, show the two-point and multiple-point statistics-based models. Brown- and yellow-colored structures represent the mound core and debris facies.
6.2 Reproduction of Facies Proportions and Two-Point Spatial Structures

In this section, we evaluate the quality of the TPS and MPS models, by comparing them against the TI (true reference), in terms of facies proportions and two-point spatial structures. Facies histograms are used to display facies proportions; facies indicator semivariograms are used to measure two-point spatial correlations.

6.2.1 Facies Histograms

Facies histograms are visual displays of facies frequency distributions. Each facies category is represented as a vertical bar, with height proportional to occupied volume. This carbonate buildup is characterized by three geological regions (chapter 3). Figure 6-2 shows the facies histograms belonging to the TI, along with those corresponding to the TPS and MPS models of each region. The top, middle, and bottom images correspond to regions 1, 2, and 3, respectively.

The facies histograms are univariate statistics that are computed by considering each node in the 3D models individually. Both TPS and MPS models were calibrated to honor this statistical information, using target global proportions (chapter 4). Based on comparison against the facies distribution of the TI, we conclude that: both models perform well in region 1; the TPS model works slightly better in region 2; and the MPS model performs better in region 3, as the TPS model overestimates the mound core and
debris facies proportions. Based on this statistical measure, both models satisfactorily capture the geology of the carbonate buildup under consideration.

Figure 6-2: Top, middle, and bottom images respectively show the facies distributions of geological regions 1, 2, 3 of the TPS and MPS models, and the TI (true reference). Facies 1, 2, and 3 represent the mound core, debris, and background facies.
6.2.2 Indicator Semivariograms

Section 5.2.1 presented the idea of capturing two-point spatial information in the form of an experimental semivariogram. After we accomplished this task, we then constructed an analytical semivariogram that fit the experimental one. The analytical semivariogram model was then used to directly convey two-point spatial facies correlations to the TPS model. In the MPS model case, the two-point spatial information was indirectly conveyed by the multiple-point patterns that were extracted using a multiple-point template.

In this section, we compare the TPS and MPS models in terms of reproduction of two-point structural information. Horizontal indicator semivariograms for facies in each region are built and compared against those of the TI (true reference).

Figure 6-3 displays the first of these comparisons. Three horizontal indicator semivariograms are shown, which model the spatial distribution of the mound core facies in region 1. The green, red, and blue semivariograms respectively correspond to the TPS model, MPS model, and TI. While both geostatistical models closely honor the TI mound core facies semivariogram, the TPS model provides a more “accurate” match.
Figure 6-3: Horizontal indicator semivariograms capturing the spatial correlation of the mound core facies in region 1. The TPS model, MPS model, and TI are shaded green, red, and blue, respectively. Lag distances are in meters. Indicator semivariograms are dimensionless.

Figure 6-4 displays indicator semivariograms that correspond to the mound core and debris facies in region 2. The semivariogram coloring used in figure 6-3 is repeated here. By visually comparing the semivariograms, we can infer that the TPS model reproduces slightly “better” the two-point spatial correlations in both facies.

Figure 6-5 displays indicator semivariograms that correspond to the mound core and debris facies in region 3. The previous semivariogram coloring is repeated here. In region 3, we observe that the TPS semivariograms provide better matches than do the MPS counterparts, especially for the debris facies (bottom image).

Based on the reproduction of TI two-point spatial correlations, the TPS model is the superior alternative to model this reef buildup.
Figure 6-4: Horizontal indicator semivariograms capturing the spatial correlation of the mound core (top image) and debris (bottom image) facies in region 2. The TPS model, MPS model, and TI are shaded blue, green, and red, respectively. Lag distances are in meters. Indicator semivariograms are dimensionless.
Figure 6-5: Horizontal indicator semivariograms capturing the spatial correlation of the mound core (top image) and debris (bottom image) facies in region 3. The TPS model, MPS model, and TI are shaded green, red, and blue, respectively. Lag distances are in meters. Indicator semivariograms are dimensionless.
6.3 Reproduction of Multiple-Point Spatial Structures

The carbonate buildup under consideration presents clearly defined geological architecture and is characterized by different heterogeneity types. According to Strebelle (2000), for complex cases such as this, two-point structural information alone is not sufficient to capture and reproduce geological features. Multiple-point structural correlations are needed. In this section, we evaluate the quality of TPS and MPS models, based on reproduction of the most important multiple-point structural patterns from the training image (true reference).

6.3.1 Truncated Multiple-Point Histograms

Multiple-point (MP) histograms contain structural patterns extracted from training images using multiple-point templates. In MP histograms, each bar represents a multiple-point pattern with its height proportional to the frequency with which the pattern was found in the TI. MP histograms normally contain thousands of patterns, and thus cannot be used to display pattern-information effectively. Truncated multiple-point (TMP) histograms are used instead. These are MP histograms that display only the most relevant multiple-point patterns (those with the higher frequencies) (Eskandari, 2008).

Figure 6-6 shows TMP histograms corresponding, from top to bottom, to regions 1, 2, and 3. Blue bars represent multiple-point patterns present in the TI. The height of each bar is proportional to the number of times its matching pattern was found. This figure was generated by first plotting the larger twenty frequency patterns from the TI. These most common patterns, often deemed to carry the most important geological information, were
ordered, from left to right, based on the number of background facies nodes. That is, the leftmost pattern consists of all background facies nodes, while the rightmost pattern has the fewer. These patterns are named indexed patterns in figure 6-6. The TI patterns were then individually searched in the MPS and TPS models. When a pattern was found in either the MPS or the TPS model, green or red bars were respectively added alongside their TI analogs. The heights of the green and red bars are, respectively, proportional to the number of times the TI patterns were observed in the MPS and TPS models.

In all the regions, the MP patterns observed in the MPS model “better” match the ones present in the TI. In region 1, where the mound cores facies forms simple mounded structures and the debris facies is not present, the MPS model provides a somewhat “closer” match to the TI patterns than does the TPS model. As the facies interactions become more complex and the geobodies more defined, the MPS model clearly captures the complexity and patterns “better.” In regions 2 and 3, the geological patterns present in the TI are reproduced, in similar proportions, in the MPS model; while most of them are not even observed in the TPS model. In agreement with Strebelle (2000), these findings suggest that the power of MPS-based modeling increases as we model more complex geological features.
Figure 6-6: Truncated multiple-point histograms. Top, middle, and bottom images show truncated multiple-point histograms corresponding to regions 1, 2, and 3. Blue, green, and red bars correspond to multiple-point patterns from the TI, MPS model, and TPS model. Indexed patterns are multiple-point patterns ordered, from left to right, based on decreasing number of background facies nodes.
6.4 Reproduction of Spatial Heterogeneity

One of the goals of geostatistics is to build models that realistically recreate geological heterogeneities. In practice, this task is very challenging because of limited access to subsurface information.

In this section, we determine the quality of the TPS and MPS models by comparing their levels of heterogeneity against that of the TI (true reference). As there is no superior theoretical measure of heterogeneity, we present different alternatives.

6.4.1 Entropy of Multiple-Point Histograms

Entropy is usually defined as a measure of uncertainty in a probabilistic model. It is also considered to be a gauge of information content (Christakos, 1990). In this section, we use entropy to quantify the variation among the multiple-point patterns (medium-scale heterogeneity). This is an extension of the bivariate entropy concept proposed by Journel and Deustch (1993) to measure geological heterogeneity. The TPS and MPS models are then compared against the TI based on their heterogeneity levels (entropies of multiple-point histograms).

For discrete random variables, entropy “H” is defined as follows:

\[ H = - \sum_{k=1}^{K} (p_k \ln(p_k)) \]  

(6-1)

where, ‘K’ is the number of discrete classes or categories, and ‘p_k’ is the probability of occurrence of class ‘k.’
We use equation 6-1 to calculate the entropy of the multiple-point patterns from the truncated multiple-point (TMP) histograms, with ‘\(p_k\)’ defined as follows:

\[
p_k = \frac{\text{Frequency of 'k' MP pattern}}{\text{Total number of MP patterns}}
\] (6-2)

Figure 6-7 shows, from top to bottom, the truncated multiple-point histograms of the TI, MPS model, and TPS model for region 1. The MP patterns were again ordered, from left to right, based on decreasing number of background facies nodes. Figures 6-8 and 6-9 display the same information for regions 2 and 3.

