The Thesis Committee for Suleen Suleen
Certifies that this is the approved version of the following thesis:

Calculation of Residual Oil Saturation for Surfactant Flood EOR
Process from Injection Well Pressure data

APPROVED BY
SUPERVISING COMMITTEE:

Supervisor:

Mukul M. Sharma

Quoc Nguyen
Calculation of Residual Oil Saturation for Surfactant EOR Process from Injection Well Pressure Data

by

Suleen Suleen, B.S.

Thesis
Presented to the Faculty of the Graduate School of
The University of Texas at Austin
in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science in Engineering

The University of Texas at Austin
August 2010
Dedication

To my Parents and Wife
Acknowledgements

I wish to extend my sincere thanks to Dr. Mukul M. Sharma for his constant support and guidance over the past two years without which this research wouldn’t have been possible. I have learnt a lot from him both in the technical and the professional spheres and intend to carry the learning along with me in every walk of life ahead. I also wish to thank Dr. Quoc Nguyen for his apt and useful suggestions and for being a big motivator during the writing of this thesis. I owe many a thanks to Dr. Michael M. Levitan (BP) for all his time and patience over answering my queries in person.

Dr. Ajay Suri provided important inputs for this research and I owe many a thanks to him. I also wish to thank Roger Terzian and Joanna Castillo for helping me with the software updates and installations that were so important and necessary for the research related work. I am thankful to SPE for maintaining such a resourceful site as SPE.org. I have made extensive use of CMG simulators and I am grateful to them for their robust software and also for helping me with the questions I had. Thanks are also due to Jin Lee who was always up for anything I ever asked a favor for.

It was the blessings of my parents and the love of my wife that kept me going and encouraged me to keep up the hard work. I thank them and all my family members for being with me in the good and the challenging times and dedicate this thesis to them. Lastly, I wish to thank my friend Nitish Koduru for his helping hand with the formatting of this document.

August 2010
Abstract

Calculation of Residual Oil Saturation for Surfactant EOR Process from Injection Well Pressure Data

Suleen Suleen M.S.E.
The University of Texas at Austin, 2010

Supervisor: Mukul M. Sharma

The objective of this thesis is to study the application of pressure transients from an injection well to determine residual oil saturation in a reservoir for a surfactant EOR process.

A published analytical solution for the multiple-region moving boundary problem of water injection in a homogeneous reservoir with radial flow geometry was studied and the model was scripted using Excel VBA. The logarithmic derivative from the late time transient response from the injection well was found to depend on the properties of the water flooded zone near the well bore. From this constant derivative value, the fluid mobility in the flushed zone can be calculated. The fluid mobility can then be further used to calculate the reservoir permeability using appropriate relative permeability curves. Water injection in a homogenous reservoir under radial flow conditions was simulated using CMG’s black oil simulator. Several cases were run to account for varying reservoir conditions and it was found that the reservoir permeability could be calculated from the pressure data using suitable relative permeability functions.
The observation that the late time derivative response from injection pressure data indicates the fluid mobility near the well bore behind the fluid front was used to calculate residual oil saturation for a surfactant EOR process. The logarithmic derivative was found to become constant at late time and was a function of injection rate, fluid viscosity, reservoir thickness, reservoir absolute permeability and relative permeability functions. If the reservoir absolute permeability and other fluid data are known, relative permeability of the injected fluid at residual oil saturation can be calculated which can be further used to evaluate residual oil saturation using appropriate relative permeability functions. Numerical simulations were made in CMG’s chemical simulator under varying reservoir conditions. It was found that relative permeability to water in the flushed zone can be calculated using the pressure response from an injection well and subsequently converted to reasonable estimates of residual oil saturation using the trapping number dependent relative permeability model.
Table of Contents

List of Tables .......................................................................................................................x

List of Figures ..................................................................................................................... xi

Chapter 1: Introduction ........................................................................................................1
    Research Objective .....................................................................................................2
    Introduction to Chapters .............................................................................................2

Chapter 2: Literature Review ...............................................................................................5
    Introduction .................................................................................................................5
    Well-Bore Pressure Response During Water Injection / Fall-Off in an Oil Reservoir: a Literature Review .................................................................6
    A New Analytical Solution Method for Two-Phase Variable Rate Problems .............10
    Evaluation of Residual Oil Saturation: A Literature Review ......................................16

Chapter 3: Implementation of the Mathematical Solution to the Multiple Region Moving Boundary Problem in Excel VBA ...............................................................30
    Introduction ...............................................................................................................30
    Mathematical Formulation of the Model ..................................................................30
    Chapter Summary .....................................................................................................34

Chapter 4: Numerical Simulation of Water Injection in Homogeneous Isotropic Reservoir with Radial Flow Geometry .....................................................................45
    Introduction ...............................................................................................................45
    CMG IMEX Simulator .............................................................................................45
    Simulation Results from Illustrative Cases ...............................................................47
    Chapter Summary .....................................................................................................53

Chapter 5: Numerical Simulation of Surfactant and Surfactant-Polymer Injection in Homogeneous Isotropic Reservoir with Radial Flow Geometry ..............................62
    Introduction ...............................................................................................................62
    CMG STARS Simulator ...........................................................................................62
    Trapping Number Dependent Relative Permeability Model ......................................65
    Simulation Results from Illustrative Cases ...............................................................69
List of Tables

TABLE 3.1: Reservoir and Fluid Data from SPE 77532 ...................................................35
TABLE 3.2: Rock Properties Data for Water Flood from SPE 77532 ..............................36
TABLE 4.1: Simulation Data ............................................................................................55
TABLE 4.2: Rock Properties Data for Water Flood (Low Capillary Number Displacement) ...............................................................................................56
TABLE 5.1: Simulation Data for Cases 1, 2, 3, 4 ...........................................................101
TABLE 5.2: Rock Properties Data for Water Flood (Low Capillary Number Displacement) .............................................................................................102
TABLE 5.3: Rock Properties Data for Surfactant Flood (High Capillary Number Displacement) .............................................................................................103
TABLE 5.4: Simulation Data for Case 7 .........................................................................104
TABLE 5.5: Simulation Data for Case 8 and Case 9 ......................................................105
List of Figures

Fig 2.1: Injection test pressure build-up: after Woodward and Thambyanayagam showing three distinct slopes ................................................................. 24

Fig 2.2: Schematics of multiple fluid-bank reservoir and water saturation profile in the reservoir, from Yeh and Agarwal ............................................. 25

Fig 2.3: Saturation vs. radial distance showing 2 shock fronts, from Bratvold and Horne ................................................................. 26

Fig 2.4: BHP vs. Log (time) and BHP-Derivative vs. Log (time) for the first injection period in the absence of an initial water bank, from Levitan ...................... 27

Fig 2.5: Variable rate schedule for the injection test: Fig 2.4 above corresponds to injection period 2 marked above, from Levitan ........................................ 28

Fig 2.6: BHP vs. Log (time) and BHP-Derivative vs. Log (time) for the 2nd and 5th injection period in the rate schedule shown in Fig 2.5, from Levitan .......... 29

Fig 3.1: Relative permeability curves for water injection process .................................................................................. 37

Fig 3.2: Fractional flow curve and shock front saturation .................................................................................. 38

Fig 3.3: Tracing the saturation profile on the fractional flow derivative plot .................................................................................. 39

Fig 3.4: X and Y axes interchange from Fig 3.3 results in familiar saturation profile with a BL shock front .................................................................. 40

Fig 3.5: Saturation profile against the transformed variable ‘x’, shock front is located at x = 1 ........................................................................... 41

Fig 3.6: Dimensionless total mobility against transformed variable ‘x’ ........................................................................... 42

Fig 3.7: Dimensionless total compressibility against transformed variable ‘x’ ........................................................................... 43

Fig 3.8: Comparison of program output with published results from Levitan’s paper SPE 77532 ........................................................................... 44

Fig 4.1: Relative permeability curves for water injection process ........................................................................... 57

Fig 4.2: DP (psi) vs. Log (time-hrs) for water injection with Swi = Swirr = 0.2 ........................................................................... 58

Fig 4.3: DP (psi) vs. Log (time-hrs) for water injection with Swi = 1-Sor = 0.75 ........................................................................... 59
Fig 4.4: $D_p$ (psi) vs. log (time-hrs) for water injection with initial water bank..............60

Fig 4.5: $D_p$ (psi) vs. log (time-hrs) for water injection with variable rate sequence .......61

Fig 5.1: Capillary desaturation curves (CDC) for oleic phase, 5.1(a) and aqueous phase. 5.1(b). At low capillary number, the residual saturations for the oleic and aqueous phases are 0.25 and 0.2 respectively. At high capillary number the residual saturation for each phase is zero. .....................................................106

Fig 5.2: Relative permeability curves from Pope et. al21 model for high capillary number Fig 5.2 (a) on Log-Log graph and 5.2 (b) on cartesian graph paper: the relative permeability curves become straight lines with zero residual phase saturations at high capillary number .........................................................107

Fig 5.3: Relative permeability curves for low capillary number 5.3 (a) and high capillary number 5.3 (b) for input to STARS. The relative permeability curves at low capillary number are Corey type curves and at high capillary number are straight lines with zero residual phase saturations..........................108

Fig 5.4: $D_p$ (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with $S_{wi} = S_{wc}$ .........................................................109

Fig 5.5: $D_p$ (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with $S_{wi} = 1-S_{orw}$ ........................................110

Fig 5.6: $D_p$ (psi) vs. Log (time-hrs) for 3 different injection sequences: Water - Surf, Water -1 D Surf - Water, Water-5 D Surf - Water ........................................111

Fig 5.7: $D_p$ (psi) vs. Log (time-hrs) for the case of water injection in reservoir with an initial bank of water resulting from surfactant flood ........................................112

Fig 5.8: $D_p$ (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with $MR = 5$.........................................................113

Fig 5.9: $D_p$ (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with $MR = 10$......................................................114

Fig 5.10: Polymer rheology, Power-law relationship between apparent viscosity and shear rate ........................................................................................................115

Fig 5.11: Viscosity vs. Distance plot at low injection rate: polymer and surfactant-polymer cases at different times.................................................................116

Fig 5.12: $D_p$ (psi) vs. Log (time-hrs) for polymer injection for 0.25 days followed by surfactant-polymer injection, the bounding curves are for continuous polymer and surfactant-polymer injection..................................................117
Fig 5.13: Top (Cartesian plot of $\Delta P_{wf}$ vs. Time); Left (Log-Log plot of $\Delta P_{wf}$ vs. Time); Right (Cartesian plot of $(\Delta P_{wf}$ vs. $t^m)$ for polymer flood ..........................118

Fig 5.14: Top (Cartesian plot of $\Delta P_{wf}$ vs. Time); Left (Log-Log plot of $\Delta P_{wf}$ vs. Time); Right (Cartesian plot of $(\Delta P_{wf}$ vs. $t^m)$ for surfactant-polymer flood ...119
Chapter 1: Introduction

Injection well testing involves an analysis of the pressure transients generated during the injection of a fluid in the reservoir. It is analogous to drawdown testing for production wells but requires special interpretation techniques when the properties of the injected fluid are different from the formation fluids, which is the case most commonly encountered. Injection well tests are commonly carried out to monitor flood performance for reservoir management in waterflood and enhanced oil recovery processes. However, recently there have been growing interests in using injection well tests for reservoir appraisal early on in the producing life of a field. Injection well tests are cleaner as they do not require the venting of hydrocarbon gases to the atmosphere which is the general norm during production well testing for reservoir appraisal.