Based on their entropies, regions 1 and 3 are, respectively, the least and most heterogeneous regions. This finding agrees well with observations made in section 6.3. Region 1 has the simplest structures (only small mounds). Region 3 has the most complex geological features (dome-like structures overlaying mounded shapes). Moreover, the entropies of the MPS model, compared to the entropies of the TPS model, are significantly closer to the entropies of the TI. TPS algorithms automatically borrow high-order moments from Gaussian distributions that maximize entropy (spatial disorder) beyond the two-point structural information from semivariograms. This basis yields incorrectly large entropy values in the TPS model (Journel, 1986). On the other hand, MPS algorithms use multiple-point spatial correlations to accurately reproduce heterogeneity levels (entropies of MP histograms). Based on this heterogeneity measure, the MPS model ‘better’ captures the geological features of this algal buildup.
Figure 6-7: Entropies of TMP histograms of region 1. Top, middle, and bottom images display TMP histograms that depict the most relevant MP patterns of the TI, MPS model, and TPS model for region 1. The height of the bars is proportional to the frequency of each pattern. The entropy of each histogram is given in the top right corner.
Figure 6-8: Entropies of TMP histograms of region 2. Top, middle, and bottom images display TMP histograms that depict the most relevant MP patterns of the TI, MPS model, and TPS model for region 2. The height of the bars is proportional to the frequency of each pattern. The entropy of each histogram is given in the top right corner.
Figure 6-9: Entropies of TMP histograms of region 3. Top, middle, and bottom images display TMP histograms that depict the most relevant MP patterns of the TI, MPS model, and TPS model for region 3. The height of the bars is proportional to the frequency of each pattern. The entropy of each histogram is given in the top right corner.
6.4.2 Flow-Capacity-Storage-Capacity Curves and Lorenz Coefficients

In this section, we use flow-capacity-storage-capacity (FC) curves and Lorenz coefficients \(L_{nw}\) to compare the TPS and MPS models against the TI. FC curves and their corresponding \(L_{nw}\) coefficients provide measures of heterogeneity using the petrophysical properties of permeability and porosity.

In this technique, reservoirs are considered to be uniform and heterogeneous; hence, they can be represented as layer-cake models (Lake, 1989). Each of the layers has its own porosity and permeability values. The main idea consists of arranging the layers based on the ratio of permeability to porosity \(\frac{k_l}{\phi_l}\) in decreasing order. As this ratio is also proportional to interstitial fluid velocity, we can define the following partial sums to calculate cumulative flow and storage capacities (Lake, 1989), as follows:

\[
F_n = \frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{N_L} k_i h_i} \quad \text{and} \quad C_n = \frac{\sum_{i=1}^{n} \phi_i h_i}{\sum_{i=1}^{N_L} \phi_i h_i}
\]

whereby ‘\(k_i\)’ and ‘\(\phi_i\)’ are layer ‘\(L\)’ permeability and porosity, ‘\(N_L\)’ is the total number of layers, and \(1 \leq n \leq N_L\).

The plot of cumulative flow capacities versus cumulative storage capacities is known as the Lorenz curve. The Lorenz curve depicts the fraction of total flow volume associated with each fraction of total pore volume (Jensen et al., 1997). From this plot, the Lorenz coefficient, a static measure of heterogeneity, is defined as follows:
\[ L_w = 2 \left\{ \int_0^1 F \, dC - \frac{1}{2} \right\} \]  

where \( \int_0^1 F \, dC \) is the area under the Lorenz curve. In this work, the following approximation, based on the trapezoidal integration rule, is used to compute the Lorenz coefficient (Lake and Jensen, 1991):

\[ L_w \approx \frac{1}{2N_L} \sum_{i=1}^{N_L} k_i \sum_{j=1}^{n} \left| k_j - k_i \right| \frac{\phi_i}{\phi_j} \]  

where \( k_i \), \( \phi_i \), and \( N_L \) are defined as in equation 6-3.

Petrophysical information about the carbonate buildup under study was not available. Fortunately, extensive petrophysical data have been tabulated and analyzed for an analog buildup, known as the Reinecke Field in West Texas (Saller et al., 2004). The Reinecke Field and the carbonate formation under analysis have similar geological architecture and lithological facies. Porosity and horizontal permeability information was borrowed from the Reinecke Field to build the FC curves for the TI, MPS model, and TPS model.

Table 6-1 displays average porosity and permeability values for each facies. The distribution of petrophysical properties within facies was not available; hence, the variability of petrophysical properties within facies is not considered. The large differences in the horizontal permeability values across facies make it important to reproduce the spatial distribution of the mound core and debris facies.
Table 6-1: Average porosity and horizontal permeability values from the Reinecke Field, assumed for the carbonate buildup under study (from Saller et al., 2004).

<table>
<thead>
<tr>
<th>Facies</th>
<th>Porosity (%)</th>
<th>Horizontal Permeability (mD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background</td>
<td>1.4</td>
<td>0.79</td>
</tr>
<tr>
<td>Mound Core</td>
<td>12</td>
<td>690.3</td>
</tr>
<tr>
<td>Debris</td>
<td>11.2</td>
<td>196.4</td>
</tr>
</tbody>
</table>

In the TI, MPS model, and TPS model, every node was assigned porosity and horizontal permeability values conditioned to their facies. Using arithmetic averages for porosities, and arithmetic and geometric averages for permeabilities, we calculated equivalent porosities and horizontal permeabilities of the corresponding 1D model, parallel to the z-axis. Using equations 6-3 and 6-5, we then built flow-capacity-storage-capacity curves and calculated their corresponding Lorenz coefficients for the TI, MPS model, and TPS model. The computer program, written in MatLab®, used to accomplish these tasks is given in appendix A.

Figure 6-10 shows FC curves for regions 1, 2, and 3. The FC curves are typical of very homogeneous reservoirs. This was determined to be an artifact of the extensive averaging of porosity and horizontal permeability required to generate the 1D model, parallel to the z-axis. To correct this artifact, we built an alternative 1D model, parallel to the x-axis. This new 1D model requires less averaging of porosity and permeability to find their equivalent values. Figure 6-11 shows the FC curves generated based on this 1D model.
Table 6-2 displays the Lorenz coefficients corresponding to the FC curves in figure 6-10. By reviewing the FC curves from figures 6-10 and 6-11, we can conclude the following:

- Regions 1 and 3 are the least and most heterogeneous regions, respectively.
- Only the MPS model accurately captures and reproduces the heterogeneity of the TI.
- The TPS model is ‘homogeneously heterogeneous.’ It lacks the valuable structural information beyond the two-point semivariogram correlations.
Figure 6-10: FC curves corresponding to the geological regions. Top, middle, and bottom images display the FC curves of regions 1, 2, and 3. The green, black, red, and blue curves correspond to a 45° line, the TI, TPS model, and MPS model.
Figure 6-11: Alternative FC curves corresponding to the geological regions. Top, middle, and bottom images display the FC curves of regions 1, 2, and 3. The green, black, red, and blue curves correspond to a 45° line, the TI, TPS model, and MPS model.
Table 6-2: Lorenz coefficients corresponding to the FC curves shown in figure 6-10.

<table>
<thead>
<tr>
<th>Lorenz Coefficients</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training Image</td>
<td>0.012</td>
<td>0.016</td>
<td>0.67</td>
</tr>
<tr>
<td>MPS Model</td>
<td>0.014</td>
<td>0.101</td>
<td>0.69</td>
</tr>
<tr>
<td>TPS Model</td>
<td>0.023</td>
<td>0.108</td>
<td>0.129</td>
</tr>
</tbody>
</table>

The TPS model was deemed the “better” model based on reproduction of the TI two-point spatial structures (indicator semivariograms).

The MPS model was deemed the “better” model based on:

- Geological similarity to the TI (visual comparison)
- Reproduction of TI multiple-point spatial structures (truncated multiple-point histograms)
- Honoring the heterogeneity level of the TI (entropy of truncated multiple-point histogram)
- Reproduction of variability (FC curves and Lorenz coefficients)

From these results, we conclude that the MPS model, as compared to the TPS model, “better” recreates the heterogeneities and well-defined geological features of this carbonate buildup.
Chapter 7: Application of Multiple-Point Statistics to Model a Carbonate Mound Complex in the Sacramento Mountain

7.1 Introduction

Carbonate buildups are characterized by curvilinear geobodies and complex facies distribution, which are highly affected by diagenesis (Lucia, 1999). In this chapter, we present a workflow to stochastically model carbonate buildups. This workflow is used to create a 3D model of the carbonate mound complex discussed in chapter 2 that is located in the Sacramento Mountains.

In chapter 6, we used two main geostatistical avenues to create stochastic geocellular models of carbonate buildups. We found that the multiple-point statistics (MPS)-based model “better” reflected structural characteristics of the training image (true reference) built in chapter 3, as compared to the two-point statistics (TPS)-based model. The MPS model was deemed “superior” based on its geological similarity to the training image (TI), its reproduction of multiple-point spatial structures, and its reproduction of heterogeneity as measured by entropy and Lorenz coefficients. The TPS model was, nevertheless, found to better reproduce the two-point spatial structure of the TI.

Based on these results, we use the SNESIM program, an MPS-based algorithm, to model the entire carbonate mound complex. The training image from chapter 3 was designed to reflect the relevant geological features of this mound complex. Therefore, the non-location-specific geological structures will be borrowed from it.
In section 7.2, we present additional features of three carbonate mound sequences that comprise the carbonate mound complex under study. The conditioning data was retrieved from outcrops in the form of facies pseudo-wells. This work is discussed in section 7.3. Finally, in section 7.4, we present the workflow proposed to model this reef buildup, along with its corresponding stochastic model.

7.2 Geological Description of the Carbonate Mound Complex

The carbonate mound complex under study is located in the eastern section of the Sacramento Mountains, near the Orogrande Basin. It consists of three mound-growth intervals, or mound sequences, superimposed on each other (Janson et al., 2003). Detailed extrapolated maps of each mound sequence can be found in Janson (2005). The bottom and top horizons of the mound growth intervals are also identified in Janson’s same work. Figure 7-1 shows, from top to bottom, the surfaces corresponding to bottom horizons of mound sequences 1, 2, and 3. The top horizons of mound sequences 1 and 2 correspond to the bottom horizons of mound sequences 2 and 3. In figure 7-1, the three previously defined regions (chapter 3) are also roughly identified. These geological regions are present in the three mound sequences in different relative proportions and locations. We can observe that each mound bottom horizon is “fairly” horizontal.
Figure 7-1: Horizons of mound sequences. Top, middle, and bottom images, respectively, correspond to the bottom horizons of mound sequences 1, 2, and 3. The pink-, green-, and blue-colored sections represent geological regions 1, 2, and 3 (from Janson, 2005).
The width of this mound complex is uniform across the mound sequences and is approximately equal to 2000 m. Table 7-1 gives the length span of each geological regions within each mound growth interval.

Table 7-1: Length of geological regions within each mound sequence.

<table>
<thead>
<tr>
<th>Region</th>
<th>Mound Sequence 1 (m)</th>
<th>Mound Sequence 2 (m)</th>
<th>Mound Sequence 3 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1</td>
<td>2000</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Region 2</td>
<td>100</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>Region 3</td>
<td>1000</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Figure 7-2 reproduces, for convenience, the training image developed in chapter 3. The top image shows the TI in perspective view. The bottom image displays its inner structure using a fence diagram. This training image is composed of three regions that are characterized by different geological architectures. Region 1 has only small mounds; Region 2 has large mounds with a thin superimposed debris layer; Region 3 has medium-sized mounds with overlaying debris dome structures. Additional structural details are given in chapter 3.

The training image contains all relevant geo-structures present within the mound complex under study.
Figure 7-2: Training image in different views. Top and bottom images, respectively, show the training image viewed in perspective and through a fence diagram. In both, brown- and yellow-colored structures represent the mound core and debris facies. In the fence diagram, the background is shaded blue.

7.3 Processing Conditioning Data

As explained in chapter 2, this particular mound complex is intersected by three canyons (Yucca Canyon, Dry Canyon, and Beeman Canyon). These canyons and their surroundings have been imaged with LIght Detection And Ranging (LIDAR) technology. Figure 7-3 shows the final LIDAR image, obtained by the merge of high-resolution local pictures (Janson, 2005).
Figure 7-3: LIDAR image of carbonate buildups under study. LIDAR point clouds are colored gray. The shades of gray correspond to the intensity of the laser return. Vegetation has a low return intensity resulting in darker points, whereas limestone cliff has a stronger return resulting in lighter points. The algal buildups are found in the cliff located in the lower part of the outcrop (arrows).

Based on measured sections of the outcrop, facies pseudo-wells were constructed (Janson, 2005). Figure 7-4 shows these facies pseudo-wells. They follow the profile of the outcrops and are used as conditioning data to guide the MPS simulations in section 7.4. The top image shows the extensive LIDAR coverage that includes the canyons and all wells. The bottom image zooms into a particular section to better display the facies pseudo-well selection.
Figure 7-4: Facies pseudo-wells used as conditioning data of MPS simulations. Top and bottom images show, respectively, all and a few facies pseudo-wells (modified from Janson, 2005). Red-, blue-, and green-colored structures represent mound core, debris, and background facies.
7.4 Modeling of Carbonate Mound Sequences

The workflow used to model the carbonate mound complex is inspired by one suggested by Caers (2005). The workflow proposed in this thesis consists of five steps:

1. Identify overall architecture and geometry in terms of horizons.
2. Using the horizons defined in step 1, build a stratigraphic grid (a grid with irregular grid blocks that conform to strata and faults).
3. Build a Cartesian grid from the stratigraphic grid. All conditioning information (e.g., facies pseudo-wells) needs to be mapped into the Cartesian grid.
4. Use a corresponding stochastic algorithm (e.g., SNESIM) to simulate properties in the uninformed nodes of the Cartesian grid.
5. Map back the simulated nodes from the Cartesian grid into the stratigraphic grid.

The rest of this section explains how we implement these steps to model the carbonate buildup.

Janson (2005) identified the bottom and top horizons of each mound sequence. These horizons, which define the overall geometry and features of this mound complex, are shown in figure 7-1 (step 1). Using linear regression techniques, we found a quadratic regression model that fit each horizon. Linear regression minimizes the linear square error between the predicted values and the available data (Blume, 2005). Equations 7-1, 7-2, and 7-3 are the quadratic models for the bottom horizons of mound sequences 1, 2,
and 3 (step 2). The correlation coefficients between the quadratic models and their corresponding horizons are 0.95, 0.96, and 0.97, respectively.

\[ \hat{Z}_1(x, y) = 2419 - 0.179x - 0.289y - 1.521 \times 10^{-5}xy - 2.514 \times 10^{-5}x^2 + 9.63 \times 10^{-6}y^2 \]  
(7-1)

\[ \hat{Z}_2(x, y) = 2283 - 0.226x - 0.284y - 1.85 \times 10^{-5}xy - 3.091 \times 10^{-5}x^2 + 7.747 \times 10^{-6}y^2 \]  
(7-2)

\[ \hat{Z}_3(x, y) = 2382 - 0.140x - 0.244y - 6.616 \times 10^{-6}xy - 1.639 \times 10^{-5}x^2 + 1.001 \times 10^{-5}y^2 \]  
(7-3)

where \( x, y, \hat{Z}_1, \hat{Z}_2, \) and \( \hat{Z}_3 \) are in meters. \( \hat{Z}_1, \hat{Z}_2, \) and \( \hat{Z}_3 \) are the predicted altitudes of bottom horizons of mound sequences 1, 2, and 3, for given \( x \) and \( y \) coordinate values.

Based on Janson (2005), the approximate lengths of the geological regions in each mound sequence are summarized in table 7-1. The length information, along with the physical dimensions of each geological region (chapter 3), were used to build three Cartesian grids that correspond to the mound sequences. The Cartesian grids were placed as such that their southeast corner matches the origin. The conditioning data, the facies pseudo-wells, were mapped into the new Cartesian grids. This mapping was accomplished by subtracting the coordinates of the corresponding bottom horizon from the coordinates of each facies pseudo-well. For example, we obtained the Cartesian coordinates of the facies pseudo-wells conditioning mound sequence 1. We accomplished this by subtracting the coordinates of mound sequence 1 bottom horizon from the stratigraphic coordinates of the facies pseudo-wells (step 3). Figure 7-5 shows the conditioning data imported into the new Cartesian grids. Some facies pseudo-wells did not map inside the Cartesian grids. This is especially true for the Cartesian grid corresponding to mound sequence 3 (bottom
image). This resulted from the fact that those facies pseudo-wells were outside of the study area, as defined by the stratigraphic grids.