Injection of a fluid which is different from the reservoir fluid results in the formation of fluid banks in the reservoir. For many years, the theory developed for interpretation of injection well tests assumed a two fluid-bank model in which an injected fluid bank existed from the well-bore into the reservoir beyond which there existed a bank of the displaced reservoir fluid. It did not account for saturation variations within the two banks. However, such a two bank model is only accurate for highly favorable mobility ratio displacements and ideally homogenous formations. More frequently, a continuous saturation variation develops between the injected and the displaced fluid banks. Recent studies and new modeling techniques account for such saturation gradients which typically develop during injection tests. These models are discussed in detail in this thesis.

A comparatively less explored application of injection well tests is in determining the residual oil saturation. The most commonly used methods of estimating residual oil
saturation are logging and tracer tests. Injection well tests could alternatively be used for determining residual oil saturation at no significant additional cost apart from that needed for injection itself. This application has been studied in detail in this thesis using both analytical solutions and numerical simulations conducted using a commercial reservoir simulator.

**RESEARCH OBJECTIVE**

The primary objective of this research is to explore the application of injection tests for determining residual oil saturation both before and after chemical flooding processes. As will be discussed later, careful monitoring and interpretation of pressure vs. time data obtained during water or surfactant injection can help estimate fluid mobilities in the flooded region which can be converted to fluid saturation values using appropriate relative permeability models.

**INTRODUCTION TO CHAPTERS**

As a prelude to this thesis, we introduce the various chapters that are to follow. Chapter 1 (this chapter) introduces injection well testing, outlines the research objectives and provides a short review of the other chapters that follow.

Chapter 2 provides a short literature survey in which we look back at the development and evolution of injection well testing methods. We review the early interpretation models that were based on the two-bank approach. We then summarize later analytical models that use a multi-bank concept and also more recent models that allow for continuous saturation variation in the water invaded region. In the second half of the chapter we review the most commonly used industry methods for finding residual oil saturation.
In Chapter 3 we summarize the analytical model published by Levitan for modeling water injection well tests allowing for continuous saturation variation in the water-flooded region. We also lay out a step-by-step procedure for programming the solution for the first injection period and present a comparison of the output from the program and the published results.

In Chapter 4 we build on the theory outlined in Chapters 2 and 3 and present multiple cases describing the application of water injection well tests in determining reservoir permeability under various reservoir conditions. We also present the results from variable rate case simulation which corroborates the fact that any rate which is maintained constant for a sufficient length of time will eventually lead to the same mobility interpretation at late time. This observation makes possible the use of water-injection well testing for interpretations of mobility in the case of variable injection rate schedules.

In Chapter 5 we present results from multiple simulations made for water and surfactant injection under various reservoir conditions and varying injection sequences. An important observation that is made in the chapter is that well-bore pressure data recorded during surfactant injection can help to calculate fluid mobilities in the surfactant flooded region. This value of fluid mobility can then be converted to fluid saturations using appropriate trapping number dependant relative permeability models which are also discussed in the chapter. Further, we make use of published techniques for analyzing the well-bore pressure response in the case of a shear thinning fluid injected into the reservoir. Polymers are invariably mixed with surfactant solutions to stabilize the displacement process and polymers have shear dependent viscosity. At high injection rates, the shear rate decreases continuously as one moves away from the well-bore till considerable distances into the reservoir. Under such conditions, the viscosity and hence
the mobility of the injected fluid is continuously varying as a function of the radial distance thereby rendering conventional straight line well test interpretation techniques inapplicable. In such cases, special interpretation techniques are discussed and cases studies have been described.

We provide the summary and conclusions in Chapter 6 of this thesis. An important conclusion drawn is that injection well testing methods seem to be quite suitable for determining important reservoir parameters such as fluid mobility and relative permeability in water injection wells. More importantly, the same approach can be used for determining residual oil saturation resulting from a surfactant flood or another enhanced oil recovery process.
Chapter 2: Literature Review

INTRODUCTION

In the first section of this chapter, a review of the prior studies related to water injection well tests has been presented. The early work involved studying the pressure response in the wellbore from isothermal injection of water in a radial homogenous reservoir using the two-bank and the multi-bank models that did not allow for a saturation gradient in the reservoir (piston-like displacement with discontinuous saturation changes). The applications and limitations of the two-bank model and the multi-bank model used for interpretation of such well tests have been discussed. This is followed by a discussion (in the second section) of a fully analytical solution to the case of two-phase displacement of oil and water in a radial homogeneous reservoir which is free from the limitations imposed by earlier models. The new model formulation is discussed along with important findings and results.

Residual oil saturation (ROS) is an important parameter that needs to be measured to access the suitability of applying any oil recovery process. In the last section of this chapter, we review the different techniques and methods used in the oil industry for the determination of ROS. We present a brief overview of several methods from coring to logging, well-testing and tracer tests that have been employed in the field for determining ROS.
WELL-BORE PRESSURE RESPONSE DURING WATER INJECTION / FALL-OFF IN AN OIL RESERVOIR: A LITERATURE REVIEW

The well-bore pressure response due to injection/fall-off of water in a radial homogeneous reservoir has been studied by various authors – Woodward and Thambbynayagam\textsuperscript{1}, Ramakrishnan and Kuchuk\textsuperscript{2}, Yeh and Agarwal\textsuperscript{3}, Abbaszadeh and Kamal\textsuperscript{4}, Bratvold and Horne\textsuperscript{5} and Levitan\textsuperscript{6}.

Woodward and Thambbynayagam\textsuperscript{1} considered injection of water in a well located in an infinite homogeneous and isotropic reservoir of constant thickness. They assumed piston-like displacement of water in which uniform, residual oil saturation existed behind the flood front and is representative of the two-bank model formulation. The mathematical solution provided by them can be used to interpret the bottom-hole pressure versus the logarithm of injection time. They suggested that such a plot would exhibit three linear sections (Fig 1.1). The first linear section corresponds to the early time when the pressure transient traverses the water bank that may have existed at the start of the injection test. The second linear section corresponds to the time when the pressure transient has moved into the in-situ reservoir fluid (still during the early injection period, when the total volume of injected water is still much smaller than the initial water bank). The third section corresponds to the zone invaded with the injected fluid behind the flood-front. In all the three linear sections of the plot, the mobility of the fluid can be determined from the slope of the linear section by using the formula:

\[
m = 162.6 \frac{q B \mu}{k h}
\]

(2.1)

Where:

\begin{align*}
m & = \text{Slope of the pressure vs. log (time) curve (psi/cycle)} \\
k & = \text{Permeability of the formation (md)} \\
h & = \text{Reservoir thickness (ft)}
\end{align*}
q = Injection rate (bbls/day)

B = Fluid formation volume factor

μ = Fluid viscosity (cp)

The slopes of the three linear sections from the pressure vs. log (time) plot can be used to calculate the mobilities of the fluids in different portions of the reservoir.

\[
\frac{k}{\mu} = 162.6 \frac{qB}{mh}
\]  

(2.2)

The first slope represents the mobility of the fluid in the fluid bank that existed before the injection test and can be used to determine the mobility of water at residual oil saturation. The second slope can be used to calculate the mobility ahead of the initial water bank and is equal to the oil mobility at residual water saturation if the initial water saturation was the irreducible water saturation in the reservoir. The third section represents the mobility behind the front and would be the same as that of the initial water bank if the fluid in the initial water bank and the injected fluid are the same.

Most of the earlier studies for water-injection well-tests such as done by Woodward and Thambyanayagam\(^1\) assumed two bank displacement systems, a water bank which extends from the well-bore to the location of the flood front, and the oil bank from the flood front till the reservoir boundary and allowing for the presence of an initial water bank at the start of the injection test that extends some distance from the well-bore into the reservoir. Such an assumption is valid only for a highly favorable mobility ratio that is \(M<<1\). However, when the mobility ratio is not favorable, the two-bank model is not valid for the analysis and interpretation of water injection well tests. There exists a saturation gradient in the region between the flood-front and the wellbore and the fluid mobilities are continuously varying in this intermediate region. Abbaszadeh and Kamal\(^4\) developed analytical solutions to the pressure fall-off problem during water injection well
tests assuming a multi-bank model, in which the saturation profile was represented as a step wise constant function of distance. The fluid mobilities are assumed constant in each of these banks.

Yeh and Aggarwal\(^3\) conducted a series of numerical experiments involving pressure transient analysis of injection wells in reservoirs with multiple fluid banks (Fig 1.2). Their numerical experiments with a reservoir simulator showed that the logarithmic derivative response from fall-off tests could be used to calculate the mobility profile, fluid bank radii and the pressure profile in the reservoir prior to shut-in.

Bratvold and Horne\(^5\) developed analytical solutions that allowed for continuous variation of saturation in the water-invaded region thereby removing the restrictions posed by the two-bank and the multi-bank models. They proposed mathematical models governing the non-isothermal injection of water in an oil reservoir and presented analytical solution techniques for the same. Their solution method is a two-step procedure. In the first step, they calculate the saturation and the temperature profiles as functions of radial distance. For building the saturation profile they consider Buckley-Leverett type displacement of incompressible fluids and included temperature effects. They discuss the effect of non-isothermal injection on the saturation profile as an additional shock related to the temperature discontinuity (Fig 1.3) in the reservoir. In the second step, they use the pre-determined saturation and temperature profiles to calculate the fluid mobilities in the reservoir as a function of radial distance and solve the pressure diffusion equation assuming compressible fluids. They use the method of superposition to develop the fall-off solution for pressure when the fronts are stationary and the problem is linear.

Several other authors have studied the transient pressure response obtained from an injection or fall-test for water injection in an oil reservoir. The ones discussed in this
section are those which are directly related to the work done in this thesis. In particular, these authors have evaluated the valuable information hidden in the pressure data that could be deciphered using different interpretation techniques to reveal the properties of the different fluid zones that typically develop during injection of water in a homogeneous, isotropic reservoir with radial flow geometry. Next, we shift our attention to the variable-rate case for water injection.

The non-linearities induced by the movement of the boundary between the oil and the water zones during variable rate injection render traditional superposition methods inapplicable. Hence, to solve this moving boundary problem under variable rate situations several approximations need to be made. These have been discussed in detail by Ramakrishnan and Kuchuk\textsuperscript{2}.

Ramakrishnan and Kuchuk\textsuperscript{2} provided a detailed analysis of several solution methods available for the moving boundary problem of water injection in a radial homogeneous reservoir considering a two-bank model for constant injection rate. They extended these methods to the general application of pressure transient analysis in a variable rate schedule using convolution and de-convolution methods. The solution scheme is approximate and has a high degree of computational complexity.

So far we have discussed the history of developments in water injection well tests. The earliest studies considered two-bank systems which slowly evolved into multi-bank treatment of the gradual saturation variation in the flooded zone which is the case most commonly experienced in typical water floods. Then we looked at the approximate solutions available for the non-linear variable rate problems in water injection well tests.

Next, we look at the complete exact analytical solution for the case of non-isothermal water injection in an oil reservoir with a varying injection rate schedule. This solution was provided by Levitan\textsuperscript{6}.
A NEW ANALYTICAL SOLUTION METHOD FOR TWO-PHASE VARIABLE RATE PROBLEMS

Introduction

In this section, we examine in detail a fully analytical solution for non-isothermal water injection with a variable injection rate that has been formulated by Levitan. The model accounts for continuous changes in the water saturation in the water invaded zone, therefore, accounting for both favorable and unfavorable mobility ratios in the displacement process.