Figure 7-5: Conditioning facies pseudo-wells mapped into Cartesian grids. Top, middle, and bottom images, respectively, correspond to mound sequences 1, 2, and 3. Red-, blue-, and green-colored structures represent mound core, debris, and background facies.
The SNESIM program, an MPS-based algorithm, was used to simulate facies in uninformed nodes for each Cartesian grid corresponding to a mound sequence. Table 7-2 shows the values of important input parameters of SNESIM. The parameter values were selected depending on the region; however, they were kept constant across mound sequences. The geological information was extracted from the training image, shown in figure 7-2, according to the geological region (step 4). Figure 7-6 shows the MPS-based models developed for each mound sequence. Figure 7-7 shows the internal structure of the MPS-based models using fence diagrams.

Table 7-2: Values of important parameters input into SNESIM to simulate mound sequences.

<table>
<thead>
<tr>
<th>Region 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
</tr>
<tr>
<td><strong>Multigrid Simulation and Number of Grids</strong></td>
</tr>
<tr>
<td><strong>Subgrid Simulation and Number of Subgrids</strong></td>
</tr>
<tr>
<td><strong>Servosystem Factor</strong></td>
</tr>
<tr>
<td><strong>Affinity/Rotation</strong></td>
</tr>
<tr>
<td><strong>Resimulation of Nodes and Resimulation Threshold</strong></td>
</tr>
<tr>
<td><strong>Number of Categories and Category Proportions</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Region 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Search Ellipsoid</strong></td>
</tr>
<tr>
<td><strong>Maximum Conditioning Data</strong></td>
</tr>
<tr>
<td>Multigrid Simulation and Number of Grids</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>Subgrid Simulation and Number of Subgrids</td>
</tr>
<tr>
<td>Servosystem Factor</td>
</tr>
<tr>
<td>Affinity/Rotation</td>
</tr>
<tr>
<td>Resimulation of Nodes and Resimulation Threshold</td>
</tr>
<tr>
<td>Number of Categories and Category Proportions</td>
</tr>
</tbody>
</table>

**Region 3**

<table>
<thead>
<tr>
<th>Search Ellipsoid</th>
<th>100 x 100 x 10 meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Conditioning Data</td>
<td>100 values</td>
</tr>
<tr>
<td>Multigrid Simulation and Number of Grids</td>
<td>Yes, 4 grids</td>
</tr>
<tr>
<td>Subgrid Simulation and Number of Subgrids</td>
<td>Yes, 4 grids</td>
</tr>
<tr>
<td>Servosystem factor</td>
<td>0</td>
</tr>
<tr>
<td>Affinity/Rotation</td>
<td>None</td>
</tr>
<tr>
<td>Resimulation of Nodes and Resimulation Threshold</td>
<td>Yes, if simulated with less than 10 conditioning nodes (threshold).</td>
</tr>
<tr>
<td>Number of Categories and Category Proportions</td>
<td>Three categories. The proportions of categories 1, 2, and 3 are 44, 48 and 8%, respectively.</td>
</tr>
</tbody>
</table>

The final step in this modeling workflow consists of mapping the simulated facies from the Cartesian grids back into the stratigraphic grids. In cases characterized by multistage deformation and erosion, transformations that mimic the folding and faulting processes of the original structures are necessary. For simple structural cases, rotations and
displacements usually suffice (Caers, 2005). Given the simplicity of the bottom horizons of each mound sequence (no folding or faulting), a simple translation of the Cartesian grid was used to place them on top of the bottom horizons. Figure 7-8 shows the finalized MPS-based models that correspond to the sequences of this carbonate mound complex.

The MPS-based models, shown in figure 7-8, were deemed “satisfactory” in the sense that they allow the merging of conditioning information provided by the facies pseudo-wells and the complex structural patterns from the geological regions of the training image.

Figures 7-9, 7-10, and 7-11, draw comparisons between the MPS-based simulations and the studied outcrops. Figure 7-9 compares their most relevant architectural features from geological regions 2 and 3. Figure 7-10 contrasts the simulated dome-like structures with the ones observed in the outcrops. Finally, the modeled mounded structures are compared against those observed in an outcrop of geological region 2 in figure 7-11.
Figure 7-6: MPS models of mound sequences developed using SNESIM. Top, middle, and bottom models respectively correspond to mound sequences 1, 2, and 3. Brown- and yellow-colored structures stand for the mound core and debris facies.
Figure 7-7: Fence diagrams of MPS models from figure 7-6. Top, middle, and bottom fence diagrams, respectively, correspond to mound sequences 1, 2, and 3. Yellow- and green-colored structures stand for the mound core and debris facies. The background is shaded blue.
Figure 7-8: MPS-based models mapped back into the stratigraphic grid. Top, middle, and bottom images, respectively, show the MPS models and base horizons for mound sequences 1, 2, and 3. Brown- and yellow-colored structures stand for the mound core and debris facies.
Figure 7-9: Comparison between cross-sections from the MPS-based model and corresponding outcrop. The blue- and green-colored structures represent the mound core and debris facies, respectively.

Figure 7-10: Comparison between dome-like structures from the MPS-based model and its outcrop analog. The blue- and green-colored structures represent the mound core and debris facies, respectively.
Figure 7-11: Comparison between mounded objects typical of geological region 2. The top image shows the simulated mounds, while the bottom highlights the corresponding mounds from an outcrop.
Chapter 8: Conclusions and Future Work

The main objective of this thesis was to stochastically capture and reproduce architectural elements often present in carbonate buildups. Two main geostatistical algorithms, the two-point statistics-based SISIM and the multiple-point statistics-based SNESIM, were used to model the geobodies and structures of a typical carbonate mound complex. Conditioning information, in the form of facies pseudo-wells, was extracted from LIDAR images of outcrops located in tree canyons that intersect this carbonate formation (Janson et al., 2007; Bellian et al., 2008).

The first step of this work was to explicitly state the interpreted heterogeneity and spatial structures present in these carbonate buildups. This was accomplished by building a training image based on a geological conceptual model that represents the mound architecture and captures all relevant spatial variations.

Having considered the recent development of multiple-point statistics (MPS), we discussed advanced MPS tools valuable in modeling complex geobodies, in this case carbonate algal buildups. Semivariograms were computed and used to convey information about geological continuity to the two-point statistics (TPS)-based model. The MPS-based model extracted the same information directly from the training image using multiple-point templates. Both models were compared, based on numerical measures of heterogeneity, spatial correlation, and structural information, against the training image (true reference). Based on the results of these comparisons, the MPS-
based model was found to closely reproduce heterogeneity level, spatial correlation, and structural information. On the other hand, the TPS-based model was found to closely reproduce only the two-point spatial correlation. As a result, the MPS-based model was deemed “superior,” and hence, it was used to model the actual mound complex using the outcrop data as conditioning. The final MPS-based model of the carbonate buildup from the Sacramento Mountains was considered “successful” because it preserved the observed architectural elements (contained in the training image), and it honored the constraining facies in the measured sections.

8.1 Research Hypothesis

The following hypothesis was proposed and tested in this thesis:

Multiple-point statistics-based models, compared to two-point statistics-based models, “better” capture and reproduce curvilinear geobodies and architectures often present in carbonate buildups.

8.2 Conclusions

This research, on stochastic characterization of carbonate buildup architectures using two- and multiple-point statistics, leads to the following conclusions:

1. The TPS-based model failed to capture the mounded shapes and dome-like structures typically found in carbonate buildups. Instead, it was found to be geologically unrealistic, looking “homogeneously heterogeneous.” This claim is grounded on extensive visual analysis using fence diagrams.
2. Training images of carbonate buildups can be difficult to build, as they contain different and distinct geological features. However, in chapter 3, this process was automated by creating a set of algorithms to individually build each geo-structure and its related architecture (Appendix A).

3. The MPS-based model, as compared to the TPS-based model, “better” captured and reproduced common geological features of carbonate buildups. The “better” capture and reproduction claim is grounded, qualitatively, on “more accurate” reproduction of realistic geological features and, quantitatively, on the results of the previously presented set of numerical measures.

4. Many advanced MPS tools proved to be necessary to model carbonate buildups. In chapter 4, we explored their individual relevance in modeling buildup architectures. Among other benefits, the MPS tools assisted in capturing geo-structures and heterogeneity that had different “volume supports,” in imposing relative spatial facies distributions, and in reproducing curvilinear geobodies.