Description of the Model

As discussed earlier, many authors studied the effect of cold water injection on the saturation profiles that develop in the reservoir. Non-isothermal effects in essence modify the water saturation profile such that it typically includes two saturation shock fronts. From the well bore, the first shock is due to the temperature discontinuity in the reservoir and the second is the standard Buckley-Leverett shock front.

Both the saturation and temperature distribution in the reservoir can be described fully as a function of radial distance with rock and fluid properties.

\[ S_w = S_w(r) \quad (2.3) \]

\[ T = T(r) \quad (2.4) \]

Next, Levitan made a variable transformation involving the radial distance from the well-bore and the total injected water volume.

\[ x = \frac{r_d}{\sqrt{2\alpha \theta(t_d)}} \quad (2.5) \]

Where:

\[ r_d = \frac{r}{r_w} = \text{Dimensionless radial distance} \quad (2.6) \]
The above transformations define saturation and temperature as a function of the transformed variable, x.

\[ S_w = S_w(x) \] 
\[ T = T(x) \]

These saturation and temperature distributions are functions of the relative permeability, water and oil viscosities and fluid and rock heat capacities. In terms of the transformed variable ‘x’ the Buckley-Leverett saturation shock front is always located at ‘x’ = 1.
Once the fluid saturation distribution has been calculated, assuming incompressible fluids and Buckley-Leverett type displacement, the pressure behavior is determined by considering the pressure diffusivity equation:

$$c_d(S_w) \frac{\partial p_d}{\partial t_d} = \frac{1}{r_d} \frac{\partial}{\partial r_d} \left[ \lambda_d(S_w) r_d \frac{\partial p_d}{\partial r_d} \right]$$  \hspace{1cm} (2.12)

Where:

$$c_d = \text{Dimensionless compressibility} = \frac{c_t(S_w)}{c_{r0}}$$

$$p_d = \text{Dimensionless pressure} = \left( p - p_i \right) \frac{2\pi h \hat{\lambda}_w}{q_0}$$

$$\hat{\lambda}_w = \text{End point water mobility}$$

$$\lambda_d(S_w) = \text{Dimensionless mobility} = \left[ \frac{kk_{rw}(S_w)}{\mu_w} + \frac{kk_{ro}(S_w)}{\mu_o} \right] \frac{1}{\hat{\lambda}_o}$$

The compressibility and mobility terms are functions of saturation and hence are also functions of ‘x’ by means of the variable transformations listed above.

Making another transformation, the diffusivity equation listed above is changed to the form as shown below.

$$\tau = \int_0^t \frac{dt}{2\alpha \theta(t)}$$  \hspace{1cm} (2.13)

$$\frac{\partial p_d}{\partial \tau} - \alpha q_d(\tau) \frac{\partial p_d}{\partial x} = \frac{1}{c_d(x) \lambda_d(x) \frac{\partial}{\partial x} \left[ \frac{\partial p_d}{\partial x} \right]}$$  \hspace{1cm} (2.14)

The above equation contains a rate term which is time dependent. This rate dependent term is zero for fall-off when the rate is equal to zero. For the injection case, the variable rate case is solved by assuming a step wise constant rate. With this
assumption, the above reduces to a variable coefficients partial differential equation that is solved by variable transformations listed below.

\[ \nu(\xi, \tau) = u(\xi) p_d(x, \tau) \]  
(2.15)

\[ f(x) = \frac{c_d(x)}{\sqrt{\lambda_d(x)}} \]  
(2.16)

\[ \xi(x) = \int_0^x f(x) \, dx \]  
(2.17)

\[ u(\xi) = \left[ \frac{c_d(x) \lambda_d(x)}{\sqrt{\lambda_d(0)}} \right]^{1/4} \sqrt{\frac{x}{\xi}} \exp \left[ \frac{aq_d}{2} \int_0^\xi xf(x) \, d\xi \right] \]  
(2.18)

With the above variable transformations, the equation is reduced to

\[ \frac{\partial \nu}{\partial \tau} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left[ \xi \frac{\partial \nu}{\partial \xi} \right] - \frac{1}{u \xi} \frac{\partial}{\partial \xi} \left[ \xi \frac{\partial u}{\partial \xi} \right] \nu \]  
(2.19)

During the first injection period when there is no water bank, the solution for the dimensionless pressure can be expressed as below and is called the self-similar solution.

\[ p_d(\xi) = q_d \int_0^\infty \frac{d\xi}{\xi u^2(\xi)} \]  
(2.20)

The logarithmic derivative from the above dimensionless pressure solution can be expressed as,

\[ \frac{\partial p}{\partial \ln(t)} = \frac{q}{4\pi h \tilde{\lambda}_w} \frac{xf(x)}{\xi u^2(\xi)} = \frac{q}{4\pi h} \begin{cases} \frac{1}{\tilde{\lambda}_o}, & t > 0 \\ \frac{1}{\tilde{\lambda}_w}, & t \rightarrow \infty \end{cases} \]  
(2.21)

The logarithmic derivative shows a peculiar trend, the early time value reflects the mobility of the oil zone ahead of the Buckley-Leverett front and the late time value reflects the mobility of the fluid in the water-invaded zone behind the front. This is particularly observed in an injection well test in which no fluid bank is present at the start of the water injection. This also implies that the pressure solution is valid only for the
first injection period. For the injection well-test in the form of multiple flow sequences with intermittent shut-in and variable flow rates, Levitan suggested the full solution. However, the information that is contained in the multi-rate test is also inherently embedded in the single rate test. This is manifested in the fact that the late time pressure derivative response following any rate that is maintained constant over a sufficiently long period of time is the same for all rates.

Fig 1.4 depicts the BHP vs. log (time) and the BHP-derivative vs. log (time) from an injection well test for the first injection period. Fig 1.4 clearly shows the presence of two slopes in the pressure-time plot, the early time slope corresponding to the uninvaded region and the late time slope corresponding to the invaded region in the reservoir. This is similar to Fig 1.1 as discussed earlier from Woodward and Thambynayagam\textsuperscript{1}. Since there is no initial water bank present at the start of the injection sequence, we do not see the first straight line seen in Fig 1.1, and the pressure transient reflects the injected fluid properties and flow conditions behind the flood-front soon after the start of injection.

Fig 1.5 and Fig 1.6 show the pressure and the logarithmic derivative response in case there is a water-bank present at the start of the injection period. It is important to note the difference between the derivative-time curves for the case of water injection with an initial water bank present to the case without it. The early time logarithmic derivative value now reflects the fluid properties in the water bank which existed at the start of the sequence. The middle portion of the curve corresponds to the properties of the uninvaded zone ahead of the Buckley-Leverett front and the late time portion of the curve corresponds to the fluid mobility for the region behind the Buckley-Leverett front in the reservoir. This typically resembles the three portion derivative-time curve (Fig 1.1) which was presented by Woodward and Thambynayagam\textsuperscript{1}. 
Another important feature of the time-derivative plot for the case of water injection with an initial water bank (which is the case in a variable rate sequence) (Fig 1.6) is that the very late time derivative value becomes equal eventually for any injection rate. This can be explained as follows. The pressure transient first propagates through the water bank that is present at the start of the injection period. It then traverses the uninvaded region till the time the total injected volume is less than the volume occupied by the water bank. At late time, the pressure transient reflects the properties of injected fluid in the invaded region behind the front which are the same for any injection rate. Hence, the late time pressure response from any injection rate that is maintained constant for a sufficiently long time is the same. In a pressure-log (time) plot this property will manifest itself in the fact that the slope of the pressure-log (time) plot at late time for any rate will be equal. It is also worthwhile to note that if the injected fluid is the same as that in the water bank, which is typically the case in waterfloods, the first and the third portions of the derivative-time curve will be similar.

So far we have reviewed the major developments in injection well testing. We looked at the particular behavior of the derivative-time curve of the bottom-hole pressure for a water injection well. Different portions of the derivative-time curve reflect the properties of different zones in the reservoir that develop during water injection well test. We shall see later that this feature of injection well testing can be utilized for two important purposes:

1. Reservoir appraisal
2. Estimating residual oil saturation in surfactant flooding EOR processes

In the application of injection well testing for reservoir appraisal, we utilize the fact that the derivative-time curve becomes similar in slope for any rate in a variable rate schedule if that rate is maintained constant for sufficient length of time. This late time
derivative value will contain information of the fluid mobility in the water invaded region behind the front. The calculated fluid mobility can be used to determine reservoir permeability using appropriate relative permeability curves. These will be discussed in detail in Chapter 3 and Chapter 4.

Injection well testing methods can also be used to determine residual oil saturation after surfactant flooding. As will be discussed in Chapter 5, pressure-time data obtained during surfactant injection can be used to calculate the fluid mobilities in the surfactant flooded region. Those fluid mobility values can be then converted to fluid saturation values using trapping number dependent relative permeability models.

We shall focus on these aspects of injection well testing in the later chapters. In the next section of this literature review look at some of the common industry methods used for determining residual oil saturation.

**EVALUATION OF RESIDUAL OIL SATURATION: A LITERATURE REVIEW**

Residual Oil Saturation (ROS) is the oil remaining in the swept part of the reservoir after the application of a secondary recovery method i.e. a waterflood. ROS is indicative of the effectiveness of waterfloods and can also be defined for surfactant flooding processes. As such it is an important consideration in planning future waterflood or EOR activities for the producing field. Many different techniques for measuring ROS have been applied in the oil industry to varying degrees of success. In this section, we present an overview of the methods used to determine ROS.

Chang et al\(^{10}\) have presented a summary of the various methods available for determining residual oil saturation. The different techniques that have been used to determine the residual oil saturation can be broadly classified as follows:

1) Single-Well ROS measurements
1. Core Analysis
   i. Conventional coring
   ii. Pressure Coring
   iii. Sponge Coring
2. Single Well Tracer Tests (SWTT)
3. Well Logging
   i. Openhole Logs
      A) Resistivity logs, Nuclear Magnetism Log (NML), Electromagnetic wave Propagation Tool (EPT), Dielectric-constant Log
   ii. Cased-Hole Logs
      A) Pulsed Neutron Capture (PNC)
      B) C/O Logs
      C) Gravity Log
4. Single Well Transient Testing Methods
   
   2) Inter-well ROS measurements
      1. Well-to-Well Tracer Test
      2. Reservoir fluid displacement and arrival of bank monitoring technique

Core analysis\textsuperscript{10} is a direct method of measuring ROS in the laboratory. Based on the type of core available, core data in the laboratory can be obtained from conventional coring, pressure coring and sponge coring. In conventional coring, the core is retrieved in simple coring tools which do not maintain the pressure and temperature conditions from
the depth the core is retrieved from. This kind of coring suffers from the fact that when the core is brought to the surface, the phase saturations in the core may have been altered due to a drastic change in pressure and temperature conditions at the surface compared to the reservoir. The oil may expand due to a decrease in pressure and simultaneously, loss of gases can lead to oil shrinkage. Water also suffers similar changes at the surface. The result is that the saturations that are measured are not indicative of the conditions that prevail in the reservoir. Pressure coring is a method in which the core retrieving tool retains the core at the same pressure as that of the reservoir. This solves the problem of fluid expansion on reduction in pressure at the surface. Pressure coring is rarely used because it is so expensive to implement. In sponge coring, a sponge-sleeve modification to the conventional core barrel is used such that the sponge absorbs the oil that bleeds from the core by expulsion due to reduction in pressure at the surface. The volume of oil expelled into the core can be retrieved in the laboratory and used to calculate the in-situ oil saturation in the core. The core can then be used to obtain residual oil saturation by conducting a waterflood.