5. Visual evaluations are largely dependent on geoscientists’ judgments. This dependency can render the ranking of stochastic models somewhat subjective. The quality of stochastic models can be alternatively quantified using a set of statistical and static heterogeneity measures proposed in this work. Using these numerical measures, we compared both stochastic models in terms of reproduction of facies proportions, two-point and multiple-point structural information, and heterogeneity levels. Additionally, we used flow-capacity-
storage-capacity curves and Lorenz coefficients to draw a rough comparison between the models in terms of flow responses.

Ideally, any given geostatistical model should be evaluated qualitatively in terms of its geological realism and then quantitatively in terms of its ability to capture and reproduce desired heterogeneities.

6. Carbonate buildups observed in the outcrop were stochastically modeled using the workflow presented in chapter 7 of this thesis. This workflow represents a clear improvement over previous methods to model carbonate buildups (chapter 2).

7. Light Detection And Ranging (LIDAR) technology can be effectively used to retrieve quantitative information from outcrops. These measurements can then be used for conditioning the stochastic models (i.e., facies pseudo-wells). More precise 3D geological information can also be inferred from LIDAR images of outcrops by interactively analyzing them through computer programs, such as Polyworks® (as compared to visual analysis of standard pictures of outcrops).

8. The mound complex analyzed in this work had very different geological characteristics. As a result, we divided it into three geological regions. Geological region 1 was characterized by simple architectural elements; while geological regions 2 and 3 exhibited complex facies interactions and intricate geo-structures. Parallel modeling of both complex and simple geologies using TPS and MPS techniques suggested the following conclusion:

In general, there is no “better” stochastic simulation technique. The “better” model or technique should be case-specific. Instead, specific questions should be
considered, such as, “Is the reservoir characterized by definite geo-structures or the lack of them?” and “Are the geo-structures relevant to flow simulation?” For example, in geological region 1, the TPS and MPS models are very similar according to the used numerical measures. Given that considerable less effort was put into building the TPS model, it is more likely to be considered the “better” alternative. On the other hand, in geological regions 2 and 3, the MPS-based model is the only one that “satisfactorily” honors the assumed geology (according to the used numerical measures). Thus, it is more likely to be considered the preferred alternative.

8.3 Future Work

In addition to having explored the capabilities of geostatistical algorithms and LIDAR technology, we have also identified some of their limitations. We believe the following areas would benefit from academic research:

1. Extraction of conditioning data from LIDAR images of outcrops. In this work, facies pseudo-wells were retrieved at key locations from LIDAR images. These wells were then used to condition the MPS models. Some structural information was not considered (figures 7-3, 7-4) because it was not interpreted geologically. Geological interpretation would have been too time-consuming, and thus could not be carried out. Systematic algorithms that could be used to expedite geological interpretation of LIDAR data would be valuable.
2. The workflow proposed in chapter 7 needs to be extended to handle complex stratigraphies. A future extension would require the development of techniques that mimic the folding and faulting processes that affect the sedimentary deposits, such as compaction and tectonism.

3. Current MPS-based algorithms deliver useful tools to stochastic modeling, but at the cost of large increases in CPU time. For example, in geological region 3, the dome-like structures with the underlying mounded geobodies had to be individually modeled. This was necessary to create, within a reasonable amount of time (approximately a day), a geocellular model that preserved the dome and embedded mounded shapes while honoring the overall dome distribution. More efficient MPS implementations are necessary to handle complex and large reservoirs.

4. Dynamic heterogeneity measures, such as Koval heterogeneity factors and longitudinal dispersion coefficients, should be included in the numerical measures that determine the quality of stochastic models. These dynamic measures would be valuable in bringing insights into the models’ flow responses. Those measures could be calculated based on flow simulations obtained from commercial reservoir simulators.
Appendix A:

Geostatistical and Related Programs Developed

Seventeen computer programs were written in MatLab®. They either implement original algorithms or perform important tasks. MatLab® programming language was selected because of its efficiency in working with vectors and matrices.

The MatLab® implementations of these programs, along with brief explanations of their functionalities, are given below:

Program One

Function sub_region( ) extracts a sub-region from a larger model.

input: data - contains the initial model.

program variables: slice_x_1, slice_y_1, slice_x_2, slice_y_2 – define the boundary of the sub-region.

output: subregion - contains the extracted smaller model.

```matlab
function subregion=sub_region(data)
    temp_1=size(data);
    temp_1=temp_1(1,1);
    num_x_slices=1;
    num_y_slices=2;
    temp_2=num_x_slices*3240+num_y_slices*1620;
    slice_x_1=48;
    slice_y_1=16;
    slice_y_2=102;
    hard_data=[0,0,0,0];
    for i=1:temp_1
        if data(i,1)==slice_x_1
            subregion=[hard_data;data(i,:)];
        end
        if data(i,2)==slice_y_1
            subregion=[hard_data;data(i,:)];
        end
        if data(i,2)==slice_y_2
            subregion=[hard_data;data(i,:)];
        end
    end
    hard_data=hard_data(2:temp_2,:);
end
```
Program Two

Function debrisReg3() creates a dome shape object in order to mimic the debris facies distribution of region 3.
input: center = [x,y] - center of dome, dim_x, dim_y - physical dimensions of regions to be simulated, top - point at which flat top of dome ends, bot - point at which flat bottom of dome starts, h - average height of dome, r - random field used to mimic roughness
output: debris_surf - 2D matrix with values corresponding to the height of the debris surface.

function debris_surf=debrisReg3(center,dim_x,dim_y,top,bot,h,r)
matrix=zeros(dim_x,dim_y);
for i=1:dim_x
    for j=1:dim_y
        dist=sqrt((i-center(1))^2+(j-center(2))^2);
        m=6/(top-bot);
        b=3-top*6/(top-bot);
        y=m*dist+b;
        z=h*(1+tanh(y));
        matrix(i,j)=z+rand()*r;
    end
end
debris_surf=matrix;
end

Program Three

Function MPH( ) builds a truncated multiple point histogram of the first input model based on a defined multiple-point template. It then searches for matching patterns in models 2 and 3. When found, it adds histogram bars with heights corresponding to the patterns frequencies.
input: Ti – training image, used as the true reference in this work, Mps – 3D model constructed based on multiple-point statistics, Tps – 3D model constructed based on two-point statistics. Dim_X, Dim_Y, Dim_Z – number of nodes in the models in x, y, and z directions, R_X, R_Y, R_Z – number of nodes in the multiple-point template in x, y, and z directions.
output: celldata – vector that contains all multiple-point patterns found, sorted in ascending order, pat – vector that contains only the fifty most important patterns, freq – vector that contains the frequencies of the fifty most important patterns.

function[celldata,pat,freq]=MPH(Ti,Mps,Tps,Dim_X,Dim_Y,Dim_Z,R_X,R_Y,R_Z)
tic
    celldata=first_step_2(Ti,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z);
    disp('First stage finished...');
    [pat,freq]=second_step_2(celldata,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z);
    disp('Second stage finished...');
    celldata2=first_step_2(Mps,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z);
    disp('Third stage finished...');
    celldata3=first_step_2(Tps,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z);
    disp('Fourth stage finished...');
    third_step_2(pat,freq,celldata2,celldata3,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z);
end
disp('Fifth stage finished.');
toc
end

Program Four

Function first_step_2() scans a 3D model using a multiple-point template to find multiple-point patterns. These patterns are then stored in a vector, sorted in ascending order.
input: Ti – any geocellular model, Dim_X, Dim_Y, Dim_Z – number of nodes in the models in x, y, and z directions, R_X, R_Y, R_Z – number of nodes in the multiple-point template in x, y, and z directions.
output: celldata – vector that contains all multiple-point patterns found, sorted in ascending order.

function celldata=first_step_2(Ti,Dim_X,Dim_Y,Dim_Z, R_X, R_Y, R_Z)
l=1;
Ti3D=zeros(Dim_X, Dim_Y, Dim_Z);
for k=1:Dim_Z
    for j=1:Dim_Y
        for i=1:Dim_X
            Ti3D(i,j,k)=Ti(l);
            l=l+1;
        end
    end
end

clear Ti;
entries=(Dim_Z-2*R_Z)*(Dim_Y-2*R_Y)*(Dim_X-2*R_X);
length=(2*R_X+1)*(2*R_Y+1)*(2*R_Z+1);
pattern=ones(1,length);
celldata=cell(entries,1);
m=1;
for k=(1+R_Z):(Dim_Z-R_Z)
    for j=(1+R_Y):(Dim_Y-R_Y)
        for i=(1+R_X):(Dim_X-R_X)
            l=1;
            for kk=(k-R_Z):(k+R_Z)
                for jj=(j-R_Y):(j+R_Y)
                    for ii=(i-R_X):(i+R_X)
                        pattern(l)=Ti3D(ii,jj,kk);
                        l=l+1;
                    end
                end
            end
            celldata(m)=cellstr(int2str(pattern));
            m=m+1;
        end
    end
end

celldata=sort(celldata);
end
Program Five

Function second_step_2() sorts multiple-point patterns based on their frequencies of occurrence in a 3D model. It then stores the fifty patterns with the highest frequencies.

Input:
- celldata – multiple-point patterns
- Dim_X, Dim_Y, Dim_Z – number of nodes in the models in x, y, and z directions
- R_X, R_Y, R_Z – number of nodes in the multiple-point template in x, y, and z directions

Output:
- pat – vector that stores the most important multiple-point patterns
- freq – vector that stores the frequencies of occurrence of the most important multiple-point patterns

```
function [pat, freq] = second_step_2(celldata, Dim_X, Dim_Y, Dim_Z, R_X, R_Y, R_Z)

j = 1;
entries = (Dim_Z - 2*R_Z) * (Dim_Y - 2*R_Y) * (Dim_X - 2*R_X);
patterns = cell(entries, 1);
frequencies = zeros(entries, 1);
freq = 1;
for i = 1 : (entries - 1)
    if isempty(strmatch(celldata(i), celldata(i+1), 'exact'))
        patterns(j) = celldata(i);
frequencies(j) = freq;
j = j + 1;
freq = 1;
    else
        freq = freq + 1;
    end
end

[freq, ind] = sort(frequencies, 'descend');
inp = 20;
pat = patterns(ind(1 : inp));
[pat, ind] = sort(pat);
freq = freq(ind);
end
```

Program Six

Function third_step_2() builds truncated multiple-point histograms of three different geocellular models in semi-log scale. Matching patterns are plotted side by side.

Input:
- pat – vector that stores the most important multiple-point patterns
- freq – vector that stores the frequencies of occurrence of the most important multiple-point patterns
- celldata2 – multiple-point patterns and corresponding frequencies from geocellular model 2
- celldata3 – multiple-point patterns and corresponding frequencies from geocellular model 3

Output:
- none

```
function third_step_2(pat, freq, celldata2, celldata3, Dim_X, Dim_Y, Dim_Z, R_X, R_Y, R_Z)

j = 1;
entries = (Dim_Z - 2*R_Z) * (Dim_Y - 2*R_Y) * (Dim_X - 2*R_X);
patterns2 = cell(entries, 1); patterns3 = patterns2;
frequencies2 = zeros(entries, 1); frequencies3 = frequencies2;
freq2 = 1;
```
for i=1:(entries-1)
    if isempty(strmatch(celldata2(i),celldata2(i+1),'exact'));
        patterns2(j)=celldata2(i);
        frequencies2(j)= freq2;
        j=j+1;
        freq2=1;
    else
        freq2=freq2+1;
    end
end
pat2=patterns2;
for i=1:20
    index=find(strcmp(pat2,pat(i)));
    if(~isempty(index))
        freq2a(i)=frequencies2(index);
    else
        freq2a(i)=0;
    end
end
freq2a=freq2a';
freq3a=freq3a';
total=sum(freq);
prob=freq/total;
prob2=freq2a/total;
prob3=freq3a/total-100;
axes('YScale','log','YMinorTick','off','YGrid','on',...'
    'YMinorGrid','off','XTick',[0 2 4 6 8 10 12 14 16 18
20],'XGrid','on','XLim',[0 21]);
box('on');
hold on;
xlabel('Indexed Patterns');
ylabel('Pattern Frequencies');
Program Seven

Function variogram( ) builds semivariogram models based on positive linear combinations of basic semivariogram structures. The basic semivariograms are spherical, exponential, Gaussian, and linear semivariograms.

input: numStruct – number of basic structures that is used to build semivariogram model, nugget – nugget effect value (short-lag heterogeneity), opt1, opt2 – coded values of first and second basic semivariogram structure, cor1, cor2 – correlation length or practical correlation length of basic semivariograms, cont1, cont2 – sill values of basic semivariograms, titl, linecolor, linethickness – formatting parameters.

output: none.

function variogram(numStruct ,nugget ,opt1 ,opt2 ,cor1 ,cor2 ,cont1 ,cont2 ,titl, linecolor, linethickness)
if (cor1 >= cor2)
  x=0:0.01:(cor1+5);
else
  x=0:0.01:(cor2+5);
end
if (numStruct == 1)
  if (opt1 == 0)
    for i=1:length(x)
      if (x(i) <= cor1)
        y(i)=1.5*x(i)/cor1-0.5*(x(i)/cor1).^3;
      else
        y(i)=1;
      end
    end
    plot(x,cont1*y+nugget,'color',color,'LineWidth',thickness);
  else
    if (opt1 == 1)
      y=1-exp(-3*x/cor1);
    end
    plot(x,cont1*y+nugget,'color',color,'LineWidth',thickness);
  else
    if (opt1 == 2)
      y=1-exp(-3*x.^2/cor1^2);
    end
    plot(x,cont1*y+nugget,'color',color,'LineWidth',thickness);
  else
    if (opt1 == 3)
      y=x;
    end
    plot(x,cont1*y+nugget,'color',color,'LineWidth',thickness);
  end
else (numStruct == 2)
    if (opt1 == 0)
        for i=1:length(x)
            if (x(i) <= cor1)
                y1(i) = 1.5*x(i)/cor1-0.5*(x(i)/cor1).^3;
            else
                y1(i) = 1;
            end
        end
    else
        if (opt1 == 1)
            y1 = 1-exp(-3*x/cor1);
        else
            if (opt1 == 2)
                y1 = 1-exp(-3*x.^2/cor1^2);
            else
                if (opt1 == 3)
                    y1 = x;
                end
            end
        end
    end
    if (opt2 == 0)
        for i=1:length(x)
            if (x(i) <= cor2)
                y2(i) = 1.5*x(i)/cor2-0.5*(x(i)/cor2).^3;
            else
                y2(i) = 1;
            end
        end
        plot(x,cont2*y2+cont1*y1+nugget,'color',color,'LineWidth',thickness);
    else
        if (opt2 == 1)
            y2 = 1-exp(-3*x/cor2);
        plot(x,cont2*y2+cont1*y1+nugget,'color',color,'LineWidth',thickness);
        else
            if (opt2 == 2)
                y2 = 1-exp(-3*x.^2/cor2^2);
                plot(x,cont2*y2+cont1*y1+nugget,'color',linecolor,'LineWidth',linethickness);
            else
                if (opt2 == 3)
                    y2 = x;
                plot(x,cont2*y2+cont1*y1+nugget,'color',linecolor,'LineWidth',linethickness);
            end
        end
    end
end
xlabel('h (lag distance)');
ylabel('Gamma');
Program Eight

Function uncertainty() builds a stochastic uncertainty model based on surface heights of the mound core plus debris surface. It also computes the point-by-point standard deviation of surface heights based on multiple-point realizations corresponding to the same region.

input: mode – specifies if task 1 or 2 will be perform, region – 3D models to be analyzed, dim_x, dim_y, dim_z– number of nodes of 3D models in the x-, y-, and z-direction, thickness – x, y, and z lengths of the nodes in 3D models.

output: Stat – stores standard deviation values

function Stat = uncertainty( mode, region, dim_x, dim_y, dim_z, thickness)
if mode == 1
    region_m=Three_D(region(:,4),dim_x, dim_y, dim_z);
    heights=zeros(dim_x, dim_y);
    for i=1:dim_x
        for j=1:dim_y
            for k=1:dim_z
                if region_m(i,j,k)~=0
                    heights(i,j)= thickness*k;
                end
            end
        end
    end
    height=[ ];
    for j=1:dim_y
        height=[height;heights(:,j)];
    end
    Stat=[mean(height), std(height)];
else if mode == 2
    temp=[ ];
    for l=1:5
        region_m=Three_D(region(:,l),dim_x, dim_y, dim_z);
        heights=zeros(dim_x, dim_y);
        for i=1:dim_x
            for j=1:dim_y
                for k=1:dim_z
                    if region_m(i,j,k)~=0
                        heights(i,j)= thickness*k;
                    end
                end
            end
        end
        height=[ ];
        for j=1:dim_y
            height=[height;heights(:,j)];
        end
    end
end
Stat=[mean(height), std(height)];
Program Nine