Single-Well Tracer Tests\textsuperscript{10,11} involve injecting a chemical tracer like ethyl-acetate with water in the reservoir and then back-flowing the well after a few days. Ethyl-acetate partitions in oil and water phases and a portion of ethyl-acetate that remains in the water phase hydrolyses to form the second tracer, ethanol. These two different chemicals, ethyl-acetate and ethanol have different partitioning coefficients and act as two different tracers. When the well is produced, these two tracers are produced at different times depending on their flow velocities which depend on the partitioning coefficients. This difference in arrival times is used to measure the residual oil saturation. The single well tracer tests have reasonable accuracy and have a considerable depth of investigation (typically 10 to 40 ft from the well bore into the reservoir). This depth of investigation
can be controlled by varying the volume of tracer injected in the reservoir. This method measures the average residual oil saturation in that part of the reservoir that is contacted by the water and is affected by the sweep efficiency during injection.

Well logging\textsuperscript{10,12,13,14,15} procedures are the most widely used methods of determining residual oil saturation. Resistivity logs have relatively deeper depths of investigation compared to some of the other logging techniques. Measurements taken by the resistivity tool can be used to determine ROS using the Archie’s equation if the formation parameters such as porosity, cementation factor and saturation exponent are known. In the Log-Inject-log technique, once the formation is logged for resistivity, the residual oil is displaced using chemicals and the resistivity log is run again after injection of brine. This reduces some of the uncertainties associated with the resistivity log\textsuperscript{10}. Nuclear Magnetic Resonance Logs (NML)\textsuperscript{10} directly measure the water saturation in the invaded zone and the virgin reservoir, thus providing an estimate of the producible oil saturation. Electromagnetic Propagation Tools (EPT) estimates ROS by measuring the phase shift and the attenuation of an electromagnetic wave propagated through the formation at a high frequency\textsuperscript{10}. The EPT log is less sensitive to changes in formation brine salinity and as such can be used for saturation measurements where the resistivity logs fail to provide accurate estimates due to unknown or low salinity. The tool has good thin bed resolution, however, the depth of investigation of this tool is only a few inches into the formation from the well bore and the results are affected by drilling mud damaged zone around the well bore. Special interpretation models are required to correct the inaccuracies induced by the short depth of investigation. Both EPT and NML have been suggested as not providing ROS values to reasonable degree of accuracy due to lack of appropriate interpretation models\textsuperscript{10}. 

19
The cased-hole logs like Pulsed Neutron Capture (PNC)\textsuperscript{10,12}, C/O (Carbon-Oxygen log)\textsuperscript{10,13} and gravity log\textsuperscript{10,14} are valuable in estimating water saturations through the metal casing. The PNC log works by emitting neutrons through the well bore into the formation and then measuring their decay rate. Hence, this log reflects the neutron absorptive properties of the formation. Since chlorine is the strongest thermal neutron absorber of the known earth elements, the PNC log responds largely to the amount of NaCl present in the formation water and can be easily correlated with the resistivity log. The emitted thermal neutrons get absorbed by the capture cross section of the formation which is the sum of the component cross sections of the rock matrix and the pore fluids according to the equation:

\[
\Sigma_b = \Sigma_{ma}(1 - \phi) + \Sigma_w S_w \phi + \Sigma_{hc}(1 - S_w)\phi
\]  

(2.22)

Where:

- \(\Sigma_b\) = Bulk capture cross section
- \(\Sigma_{ma}\) = Matrix capture cross section
- \(\Sigma_w\) = Water capture cross section
- \(\Sigma_{hc}\) = Hydrocarbon capture cross section

To determine \(S_w\) from the above equation, four parameters need to be known: \(\Sigma_{ma}, \Sigma_w, \Sigma_{hc}, \phi\). Conventionally, the water and hydrocarbon cross sections are determined from the laboratory analysis of the fluids and use of published data on cross-sections, the porosity is obtained from conventional core analysis or porosity logs. However, the matrix capture cross section is the least certain. To eliminate this uncertainty, Log-Inject-Log techniques were developed in which the formation was once logged for the neutron capture response, and then flushed with either water of known salinity and capture cross section or chemicals that could displace the remaining oil. When water is injected after first log, the second log measurement gave two simultaneous
equations from which the water saturation could be calculated after eliminating the uncertain matrix capture cross section from the two equations. In case of chemical injection following the first log, it is assumed that all the remaining oil is displaced so that the zone near the well bore is fully saturated with water. The second log measurement will result in two simultaneous equations as above and could be solved for the water saturation prior to the first log.

The C/O log (Carbon/Oxygen log) works on the principle that the nuclei of different elements react in predictable ways when bombarded with high energy neutrons. Different elements can thus be identified by the energy levels of the gamma rays that are released as a result of neutron bombardment. From these measurements, the C/O ratio can be determined which effectively indicates the presence or absence of hydrocarbons\textsuperscript{10,13}. The C/O log is insensitive to the chloride content of the formation water, hence it can be used in wells where PNC is not applicable as the latter relies on the water saturations measured on the basis of neutron capture primarily by chloride ions in the formation water.

The gravimetric logging technique for the determination of ROS uses a borehole gravity meter that measures the vertical component of the acceleration due to earth’s gravity (g) at two preselected positions along the wellbore with known vertical separation\textsuperscript{14}. From these measurements the bulk density of the formation volume over an investigation radius of approx. 50 ft can be known. The ROS determination using this method needs a base measurement for the bulk density early in the producing life of the well and a measurement at the time when the ROS is required. The difference in the two bulk densities can be used to evaluate the change in water saturation in the formation from which ROS can be calculated. The advantages of this method are that the ROS measurements are independent of near well bore effects like hole size, near well-bore
damage etc. The disadvantages are that a base measurement is needed early on in the life of the well and that the vertical resolution is small, approx. 10 ft.

The use of transient well testing has also been reported in the literature as having the potential to give reasonable estimates of the ROS\textsuperscript{10,15}. The method involves measuring the slope of the traditional BHP vs. Log (time) plots and using it to determine effective permeability to the mobile phase near the well bore. This could be converted to saturation values using appropriate relative permeability curves. This has already been discussed in detail earlier in this chapter.

In the inter-well ROS measurement with tracer tests, two or more tracers of different partitioning coefficients between oil and water phases are injected in the formation. As the fluid carrying the tracers moves through the reservoir, one tracer is retarded more than the other. By monitoring the degree of separation of these tracers at the observation well, an estimate of the inter-well ROS can be made\textsuperscript{10}.

In US Patent 3874451\textsuperscript{16}, the inventors Stanley and Robert have described a method to determine the oil saturation in a reservoir, preferably the ROS after waterflood. In this method, a fluid is injected in the reservoir which is capable of displacing both oil and water and forms a water-oil bank. This displacing fluid can typically be a surfactant solution. The arrival of the water-oil bank at the observation well can be determined by the change in the slope of the build-up bottom-hole pressure profile in the observation. The theory is based on the fractional flow of oil and water and results in the following equation to determine ROS:

\[
S_{oi} = f_{ob} (1 - V_i) + f_{oi} V_i
\]

(2.23)

Where:

\(S_{oi}\) = Initial oil saturation (is equal to \(S_{orw}\) after waterflood)

\(f_{ob}\) = fractional flow of oil in the oil water bank
\[ V_i = \text{Pore volume of displacing fluid injected based on the radial volume between injector and observation well.} \]

\[ f_{oi} = \text{Fractional flow of oil before the application of this method (} = \text{zero after waterflood,} \quad = 1 \text{ for } S_{wi} = S_{wc} \) \]

The fractional flow of oil in the water-oil bank can be determined by core-floods done in the laboratory or can be determined from the producing water-oil ratios in the observation well. Similarly, the initial fractional flow of oil is known from water-oil ratios prior to the application of this method. The total volume of injected fluid can be measured by measuring the flow rate at the surface and is a known parameter. Hence, by the application of the above equation, the oil saturation prior to the application of this method can be known.

In this section we very briefly reviewed the various methods and techniques which are available to determine the oil saturation in the reservoir. Most of these methods were developed to determine ROS after the waterflood primary recovery so that appropriate economic analysis for tertiary recovery processes could be undertaken. In the next few chapters, we present a method to determine ROS before and after a planned surfactant flood is conducted in a reservoir.
Fig 2.1: Injection test pressure build-up: after Woodward and Thambyanayagam showing three distinct slopes.

Slope, $m_1 = \frac{162.6qB_w\mu_w}{k_{wh}}$

Slope, $m_2 = \frac{162.6qB_w\mu_o}{k_{oh}}$
Fig 2.2: Schematics of multiple fluid-bank reservoir and water saturation profile in the reservoir, from Yeh and Agarwal.
Fig 2.3: Saturation vs. radial distance showing 2 shock fronts, from Bratvold and Horne.
Fig 2.4: BHP vs. Log (time) and BHP-Derivative vs. Log (time) for the first injection period in the absence of an initial water bank, from Levitan.
Fig 2.5: Variable rate schedule for the injection test: Fig 2.4 above corresponds to injection period 2 marked above, from Levitan\textsuperscript{6}. 
Fig 2.6: BHP vs. Log (time) and BHP-Derivative vs. Log (time) for the 2nd and 5th injection period in the rate schedule shown in Fig 2.5, from Levitan\textsuperscript{6}
Chapter 3: Implementation of the Mathematical Solution to the Multiple Region Moving Boundary Problem in Excel VBA

INTRODUCTION

The analytical solution to the multiple region moving boundary problem given by Levitan\(^6\) (as discussed in Chapter 2), was implemented in Excel VBA. In this chapter we present the mathematical simplification of the equations presented earlier and also discuss the algorithm used for programming the solution. Results of this analytical model allow us to access the importance of different parameters and to verify the results of the numerical model for more complex cases presented in the next chapter.

MATHEMATICAL FORMULATION OF THE MODEL

**STEP 1: Plot \( S_w \) as a function of the transformed variable \( X \). \( S_w = F(x) \)**

From Eq. (2.5),

\[
x = \frac{r_d}{\sqrt{2\alpha \theta(t_d)}}
\]

Substituting the expressions for ‘\( r_d \)’, ‘\( \alpha \)’ and ‘\( \theta(t_d) \)’ from Eq. 2.6, 2.7, 2.8 and 2.9 in Eq. 2.5 we get

\[
x = \frac{1}{x_f} \sqrt{\frac{\pi r^2 h \phi}{q*t}}
\]

(3.1)

Where:

- \( x_f \) is the normalizing factor such that the fluid front is always located at \( x=1 \).
- ‘\( q \)’ is the injection rate
- ‘\( t \)’ is the time

Now, for the radial form of the Buckley-Leverett equation,
\[
\frac{dr}{dt} = \frac{q}{2\pi r h \phi} \frac{dF_w}{dS_w}
\]  

(3.2)

Where:

\(F_w\) = the fractional flow of water

\(S_w\) = water saturation

\(r\) = radial distance of the front location with saturation \(S_w\)

Integrating Eq. 3.2,

\[
\frac{\pi r^2 h \phi}{q} = \frac{dF_w}{dS_w}
\]  

(3.3)

Using Eq. 3.3, Eq. 3.2 can be written as:

\[
[x]_{S_w} = \frac{1}{x_f} \sqrt{\frac{dF_w}{dS_w}}_{S_w=S_wf}
\]  

(3.4)

Where:

\[
x_f = \sqrt{\frac{dF_w}{dS_w}}_{S_w=S_wf}
\]  

(3.5)

Where, \(S_{wf}\) is the shock front saturation evaluated as the point of tangency for a tangent drawn from the initial saturation to the fractional flow curve (Fig 3.2).

The derivative \((dF_w/dS_w)\) corresponding to a saturation \((S_w)\) is available from the fractional flow theory. Therefore, using Equations 3.4 and 3.5, an array of saturation values can be converted to an array of the transformed variable ‘x’. Once we have the two corresponding arrays of ‘\(S_w\)’ and ‘x’, we can plot \(S_w\) as a function of the transformed variable x. This is shown in Fig 3.3, Fig 3.4, and Fig 3.5.
**STEP 2: Evaluate total mobility and total compressibility as functions of ‘x’**

To every saturation value $S_w$ corresponds a dimensionless total mobility and dimensionless total compressibility defined as:

$$\lambda_d(S_w) = \left[\frac{k \cdot k_{rw}(S_w)}{\mu_w} + \frac{k \cdot k_{ro}(S_w)}{\mu_o}\right] \frac{1}{\lambda_o}$$  \hspace{1cm} (3.6)

Where:

$$\lambda_o = \frac{k \cdot k_{ro}(S_{wc})}{\mu_o}$$

$\mu_w$ = Water viscosity (cp)

$\mu_o$ = Oil viscosity (cp)

$k$ = Absolute reservoir permeability (md)

$S_{wc}$ = Connate water saturation

And:

$$c_d(S_w) = \frac{c_i(S_w)}{c_{to}}$$  \hspace{1cm} (3.7)

Where:

$c_{to} = c_i(S_{wc})$

Using equations 3.6 and 3.7, total dimensionless mobility and compressibility can be plotted as functions of ‘x’ as shown in Fig 3.6 and Fig 3.7.

**STEP 3: Calculation of other dimensionless parameters as functions of ‘x’**

Using dimensionless total mobility and compressibility calculated as functions of x, we calculate several other dimensionless parameters as listed below:

$$f(x) = \sqrt{\frac{c_d(x)}{\lambda_d(x)}}$$  \hspace{1cm} (3.8)

$$\bar{\xi}(x) = \int_0^x f(x) dx$$  \hspace{1cm} (3.9)
**STEP 4: Calculation of other dimensionless parameters as functions of \( X \)**

Next we calculate the dimensionless pressure and the logarithmic pressure derivative as below:

\[
p_d(\xi) = q_d \int_{\xi}^{\infty} \frac{d\xi}{\xi u^2(\xi)}
\]

(3.11)

\[
\frac{\partial p}{\partial \ln(t)} = \frac{q}{4\pi h \hat{\lambda}_w} \frac{xf(x)}{\xi u^2(\xi)} = \frac{q}{4\pi h} \left\{ \begin{array}{ll}
1/\hat{\lambda}_o, & t \to 0 \\
1/\hat{\lambda}_w, & t \to \infty
\end{array} \right\}
\]

(3.12)

Where:

\[
p_d = (p - p_i) \frac{2\pi h \hat{\lambda}_w}{q_o}
\]

\[
\hat{\lambda}_w = \frac{kk_{rw}(S_{or})}{\mu_w}
\]

It is important to note that all the dimensionless parameters and the resulting pressure equations 3.11 and 3.12 have been obtained as functions of ‘\( x \)’. ‘\( x \)’ itself is a function of the radial distance and time and is given by Eq. 3.4. Hence, for a fixed radial distance, \( r = r_w \), as time varies so does the value of ‘\( x \)’. Thus corresponding to each time, there is a value of ‘\( x \)’ and hence a corresponding value of the dimensionless pressure and logarithmic derivative given by Eq. 3.11 and 3.12. We can thus obtain a pressure vs. time response at the wellbore. This is shown in Fig 3.8. The curves obtained from the program were matched with the curves published by Levitan\(^6\).
CHAPTER SUMMARY

In this chapter we presented a simplification of the mathematical solution to the multiple region moving boundary problem given by Levitan\textsuperscript{6}. We also described a step by step procedure for calculating the pressure response at the wellbore during water injection in a homogeneous reservoir with radial flow geometry. It is important to note, however, that this solution is valid only for the first injection period when there is no water bank at the start of the injection sequence. However, the late time logarithmic pressure derivative for the first and any subsequent injection period is the same and is given by equation 3.12. This fact was explained in Chapter 2 and it can be utilized for the calculation of reservoir permeability using injection pressure data. This application of injection well testing is described in detail in the next chapter.
TABLE 3.1: Reservoir and Fluid Data from SPE 77532

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness</td>
<td>h</td>
<td>ft</td>
<td>100</td>
</tr>
<tr>
<td>Res. Permeability</td>
<td>k</td>
<td>md</td>
<td>200</td>
</tr>
<tr>
<td>Injection Rate</td>
<td>Q</td>
<td>BPD</td>
<td>500</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>(\mu_w)</td>
<td>cp</td>
<td>0.25</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>(\mu_o)</td>
<td>cp</td>
<td>0.3</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>(C_w)</td>
<td>Psi(^{-1})</td>
<td>3.00E-06</td>
</tr>
<tr>
<td>Oil Compressibility</td>
<td>(C_o)</td>
<td>Psi(^{-1})</td>
<td>9.00E-06</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>(C_r)</td>
<td>Psi(^{-1})</td>
<td>5.00E-06</td>
</tr>
</tbody>
</table>
TABLE 3.2: Rock Properties Data for Water Flood from SPE 77532

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-point relative permeability to water in water flood</td>
<td>$k_{rw}^o$</td>
</tr>
<tr>
<td>End-point relative permeability to oil in water flood</td>
<td>$k_{ro}^o$</td>
</tr>
<tr>
<td>Saturation exponent for water</td>
<td>$n_w$</td>
</tr>
<tr>
<td>Saturation exponent for oil</td>
<td>$n_o$</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>$S_{wirr}$</td>
</tr>
<tr>
<td>Residual oil saturation</td>
<td>$S_{orw}$</td>
</tr>
</tbody>
</table>
Fig 3.1: Relative permeability curves for water injection process
Fig 3.2: Fractional flow curve and shock front saturation

$F_w = \text{Fractional Flow of Water}$

$S_w = \text{Sw} = \text{Shock front saturation}$

$S_w = \text{Sw} = \text{Initial water saturation}$
Fig 3.3: Tracing the saturation profile on the fractional flow derivative plot
Fig 3.4: X and Y axes interchange from Fig 3.3 results in familiar saturation profile with a BL shock front.
Fig 3.5: Saturation profile against the transformed variable ‘x’, shock front is located at x = 1
Fig. 3.6: Dimensionless total mobility against transformed variable ‘x’
Fig 3.7: Dimensionless total compressibility against transformed variable ‘x’

\[ C_{td} = \text{Dimensionless total compressibility} \]

\[ X (\text{Levitan's transformed variable}) \]
Fig 3.8: Comparison of implemented Excel program with published results from Levitan’s paper SPE 77532
Chapter 4: Numerical Simulation of Water Injection in Homogeneous Isotropic Reservoir with Radial Flow Geometry

INTRODUCTION

In this chapter we discuss the applications of injection well testing methods in determining reservoir permeability. We begin by formulating a numerical model and then present several results from the numerical simulations of water injection in a homogeneous reservoir with radial flow geometry. These results are compared with the analytical solution. Injection well testing methods can be put to good use for appraisal wells. Production well testing involves elaborate testing facilities on the drilling rig and also results in release of burnt or un-burnt hydrocarbons to the atmosphere. Injection well testing overcomes these drawbacks and is also a cleaner way of reservoir appraisal.

CMG IMEX SIMULATOR

Introduction

IMEX is a black-oil simulator developed by CMG. IMEX can be run in the fully implicit mode or the adaptive implicit-explicit mode which only solves a few selected well blocks parameters implicitly and hence runs faster. All the numerical simulation results that are presented in this chapter were run fully implicitly in IMEX.

Description of the Simulator Data Sections

Gridding

The reservoir was gridded radially with the well in the center of the grid. The grid sizes were taken to be small (of the order of 0.01 ft) near the well-bore and were steadily made coarser as the distance from the well-bore increased. To optimize the run time, the grids were kept relatively fine out to the distance where the fluid front would have
reached during the simulation time and coarse grids were used beyond that distance. Also, for well testing purposes, radial grids are more suitable than the Cartesian grids for greater accuracy and faster convergence.

**Reservoir Dimension and Boundary Conditions**

The reservoir dimension was kept so as to keep the flow regime in the transient state or the infinite-acting state during the simulated time period. No-flow boundary-condition was used for the simulation runs but we do not see the boundary effects during the simulated time period in any of the results that are presented in this chapter.

**Fluid Description and Phase Behavior**

Oil-water model option from IMEX was used for the simulation runs. It uses a two-phase, oil and water model, with no modeling of free gas or variation in solution gas.

**Rock Properties**

Corey type relative permeability curves were used for low trapping number water-flood displacement processes. The end-point relative permeability values and the residual saturations will be discussed on a case-by-case basis later in this section.

**Initial Conditions**

Several simulations were made under varying initial conditions. The initial oil and water saturations were varied from residual water saturation to residual oil saturation after water-flood at the start of the water injection process. These will be discussed with the case studies later in the chapter.

**Well Model**

IMEX uses the radial well model for calculating well-bore-pressures for radial grid geometry.
\[ Q_I = \frac{\lambda_l h (P_w - P_{wb}) * 2\pi k}{(\ln \frac{r_e}{r_w} + S)} \]

Where:

- \( Q \) = the injection rate
- \( P_w \) = the well-bore pressure and
- \( P_{wb} \) = the well block pressure.
- \( R_e \) = the effective block radius
- \( R_w \) = the well-bore radius
- \( k \) = the absolute permeability
- \( H \) = the reservoir thickness
- \( \lambda_l \) = the fluid mobility
- \( S \) = Skin factor

This radial inflow model couples the pressure in the well-bore to the average grid-block pressure. This inflow model is similar to the Peaceman well bore model for cartesian grids. It assumes an equivalent radius \( r_e \), which is the effective radial distance at which the well-block pressure is acting and then couples it with the radial Darcy inflow equation for steady state flow.

**Simulation Results from Illustrative Cases**

In this section we discuss the set-up, formulation and results from several simulation runs that were made for different cases of water injection in a homogeneous reservoir with radial flow geometry. For each case, important simulation data and pressure-time plots are given followed by a detailed discussion on the results that were
obtained. Calculation of reservoir permeability from the pressure-time data is then presented.

**Case 1: Water Injection in a Reservoir with Connate Water Saturation**

The initial water saturation in the reservoir was assumed to be the connate saturation, \( S_{wi} = S_{wc} \). Other important simulation data and relative permeability data are presented in table 4.1, Table 4.2 and Fig 4.1.