Function matrix_3D( ) stores a geocellular model in GSLIB format in a 3D matrix.
input: Ti – geocellular model, Dim_x, Dim_y, Dim_z– number of nodes of 3D models in the x-, y-, and z-direction.
output: out – matrix that contains the geocellular model

function out=matrix_3D(Ti,Dim_X,Dim_Y,Dim_Z)
l=1;
for k=1:Dim_Z
    for j=1:Dim_Y
        for i=1:Dim_X
            Ti3D(i,j,k)=Ti(l);
            l=l+1;
        end
    end
    out=Ti3D;
end

Program Ten

Function template_3D( ) builds an ellipsoidal multiple-point template given major, medium, and minor axis.
input: major, medium, and minor – axes describing the geometry of an ellipsoidal template.
output: out – vector that contains the coordinates of nodes within the multiple-point template.

function out=template(major,medium,minor)
size=major*minor*medium;
out=zeros(size,4);
l=1;
for k=-minor:minor
    for j=-medium:medium
        for i=-major:major
            out(1,l)=i;
            out(1,2)=j;
            out(1,3)=k;
            if ((i/major)^2+(j/medium)^2+(k/minor)^2)<=1
                out(1,4)=1;
            end
        end
    end
end
Program Eleven

Function SurftoVol( ) defines 3 subregions in grid based on two surfaces, geological boundaries.
input: mc, db – matrices of heights of the mound and debris surfaces, nod_z – number of nodes in the z-direction, del_x, del_y, del_z – length of each node in the x-, y-, and z-directions.
output: ps – vector that contains coordinates plus facies [x,y,z,facies], grid – vector that contains only facies (SGEMS format).

```matlab
function [ps,grid]=SurftoVol(mc,db,nod_z,del_x,del_y,del_z)
[nod_x,nod_y]=size(mc);
X=zeros(nod_x*nod_y*nod_z,1);
Y=zeros(nod_x*nod_y*nod_z,1);
Z=zeros(nod_x*nod_y*nod_z,1);
Facies=zeros(nod_x*nod_y*nod_z,1);
for k=1:nod_z
    for j=1:nod_y
        for i=1:nod_x
            X(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=(i-1)*del_x;
            Y(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=(j-1)*del_y;
            Z(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=(k-1)*del_z;
            if k*del_z <= mc(i,j)
                Facies(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=2;
            elseif k*del_z <= db(i,j)
                Facies(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=1;
            end
            if ((k*del_z > mc(i,j)) && (k*del_z > db(i,j)) && k==1)
                Facies(i+(j-1)*nod_x+(k-1)*nod_x*nod_y)=1;
            end
        end
    end
    ps=[X,Y,Z,Facies];
    grid=Facies;
end
```

Program Twelve

Function Plot_Well( ) plots a well in 3D space based on its coordinates.
input: well – matrix that contains multiple wells. The wells are specified by their coordinates and facies.
output: none
function Plot_Well(well)
scatter3(well(:,2),well(:,3),well(:,4),5,well(:,5),'Marker','Square');
xlabel('X-Coordinate');
ylabel('Y-Coordinate');
zlabel('Depth');
title('Well Facies');
colorbar;
set(gca,'DataAspectRatio',[1 1 1])
end

Program Thirteen

Function mound_core() creates a mound core surface using a hyperbolic tangent function.
input: sgsim – 2D Gaussian random field, dim_x, dim_y – physical extent of model in x- and y- directions, h, v, s – scaling, areal density, and steepness parameters (see chapter 3 for explanation).
output: mound_surf – matrix with the height of the mound core surface.

function mound_surf=mound_core(sgsim, dim_x,dim_y,h,v,s)
sgsim=format2D(sgsim,dim_x,dim_y);
for j=1:dim_y
    for i=1:dim_x
        if sgsim(i,j)<0
            sgsim(i,j)=0;
        end
    end
end
mound_surf=h/1.5*(1+tanh(s*(v+sgsim)));
end

Program Fourteen

Function Indicator_Coding() indicator-code a geocellular model based on lithological facies.
input: data – geocellular model in SGEMS format, facies – lithological facies to be used to indicator-code.
output: ind_model – indicator-code geocellular model.

function ind_model=Indicator_Coding(data, facies)
rows=length(data);
temp=zeros([rows,1]);
for i = 1:rows
    if (data(i)== facies)
        temp(i)=1;
    end
end
ind_model=temp;
end
Program Fifteen

Function SGEMS_formatting( ) changes the format of a geocellular model in SGEMS format into coordinates-facies format [coordinates, facies]. The later format is then stored in a vector.

input: Ti – geocellular model in SGEMS format, Dim_x, Dim_y, Dim_z – number of nodes in the x-, y-, and z- directions.

output: model – geocellular model in coordinates-facies format.

```matlab
function model=SGEMS_formatting(Ti,Dim_X,Dim_Y,Dim_Z)
l=1;
out=zeros(length(Ti),4);
Min=min(Ti);
Max=max(Ti);
for k=1:Dim_Z
    for j=1:Dim_Y
        for i=1:Dim_X
            out(l,1)=i-1;
            out(l,2)=j-1;
            out(l,3)=k-1;
            out(l,4)=Ti(l);
            l=l+1;
        end
    end
end
model = out;
end
```

Program Sixteen

Function FC_Curves( ) finds equivalent layer-cake models corresponding to geocellular models. It then calculates and builds flow-capacity-storage-capacity curves and computes Lorenz coefficients.

input: Ti – geocellular model in SGEMS format, Dim_x, Dim_y, Dim_z – number of nodes in the x-, y-, and z- directions, f_0, f_1, f_2 – facies and their petrophysical properties [facies_code, porosity, horizontal permeability], thickness – thickness of a layer in the layer-cake model.

output: Lorenz – model’s Lorenz coefficient of heterogeneity.

```matlab
function Lorenz=FC_Curves(Ti,Dim_x,Dim_y,Dim_z, f_0, f_1,f_2, thickness,titl)
Ti3D=Three_D(Ti,Dim_x,Dim_y,Dim_z);
Ti3DPor=zeros(Dim_x,Dim_y,Dim_z);
Ti3DPerm=zeros(Dim_x,Dim_y,Dim_z);
for k=1:Dim_Z
    for j=1:Dim_Y
        for i=1:Dim_X
            if(Ti3D(i,j,k)==f_0(1))
                Ti3DPor(i,j,k)=f_0(2);
                Ti3DPerm(i,j,k)=f_0(3);
            else
                if(Ti3D(i,j,k)==f_1(1))
                    Ti3DPor(i,j,k)=f_1(2);
                else
                    if(Ti3D(i,j,k)==f_2(1))
                        Ti3DPor(i,j,k)=f_2(2);
                    end
                end
            end
        end
    end
end
```
\text{Ti3DPerm}(i,j,k) = f_1(3); \\
\text{else} \\
\quad \text{if} (\text{Ti3D}(i,j,k) == f_2(1)) \\
\quad\quad \text{Ti3DPor}(i,j,k) = f_2(2); \\
\quad\quad \text{Ti3DPerm}(i,j,k) = f_2(3); \\
\text{end} \\
\text{end} \\
\text{end} \\
\text{end} \\
\text{end} \\
\text{end} \\
\text{Ti2DPor} = \text{zeros(Dim}_x, \text{Dim}_y); \\
\text{Ti2DPerm} = \text{zeros(Dim}_x, \text{Dim}_y); \\
\text{for } j=1: \text{Dim}_y \\
\quad \text{for } i=1: \text{Dim}_x \\
\quad\quad \text{Ti2DPor}(i,j) = \text{mean}(\text{Ti3DPor}(i,j,:)); \\
\quad\quad \text{Ti2DPerm}(i,j) = \text{harmmean}(\text{Ti3DPerm}(i,j,:)); \\
\quad \text{end} \\
\text{end} \\
\text{Ti1DPor} = \text{zeros(Dim}_x,1); \\
\text{Ti1DPerm} = \text{zeros(Dim}_x,1); \\
\text{for } i=1: \text{Dim}_x \\
\quad \text{Ti1DPor}(i) = \text{mean}(\text{Ti2DPor}(i,:)); \\
\quad \text{Ti1DPerm}(i) = \text{mean}(\text{Ti2DPerm}(i,:)); \\
\text{end} \\
\text{Ti1DR} = \text{Ti1DPerm} ./ \text{Ti1DPor}; \\
[\text{Ti1DR}, \text{ind}] = \text{sort( Ti1DR, 'descend' );} \\
\text{Ti1DPorosity} = \text{Ti1DPor( ind);} \\
\text{Ti1Permeability} = \text{Ti1DPerm( ind);} \\
\text{size}_x = \text{ones(Dim}_x,1) * \text{size}_X; \\
\text{X} = \text{sum( Ti1DPorosity .* size}_x); \\
\text{Y} = \text{sum( Ti1Permeability .* size}_x); \\
\text{x}(1) = 0; \text{ y}(1) = 0; \\
\text{for } i=1: \text{Dim}_x \\
\quad \text{x}(i+1) = \text{sum( Ti1DPorosity(1:i).* size}_x(1:i))/X; \\
\quad \text{y}(i+1) = \text{sum( Ti1Permeability(1:i).* size}_x(1:i))/Y; \\
\text{end} \\
\text{plot(x, y, '-^', 'MarkerFaceColor', 'blue', 'color', 'blue', 'LineWidth', 2,} \\
\quad 'MarkerSize', 5) \\
\text{xlabel('Storage Capacity (C)');} \\
\text{ylabel('Flow Capacity (F)');} \\
\text{title(titl);} \\
\text{hold on;} \\
\text{Lorentz = 2*(trapz(x,y) - 0.5);} \\
\text{end} \\