**Results and Discussion:**

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 4.2. From the graph, a linear late time graph is evident. The slope of this late time curve can be calculated as:

\[
\frac{\partial p}{\partial \log t} = 660 - 562 = 98 \text{ psi / cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 42.6 \text{ psi / cycle}
\]

**Calculation of reservoir permeability:** The late time derivative data reveal the fluid flow conditions near the well and behind the fluid front. In this case, these conditions are mobile water at residual oil saturation.

The reservoir permeability can be calculated using the relationship:

\[
\frac{\partial p}{\partial \ln t} = \frac{q * B * \mu_w}{4\pi h * k * k_{rw-w}^o}
\]

(4.1)

Where:

- \( B \) is the water formation volume factor = 1.0
- \( Q \) is the injection rate = 500 bpd
- \( H \) is reservoir thickness = 20 ft
\( \mu_w \) is water viscosity = 1 cp

\( k_{rw}^0 \) -water = end point relative permeability of water = 0.2

From the above relationship, reservoir permeability was calculated as:

\[
k = \frac{q \cdot B \cdot \mu_w}{4\pi h \cdot \frac{\partial p}{\partial \ln t} \cdot k_{rw}^o \cdot \text{water}} = \frac{70.716 \cdot 500 \cdot 1 \cdot 1}{20 \cdot 42.6 \cdot 0.2} = 207.5 \text{ md} \quad (4.2)
\]

The calculated value of the reservoir permeability is in good agreement with the input value of 200 md.

This example shows how to calculate the reservoir permeability using the injection pressure data for the case of low capillary number displacements like water-flood. In this example, the initial water saturation was assumed to be the irreducible water saturation. In the next example, we take a look at the case of water injection in a reservoir that has been under water flood for a considerably long period such that all the mobile oil near the well-bore has been displaced by the water.

**Case 2: Water Injection in a Reservoir with Residual Oil Saturation**

The reservoir was assumed to be at the residual oil saturation, \( S_{wi} = 1 - S_{or} \). Other important simulation data and relative permeability data are presented in table 4.1, Table 4.2 and Fig 4.1.

**Results and Discussion:**

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 4.3. From the graph, a linear late time graph is evident. The slope of this late time curve can be calculated as:

\[
\text{Slope-Late Time} \quad \frac{\partial p}{\partial \log t} = 700 - 601 = 99 \text{ psi / cycle}
\]
\[
\frac{\partial p}{\partial \log t} = 43 \text{ psi / cycle}
\]

*Calculation of reservoir permeability:* The late time derivative data reveal the fluid flow conditions near the well and behind the fluid front. In this case, these conditions are mobile water at residual oil saturation.

The reservoir permeability can be calculated using equation 4.2 described earlier as:

\[
k = \frac{q \cdot B \cdot \mu_w}{4\pi h \cdot \frac{\partial p}{\partial \ln t} \cdot k_{rw}^o} = \frac{70.716 \cdot 500 \cdot 1 \cdot 1}{20 \cdot 40 \cdot 0.2} = 205.6 \text{ md}
\]

The calculated value of the reservoir permeability is in good agreement with the input value of 200 md.

This example shows that the initial water saturation has no effect on the interpretation of results, a fact that will be revisited while studying the pressure response during surfactant injection. Hence, the discussed method of calculating the reservoir permeability can be utilized at any stage during the life of an injection well.

**Case 3: Water Injection in a Reservoir with an Initial Water Bank**

This example represents the case of in which injection had been carried out for a considerable period of time such that a water bank exists at the start of the injection test. The initial water saturation distribution was set as \(S_{wi} = 1 - S_{or}\) for the first 500 ft from the wellbore and \(S_{wi} = S_{wc}\) thereafter. Other important simulation data and relative permeability data are presented in Table 4.1, Table 4.2 and Fig 4.1.

**Results and Discussion:**

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 4.4. From the graph two linear sections are evident. The early
time slope reflects the fluid and reservoir properties during the time the pressure transient travels through the initial water bank. After that, the pressure response exhibits properties of the reservoir beyond the initial water bank further away from the well. The early-time slope can be calculated as:

\[
\text{Slope-Early Time} = \frac{\partial p}{\partial \log t} = 600 - 500 = 100 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \log t} = 43.43 \text{ psi / cycle}
\]

*Calculation of reservoir permeability:* The early time derivative data reveal the fluid flow conditions near the well and behind the fluid front. In this case, these conditions are mobile water at residual oil saturation.

The reservoir permeability can be calculated using equation 4.2 described earlier as:

\[
k = \frac{q \ast B \ast \mu_w}{4\pi h \ast \frac{\partial p}{\partial \ln t} \ast k_{rw - water}^*} = \frac{70.716 \ast 500 \ast 1 \ast 1}{20 \ast 43.43 \ast 0.2} = 203.5 \text{ md}
\]

The calculated value of the reservoir permeability is in good agreement with the input value of 200 md.

This example shows that in the case of a reservoir with an initial water bank, early injection data can be used to calculate reservoir permeability. It is important to note that the duration of this early response depends on the size of the water bank and the fluid properties. For the fluid and rock data set used in this simulation, this transient pressure response lasts for almost 10 hrs as shown in Fig. 4.4. Woodward and Thambynayagam suggest calculations for estimating the duration for which the early pressure transient will represent the properties of the water bank through which it traverses.
Case 4: Water Injection with Variable-Rate Injection Sequence

A variable rate injection schedule was followed as shown in Fig 4.5. The initial water saturation in the reservoir was assumed to be the connate water saturation, $S_{wi} = S_{wc}$. Other important simulation data and relative permeability data are presented in table 4.1, Table 4.2 and Fig 4.1.

Results and Discussion:

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 4.5. It is evident from the graph that we observe straight line segments for all the three injection periods. The slopes for the 1st and 3rd injection periods have been calculated as below:

Slope-Period 1

\[
\frac{\partial p}{\partial \log t} = 50 - 40 = 10 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 4.34 \text{ psi / cycle}
\]

Reservoir permeability can be calculated using Eq. 4.2:

\[
k = \frac{q * B * \mu_w}{4 \pi h * \frac{\partial p}{\partial \ln t} * k_{rw -water}^{o}} = \frac{70.716 * 50 * 1 * 1}{20 * 4.34 * 0.2} = 203.7 \text{ md}
\]

Slope-Period 3

\[
\frac{\partial p}{\partial \log t} = 317 - 276 = 41 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 17.8 \text{ psi / cycle}
\]

Reservoir permeability can be calculated using Eq. 4.2:
Thus we find that the reservoir permeability calculated using the logarithmic derivative (slope of the curve) for both the injection periods are close to the input value to the simulator of 200 md. It is important to note that a variable injection rate schedule was followed. For the first injection period, the pressure profile began reflecting the reservoir properties in the flooded zone soon after the start of injection. For the third injection period, the pressure profile assumes a slope that is indicative of the reservoir properties at late time. This is in accordance with the discussion that was presented in Chapter 3 and Chapter 2 while discussing the nature of the solution presented by Levitan\textsuperscript{6}. For any given injection rate that is kept constant for a sufficient period of time in a variable rate schedule, the pressure profile at late time is the same and is indicative of the reservoir properties behind the fluid front. The late time logarithmic derivative for any such injection period is given by equation 3.12. The same fact was utilized above in calculating reservoir permeability from late time pressure derivative data.

**CHAPTER SUMMARY**

In this chapter, we studied the application of water injection well testing methods for determining reservoir appraisal parameters. Water injection well test can be taken up for appraisal wells as well as for old injection wells in a producing field. This also implies that the presence of an initial water bank will not affect the interpretation of results and pressure interpretation will still lead to correct estimates of reservoir permeability as we saw for case 3. We also looked at the case of variable rate schedule and noticed that the late time pressure data for any injection rate maintained constant for a sufficiently long period of time can be used to calculate the reservoir permeability.
One underlying assumption in the application of injection well testing methods to reservoir appraisal is that the rock properties in terms of the relative permeability curves are known. Relative permeability data can be easily obtained from routine core analysis and fair estimates of the relative permeability curves are generally available for a producing field. In the next chapter, we take a look at the application of well testing methods for determining residual oil saturation after a surfactant flooding process.
TABLE 4.1: Simulation Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness</td>
<td>H</td>
<td>ft</td>
<td>20</td>
</tr>
<tr>
<td>Reservoir Permeability</td>
<td>k</td>
<td>md</td>
<td>200</td>
</tr>
<tr>
<td>Injection Rate</td>
<td>Q</td>
<td>BPD</td>
<td>500</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>µ_w</td>
<td>cp</td>
<td>1</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>µ_o</td>
<td>cp</td>
<td>3</td>
</tr>
<tr>
<td>Skin</td>
<td>S</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>C_w</td>
<td>Psi⁻¹</td>
<td>3.00E-06</td>
</tr>
<tr>
<td>Oil Compressibility</td>
<td>C_o</td>
<td>Psi⁻¹</td>
<td>9.00E-06</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>C_r</td>
<td>Psi⁻¹</td>
<td>5.00E-06</td>
</tr>
</tbody>
</table>

Note: Case 4 was a variable rate sequence simulation (Fig 4.5)
TABLE 4.2: Rock Properties Data for Water Flood (Low Capillary Number Displacement)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-point relative permeability to water in water flood</td>
<td>$k_{rw}^{o_{water}}$</td>
<td>0.2</td>
</tr>
<tr>
<td>End-point relative permeability to oil in water flood</td>
<td>$k_{ro}^{o_{water}}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Saturation exponent for water</td>
<td>$n_w$</td>
<td>2</td>
</tr>
<tr>
<td>Saturation exponent for oil</td>
<td>$n_o$</td>
<td>2</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>$S_{wirr}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Residual oil saturation</td>
<td>$S_{orw}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Fig 4.1: Relative permeability curves for water injection process
Fig 4.2: DP (psi) vs. Log (time-hrs) for water injection with $S_{wi} = S_{wir} = 0.2$
Fig 4.3: DP (psi) vs. Log (time-hrs) for water injection with Swi = 1-Sor = 0.75
Fig 4.4: $D_p$ (psi) vs. log (time-hrs) for water injection with initial water bank

$D_p = BHP - 5000$ (psi)
Fig 4.5: Dp (psi) vs. log (time-hrs) for water injection with variable rate sequence
Chapter 5: Numerical Simulation of Surfactant and Surfactant-Polymer Injection in Homogeneous Isotropic Reservoir with Radial Flow Geometry

INTRODUCTION

In this chapter we discuss the applications of injection well testing methods in determining residual oil saturation remaining before and after a chemical flooding process. First we take a look at the model formulation and present several results from the numerical simulations of surfactant injection in a homogeneous reservoir with radial flow geometry. We then discuss the analysis technique used for interpreting injection well tests performed with non-Newtonian fluids i.e. polymers.

CMG STARS SIMULATOR

STARS is a three-phase multi-component simulator that was developed by CMG to simulate advanced processes like chemical/polymer flooding, thermal applications, steam injection, fireflood etc. STARS can be run in the fully implicit mode which is the most accurate or the adaptive implicit mode which only solves a few selected well block parameters implicitly and hence runs faster. All the numerical simulation results that are presented in this chapter were run fully implicitly in STARS.

Description of the Simulator Data Sections

Gridding

The reservoir was gridded radially with the well in the center of the grid. The grid sizes were taken to be small (of the order of 0.01 ft) near the well-bore and were steadily made coarser as the distance from the well-bore increased. To optimize the running time, the grids were kept relatively fine up to the fluid front and coarse grids were used beyond
that distance. Also, for well testing purposes, radial grids are more suitable than Cartesian grids for greater accuracy and faster convergence.