\textbf{Program Seventeen} \\

Function extract() mines horizons parallel to xy, xz, and yz planes as well, as wells in the vertical direction. If option = 0, then a horizon parallel to yz is extracted. If option = 1, then a cross-section parallel
to xz is extracted. If option = 2, then a cross-section parallel to xy is extracted. If option = 3, then a vertical well is extracted. If option = 4, then a subregion, composed of xz planes, is extracted.

Input: Ti – geocellular model in SGEMS format, option – specifies to the program which task to perform, plane – specifies which plane to extract, wellx, welly – coordinates of vertical well to be extracted, Dim_x, Dim_y, Dim_z – number of nodes in the x-, y-, and z-directions.

Output: structure – vertical well, horizon, or subregion mined from geocellular model.

function structure = extract(Ti, option, plane, wellx, welly, Dim_x, Dim_y, Dim_z, number)
    ti = formatting(Ti, Dim_x, Dim_y, Dim_z);
    temp = length(ti);
    j = 1;
    if option == 0
        yzplane = zeros(length(ti), 4);
        for i = 1:temp
            if ti(i, 1) == plane
                yzplane(j, :) = ti(i, :);
                j = j + 1;
            end
        end
        yzplane = yzplane(1:j-1,:);
        structure = yzplane;
    else if option == 1
        xzplane = zeros(length(ti), 4);
        for i = 1:temp
            if ti(i, 2) == plane
                xzplane(j, :) = ti(i, :);
                j = j + 1;
            end
        end
        xzplane = xzplane(1:j-1,:);
        structure = xzplane;
    else if option == 2
        xyplane = zeros(length(ti), 4);
        for i = 1:temp
            if ti(i, 3) == plane
                xyplane(j, :) = ti(i, :);
                j = j + 1;
            end
        end
        xyplane = xyplane(1:j-1,:);
        structure = xyplane;
    else if option == 3
        vertwell = zeros(length(ti), 4);
        for i = 1:temp
            if ti(i, 1) == wellx && ti(i, 2) == welly
                vertwell(j, :) = ti(i, :);
                j = j + 1;
            end
        end
        vertwell = vertwell(1:j-1,:);
        structure = vertwell;
    else if option == 4
xzsubregion=zeros(length(ti),4);
for i=1:temp
    if (ti(i,2) >= plane-1 && ti(i,2) <= plane+number-1)
        xzsubregion(j,:)=ti(i,:);
        j=j+1;
    end
end
xzsubregion = xzsubregion(1:j-1,:);
structure = xzsubregion;
Appendix B:

Application of Multiple-Point Statistics to Karst Modeling

Karsts are the product of massive subsurface dissolution caused by somewhat acidic water draining through limestone or dolomite rocks. As water continues to circulate, more rock is dissolved, leading to the formation of caves and large fractures.

The particular structures of karsts are largely dependent on the water underground drainage system. This system is in turn dependent on the depositional environment. A set of karstic formations typical of most common depositional environments have been compiled by the Bureau of Economic Geology (BEG). Cave surveys, which are point-by-point mappings of the inner architecture of karsts, are also available for each formation.

The concept of using multiple-point statistics (MPS)-based algorithms to model karstic formations is briefly presented in this section. We want to explore the potential of MPS techniques to capture the seemingly random geo-structures found in karsts. A karstic formation from South Dakota, known as the Wind Cave, is used as the case study.

The Wind Cave is one of the largest caves in the world. It is characterized by very complex structural patterns that have been extensively mapped. Figure B-1 shows an extensive cave survey of Wind Cave.
In this work, this cave survey is used as the training image (TI). The multiple-point geological patterns are extracted from the TI using two multiple-point templates shown in figure B-2.

Figure B-2: Ellipsoidal multiple-point template used for karst modeling.
Some conditioning data is extracted from the training image to form pseudo-wells. This is done to imitate real-life cases, where conditioning data is available from wells. Figure B-3 displays ten pseudo-wells used.

![Figure B-3: Pseudo-wells used to condition MPS model.](image)

We then use the SNESIM program to generate two MPS-based models. Table B-1 shows the values assigned to the SNESIM parameters.

Table B-1: Values of important parameters input into SNESIM to simulate karst architectures. Category 1 and 2 refer to karst and background facies.

<table>
<thead>
<tr>
<th>Search Ellipsoid</th>
<th>50 x 20 x 10 meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Conditioning Data</td>
<td>80 values</td>
</tr>
<tr>
<td>Multigrid Simulation and Number of Grids</td>
<td>Yes, 4 grids.</td>
</tr>
<tr>
<td>Subgrid Simulation and Number of Subgrids</td>
<td>Yes, 4 grids.</td>
</tr>
<tr>
<td>Servosystem Factor</td>
<td>0</td>
</tr>
<tr>
<td>Affinity/Rotation</td>
<td>none</td>
</tr>
</tbody>
</table>
Resimulation of Nodes and Resimulation Threshold  | yes  
--- | ---  
Number of Categories and Category Proportions  | 2 categories. The proportions of categories 1 and 2 are 36 and 64%, respectively.

The MPS-based models are given in figure B-4. The top and bottom models, respectively, correspond to the circular and elliptical multiple-point templates.

Figure B-4: MPS-based models. Top and bottom models correspond to the circular and elliptical templates.
Based on visual comparison of the MPS models against the training image, we conclude that while they share some similar geo-structures, the architecture of the TI is somewhat different. A more optimal selection of the multiple-point template or a different MPS algorithm might be necessary to achieve a “successful” characterization.
Appendix C:

Vertical Semivariogram Modeling for Carbonate Buildups in the Sacramento Mountains

Horizontal facies indicator semivariograms of the carbonate buildups under study were given in chapter 4. The horizontal semivariograms were deemed the most important ones as the facies are better correlated in the horizontal directions. For the sake of completeness, in this section, we present the vertical indicator semivariograms for the mound core and debris facies in each region.

Figure C-1 shows the vertical mound core indicator semivariogram in region 1.

Figure C-1: Vertical indicator semivariogram for mound core facies in region 1. The red marks are points from the experimental semivariogram. Lag distances are in meters. Indicator semivariogram values are dimensionless.
Figure C-2 shows, from top to bottom, the vertical indicator semivariograms for the mound core and debris facies in region 2. Figure C-3 shows, from top to bottom, the vertical indicator semivariograms for the mound core and debris facies in region 3.

Figure C-2: Vertical indicator semivariograms for mound core (top image) and debris (bottom image) facies in region 2. The red marks are points from the experimental semivariogram. Lag distances are in meters. Indicator semivariogram values are dimensionless.
Figure C-3: Vertical indicator semivariograms for mound core (top image) and debris (bottom image) facies in region 3. The red marks are points from the experimental semivariogram. Lag distances are in meters. Indicator semivariogram values are dimensionless.
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