**Reservoir Dimensions and Boundary Conditions**

The reservoir dimensions were kept large so as to keep the flow regime in the transient state or the infinite-acting state during the simulated time period. No-flow boundary-condition was used for the simulation runs but we do not see the boundary effects during the simulated time period in any of the results that are presented in this chapter.

**Fluid Description and Phase Behavior**

STARS is a multi-component, multi-phase, compositional simulator that uses the phase equilibrium ratios (Ki) to define the phase behavior. In modeling surfactant-floods, the 3 components are water, oil and surfactant. In case a surfactant-polymer flood is to be simulated, polymer is added as the fourth component to the system. The three phases are the aqueous phase, the oleic phase and the gaseous phase. STARS models the phase behavior in the surfactant displacement process as a two-minus or a lower phase microemulsion. The oil component can partition into the aqueous phase in the presence of surfactant. The partitioning of oil in the aqueous phase results in lowered IFT between the aqueous phase and the oil phase. This results in a high capillary number displacement process which helps in the mobilization of oil from the pore spaces of the rock.

Gas-liquid and liquid-liquid “Ki” values are entered for each component ‘i’. If a component does not partition into a particular phase, the corresponding “Ki” value is entered as zero. For example, to disallow the partitioning of water, surfactant and polymer into the gas phase, the corresponding gas-liquid “Ki” values are set to zero. The
liquid-liquid “Ki” values are used to model the partitioning of oil in the aqueous phase in the presence of surfactant.

**Rock Properties**

STARS uses a trapping number dependent relative permeability model which is based on the work done by van Quy and Labrid\textsuperscript{17} and Amaefule and Handy\textsuperscript{18}. This model will be discussed in detail later in this chapter along with another trapping number dependant relative permeability model by Pope et al\textsuperscript{19}.

Corey type relative permeability curves were used for low trapping number water-flood displacement processes. The end-point relative permeability values and the residual saturations will be discussed on a case-by-case basis later in this section. Straight line relative permeability curves were used for the ultra-low interfacial tension between the displacing and displaced phases as in the case of surfactant flooding. This is based on the trapping number dependent relative permeability model which will be discussed later.

**Initial Conditions**

Several simulations were run under varying initial conditions. The initial oil and water saturations were varied from residual water saturation to residual oil saturation after water-flood at the start of the surfactant injection process. These will be discussed with the case studies later in the chapter.

**Well Model**

We used a radial well model for calculating well-bore-pressures for the single well radial grid geometry.

\[
Q = \frac{\lambda h (P_w - P_{wb}) \times 2\pi k}{(\ln \frac{r_e}{r_w}) + S} \quad (5.1)
\]

Where:
\[ Q = \text{the injection rate} \]
\[ P_w = \text{the well-bore pressure and} \]
\[ P_{wb} = \text{the well block pressure}. \]
\[ R_e = \text{the effective block radius} \]
\[ R_w = \text{the well-bore radius} \]
\[ k = \text{the absolute permeability} \]
\[ H = \text{the reservoir thickness} \]
\[ \lambda_I \text{ = the fluid mobility} \]
\[ S = \text{Skin factor} \]

This radial inflow model couples the pressure in the well-bore to the average grid-block pressure. This inflow model is similar to the Peaceman well bore model for Cartesian grids. It assumes an equivalent radius \( r_e \), which is the effective radial distance at which the well-block pressure is acting and then couples it with the radial Darcy inflow equation for steady state flow.

**TRAPPING NUMBER DEPENDENT RELATIVE PERMEABILITY MODEL**

In the presence of surfactants, the assumption that the rock-fluid properties are functions only of fluid saturations is not sufficient to fully describe the observed flow behavior. In these cases, models which allow the interpolation between relative permeability data sets prove to be useful. STARS uses such an interpolation model based on the work done by Van Quy and Labrid\(^{17}\) and Amaefule and Handy\(^{18}\). Their work demonstrated that ultra-low IFT values lower the residual saturations of the phases and straighten the relative permeability curves. Van Quy and Labrid\(^{17}\) measured four pairs of relative permeability curves. These correspond to two extreme and two intermediate situations. The extreme conditions represent oil displaced by water, and water and oil displaced by the microemulsion. They noted that the permeabilities depended on two
parameters— the saturation, $S_w$ and the capillary number defined as $N_c = \frac{\mu_w V}{\sigma}$ where ‘$\nu$’ represents the ratio of the injection flow-rate to the constant cross section of the porous formation and ‘$\sigma$’ is the interfacial tension. Their findings revealed that oil could be completely displaced for a capillary number given by $\ln (N_c) > -6.725$ or $N_c > 1.2E-3$ and partially displaced if $-8.255 < \ln (N_c) < -6.725$ or $2.6E-4 < (N_c) < 1.2E-3$. The critical capillary numbers can be summarized as below (from STARS 2009.10 Manual):

- $N_c = 6.0E-8$ (Both water and oil residual saturations start to decrease)
- $N_c = 2.6E-4$ (Intermediate curves from van Quy and Labrid)
- $N_c = 1.2E-3$ (Residual oil saturation reaches zero)
- $N_c = 2.3E-1$ (Residual water saturation reaches zero and relative permeabilities are straight lines)

The effect of decreasing residual phase saturations with increasing capillary number can be modeled using two sets of relative permeability curves and using suitable interpolation parameters, as is done in STARS. For such a model two sets of relative permeability curves are entered.

One set corresponds to high interfacial tension in the absence of surfactant. If the capillary number is below this critical capillary number given by $\ln (N_c) = \log_{10} (6.0E-8)$ then the relative permeability curves corresponding to high interfacial tension are used and the residual phase saturations do not decrease. This critical capillary number is entered in STARS using the parameter $DTRAP_{w(1)} = DTRAP_{nw(1)} = \log_{10} (6.0E-8)$ where the subscript ‘‘$W$’’ stands for the wetting phase and ‘‘NW’’ for the non-wetting phase.

The other set of relative permeability curves corresponds to ultra-low interfacial tension values and these are used if the capillary number is more than the critical
capillary number. These critical capillary numbers for zero residual saturations are entered in STARS using \( D\text{TRAP}_W(2) = \log_{10}(2.3\times10^{-1}) \) and \( D\text{TRAP}_{NW}(2) = \log_{10}(1.2\times10^{-3}) \). For any intermediate capillary numbers, an interpolation scheme is followed. The interpolation parameters are calculated as:

\[
\alpha_W = \frac{\log_{10}(N_C) - D\text{TRAP}_W(1)}{D\text{TRAP}_W(2) - D\text{TRAP}_W(1)}, \quad \alpha_{NW} = \frac{\log_{10}(N_C) - D\text{TRAP}_{NW}(1)}{D\text{TRAP}_{NW}(2) - D\text{TRAP}_{NW}(1)}
\]

\[
\gamma_W = \alpha_W^{\beta_W}, \quad \gamma_{NW} = \alpha_{NW}^{\beta_{NW}}
\]

Where \( \beta_W \) and \( \beta_{NW} \) are the curvature interpolation parameters generally assumed to be 1 which would imply the slopes of the interpolated relative permeability curves change at the same rate as the endpoint values. Next, these calculated parameters are used to interpolate the relative permeabilities as shown below:

\[
k_{rw} = k_{rw}(1).(1 - \gamma_W) + k_{rw}(2).\gamma_W
\]

\[
k_{rnw} = k_{rnw}(1).(1 - \gamma_{NW}) + k_{rnw}(2).\gamma_{NW}
\]

Where \( k_{rw}(1,2) \) and \( k_{rnw}(1,2) \) are the relative permeability values taken from the two sets of relative permeability curves entered for low and high critical capillary numbers receptively.

For converting the permeability values determined from the well-tests to phase saturation values, another trapping number dependent relative permeability model given by Pope et. al\(^{19} \) was used. This model was developed to account for the decreased gas relative permeability due to condensate drop-out and build-up near the well bore in gas-condensate wells that show a decrease in production once the pressure falls below the dew point pressure. They suggested that the residual saturation is based on the trapping number as shown below:
\[ S_{tr} = \min(S_{lr}, S_{lr}^{high} + \frac{S_{lr}^{low} - S_{lr}^{high}}{1 + T_l(N_{TI})^{\tau_l}}) \]  

(5.6)

Where:
- \( S_{lr}^{high} \) = Residual saturation of phase ‘l’ at high trapping number.
- \( S_{lr}^{low} \) = Residual saturation of phase ‘l’ at low trapping number.
- \( T_l, \tau_l \) = Matching parameters
- \( S_{lr} \) = Residual saturation of phase ‘l’ at any trapping number.

The next step is to correlate the endpoint relative permeability with the trapping number.

\[ k_{rl}^0 = k_{rl}^{0\text{low}} + \frac{S_{lr}^{low} - S_{lr}^{high}}{S_{lr}^{high} - S_{lr}^{low}}(k_{rl}^{0\text{high}} - k_{rl}^{0\text{low}}) \]  

(5.7)

Where:
- \( k_{rl}^{0\text{low}} \) = Endpoint relative permeability of phase ‘l’ at low trapping number.
- \( k_{rl}^{0\text{high}} \) = Endpoint relative permeability of phase ‘l’ at high trapping number.
- \( S_{lr}^{low} \) = Residual saturation of the conjugate phase (the conjugate phase of water is oil) at low trapping number.
- \( S_{lr}^{high} \) = Residual saturation of the conjugate phase (the conjugate phase of water is oil) at high trapping number.
- \( k_{rl}^0 \) = Endpoint relative permeability of phase ‘l’ at any trapping number.

Having determined the residual phase saturations and the endpoint relative permeability the relative permeabilities are calculated as shown below:

\[ \log_{10} k_{rl} = \log k_{rl}^0 + \log S_l + \frac{\log(k_{rl}/k_{rl}^0) - \log S_l}{1 + T_l(N_{TI})^{\tau_l}} \]  

(5.8)

Where the normalized saturation is given by:
\[ \log \bar{S}_l = \frac{S_l - S_{br}}{1 - \sum_{l=1}^{n_p} S_{br}} \] (5.9)

Where:
\( n_p \) = Total number of phases present.
\( T_l \) = Matching parameter
\( \tau \) = Matching parameter

Fig 5.1 (a) and 5.1 (b) show example capillary desaturation curves which were calculated based on the Pope et. al\(^{19}\) model described above. High residual phase saturation at low capillary number correspond to high IFT between phases as is the case of water-oil or polymer-oil displacements. Similarly, low residual phase saturations at high capillary number correspond to low IFT between phases as is the case in surfactant-oil or surfactant-polymer-oil displacement processes. Fig 5.2 (a) and Fig 5.2 (b) show typical relative permeability curves calculated using this model at high capillary numbers on log-log and Cartesian scales respectively. It is important to note that the relative permeability curves straighten out with zero residual phase saturations at ultra-low IFT which results in high capillary numbers. Both the relative permeability models, discussed above, one by Van Quy and Labrid\(^{17}\) and the other by Pope et al\(^{19}\) give the relative permeability curves at ultra-low interfacial tensions as straight lines with zero residual saturations.

**Simulation Results from Illustrative Cases**

In this section we discuss the set-up, formulation and results from the several simulation runs that were made for different cases of surfactant injection in a homogeneous reservoir with radial flow geometry. For each case, a point wise description of the simulation injection sequence (water-surf, water-surf-water etc), relevant
application, important simulation data and pressure-time plots are given followed by a detailed discussion on the results that were obtained. For a few cases, example calculations for the determination of residual oil saturation after surfactant-flood from the pressure derivative data are presented. Similar calculations are implied for other case studies.

**Case 1: Reservoir at Connate Water Saturation**

Water was injected into the well for a period of 6 hrs followed by surfactant injection. The initial water injection test sequence was to establish a baseline mobility value for injected water near the well-bore. This was followed with surfactant injection which further lowers the oil saturation near the well-bore and enhances the mobility of water. This change in mobility can be seen as a change in slope of the pressure-time curve for the two injection periods. The initial water saturation in the reservoir was assumed to be the irreducible water saturation, $S_{wi} = S_{wirr}$. This method of determining residual oil saturation can be used for new wells that are to be tested for evaluating surfactant performance in lowering residual oil saturation. Although surfactant injection will most likely be carried out as a tertiary recovery process, surfactant-flood evaluation may be taken up during the early stages of the life of the well (as in this case at the very beginning of production). As we shall see later, the initial water saturation does not affect the interpretation of the results and this example will also serve to establish that when compared to Case 2.

The important simulation input data are presented in Table 5.1. The relative permeability curves at low capillary number (water displacing oil) are Corey Type curves as presented in Fig 5.3 (a). Straight line relative permeability curves with zero residual saturation for both oil and water were assumed for ultra low IFT values Fig 5.3 (b). These straight line curves were used for a high capillary number, surfactant displacement
process. The end points, exponents and critical saturations used for developing the two sets of relative permeability curves are given in Table 5.2 and Table 5.3.

**Results and Discussion:**

The plot of the incremental well-bore pressure versus log (time) is shown in Fig 5.4. From the graph, two linear sections are evident. The slopes of these linear sections can be calculated as below:

\[
\text{Slope}_{\text{Early Time}} \frac{\partial p}{\partial \log t} = 418-320 = 98 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 42.56 \text{ psi / Log}_e \text{ Cycle}
\]

\[
\text{Slope}_{\text{Late Time}} \frac{\partial p}{\partial \log t} = 361-340 = 21 \text{ psi / Cycle)
\]

\[
\frac{\partial p}{\partial \ln t} = 9.12 \text{ psi / Log}_e \text{ Cycle}
\]

The injection sequence consisted of water injection for 0.25 days followed by surfactant injection. As discussed earlier in the literature review\(^1\), the pressure-log (time) plot for injection pressure data can be divided into three regions, very-early time, early-time and late time. In this case, the very-early time and early-time sections correspond to the water injection which was carried out for 0.25 days or 6 hours. The late-time response corresponds to the surfactant injection period. The very-early time section reflects the properties of the fluid filled reservoir ahead of the front and far away from the well which in this case is reservoir rock at residual water saturation. The duration of this response is very short and has been omitted from the current plot. Only early-time and late-time responses have been shown in the figure.

The early time derivative data reveals the fluid flow conditions near the well and behind the fluid front. In this case, these conditions correspond to a residual oil saturation. The expected logarithmic derivative for the early-time injection period is:
\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-water}} = 70.716 \times 500 / (20 \times 200 \times 0.2 / 1)
\]

\[
= 44.2 \text{ psi / Cycle.}
\]

From the pressure profile, the calculated derivative value is 42.56 psi / \(\log_e\text{ Cycle}\). To calculate the water saturation corresponding to the observed value of the logarithmic derivative, we first calculate the end point relative mobility to water at residual oil saturation after water-flood using the following formula:

\[
k_{rw-water} = \frac{q}{4\pi h \frac{k}{\mu} \frac{\partial p}{\partial \ln t}} = 70.716 \times 500 / (20 \times 200 \times 42.56 / 1) = 0.207
\]

From the Corey type relative permeability curves at high IFT or low capillary number Fig 5.3 (b), we get the water saturation corresponding to the end point relative permeability as calculated above to be 0.75 which leads to the residual oil saturation after water flood to be calculated as 1 - 0.75 = 0.25. This is equal to the input provided to the simulator in terms of the relative permeability curves. It is to be noted that the correct value of end point relative permeability to water at residual oil saturation during water-flood is 0.20 which is very close to the observed value of 0.207 calculated from the pressure-log (time) curve.

After 6 hrs, we switch to surfactant injection. The surfactant reduces the residual oil saturation in the reservoir from its initial value of 0.25 to 0.0. This is manifested in the late-time derivative response which represents the fluid mobility in the surfactant-flooded water saturated zone at residual oil saturation behind the fluid front. The expected logarithmic derivative can be calculated as:

\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-surf}} = 70.716 \times 500 / (20 \times 200 \times 1 / 1) = 8.84 \text{ psi / Cycle}
\]
From the pressure profile, the calculated derivative value = 9.12 psi / Cycle. To calculate the water saturation corresponding to the observed value of the logarithmic derivative, we first calculate the end point relative mobility to water at residual oil saturation after surfactant-flood using the following formula:

\[
k_{rw - surf} = \frac{q}{4\pi h \cdot \frac{k \cdot \partial p}{\mu \cdot \partial \ln t}} = \frac{70.716 \cdot 500}{(20 \cdot 200 \cdot 9.12 / 1)} = 0.97
\]

From straight line relative permeability curves at high capillary number as shown in Fig 5.3 (b), we get the water saturation corresponding to the end point relative permeability as calculated above to be 0.97 which leads to the residual oil saturation after surfactant-flood to be calculated as 1 – 0.97 = 0.03. It is to be noted that the correct value of end point relative permeability to water at residual oil saturation after surfactant-flood is 1.00 which is very close to the observed value of 0.97 calculated from the pressure-log (time) curve. Also, the interpreted residual oil saturation of 0.03 is very close to the actual input value of 0.00 to the simulator.

From the above calculations we see that there are contrasting slopes for the two portions of the pressure-log (time) curve which suggest a considerable improvement in the water phase mobility near the well-bore post surfactant-flood. This improvement in the water mobility is brought about by the decrease in the residual oil saturation and a corresponding increase in the relative permeability of water. We can also see that the phase mobility values can be converted to phase saturation values using appropriate relative permeability curves. This example illustrates the method involved in calculating residual oil saturation after surfactant-flood using the pressure – log (time) plot.
Case 2: Reservoir at Residual Oil Saturation after Water Flood

Water was injected for 6 hrs followed by surfactant injection. The change in mobility from water injection to surfactant injection can be seen as a change in slope of the pressure-time curve for the two injection periods. The initial water saturation in the reservoir was assumed to be the end-of-water-flood water saturation, \( S_{wi} = 1 - S_{orw} \).

This method of determining residual oil saturation can be used for wells that have been under water-flood for a long period of time and are to be tested for evaluating surfactant performance in lowering residual oil saturation. This is the classical case of surfactant-flood as a tertiary recovery process which is carried out at the end of a water-flood. This example will also serve to compare with Case 1 and establish that initial water saturation has no effect on the interpretation of results.

The important simulation data are presented in Table 5.1, Table 5.2, Table 5.3 and Fig 5.3 (a) and Fig 5.3 (b).

Results and Discussion:

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 5.5. From the graph, two linear sections are evident. The slopes of these linear sections can be calculated as below:

\[
\text{Slope}_{\text{Early Time}} \quad \frac{\partial p}{\partial \log t} = 520-422 = 98 \text{ psi / Cycle} \\
\frac{\partial p}{\partial \ln t} = 42.56 \text{ psi / Log}_e \text{ Cycle}
\]

\[
\text{Slope}_{\text{Late Time}} \quad \frac{\partial p}{\partial \log t} = 426-405 = 21 \text{ psi / Cycle} \\
\frac{\partial p}{\partial \ln t} = 9.12 \text{ psi / Cycle}
\]

The injection sequence consisted of water injection for 0.25 days followed by surfactant injection. The pressure-log (time) plot can be divided into three regions: very-
early time, early-time and late time. As was done for Case 1 Only early-time and late-
time responses have been shown in Fig 5.5.

The early time derivative data reveals the fluid flow conditions near the well and
behind the fluid front. Similar to Case 1, these conditions are mobile water at residual oil
saturation. The expected logarithmic derivative for the early injection period is:

\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \lambda_{w-water}} = 70.716 \times 500 / (20\times200\times0.2 / 1) = 44.2 \text{ psi / Cycle}
\]

From the pressure profile, the calculated derivative value is 42.56 psi. To
calculate the water saturation corresponding to the observed value of the logarithmic
derivative, we first calculate the end point relative mobility to water at residual oil
saturation after water-flood using the following formula:

\[
k_{rw-water} = \frac{q}{4\pi h \frac{k}{\mu} \frac{\partial p}{\partial \ln t}} = 70.716 \times 500 / (20\times200\times42.56 / 1) = 0.207
\]

Using Corey type relative permeability curves for low capillary number
displacement the water saturation in the water-flooded region behind the front was
calculated as 0.75 which results in the calculated residual oil saturation after water-flood
as 1 – 0.25 = 0.75.

After 6 hrs, we switch to surfactant injection. The surfactant reduces the residual
oil saturation in the reservoir from its initial value of 0.25 to 0.0. This is manifested in the
late-time derivative response which represents the fluid mobility in the surfactant-flooded
water saturated zone at residual oil saturation behind the Buckley-Leverett front. The
expected logarithmic derivative can be calculated as

\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \lambda_{w-surf}} = 70.716 \times 500 / (20\times200\times1 / 1) = 8.84 \text{ psi / Cycle}
\]

From the pressure profile, the calculated derivative value = 9.12 psi. To calculate
the water saturation corresponding to the observed value of the logarithmic derivative, we
first calculate the end point relative mobility to water at residual oil saturation after surfactant-flood using the following formula:

\[
k_{rw}^{*} = \frac{q}{4\pi h \frac{k^*}{\mu} \frac{\partial p}{\partial \ln t}} = 70.716 * 500 / (20*200*9.12 / 1) = 0.97
\]

Using the straight line relative permeability curves for high capillary number displacement, the water saturation corresponding to the end point mobility calculated above was found to be 0.97 which results in the calculated residual oil saturation after surfactant-flood as 0.03.

The above calculations confirm that there are different slopes for the two portions of the pressure-log (time) curve which suggest a considerable improvement in the water phase mobility near the well-bore post surfactant-flood. This improvement in the water mobility is brought about by the decrease in the residual oil saturation and a corresponding increase in the relative permeability of water. When compared to Case 1, we see that the absolute differential pressure at the well-bore is more in Case 2, when the initial conditions are \(S_{wi} = 1 - S_{orw}\). This is because water is less compressible than oil and injecting in a reservoir filled primarily with water will lead to higher well-bore pressures than when injecting in a reservoir which has high oil saturation. Even though the absolute pressures are different, it is important to note that the slope of the two portions of the pressure-log (time) curve for Case 2 are exactly same as the corresponding values for Case 1. This reaffirms the point stated while discussing Case 1 that the interpretation of the pressure-log (time) curve is independent of the initial water saturation. Case 1 and Case 2 are the two extreme cases of variation in initial water saturation.
Case 3: Water Injection Followed by Injection of a Small Volume of Surfactant

Water was injected initially to establish baseline mobility followed by surfactant injection which was carried out for 1 day (Curve 3, Fig 5.6) or 5 days (Curve 2, Fig 5.6). This was again followed by water injection for the remaining simulation time. The reservoir was assumed to be initially at irreducible water saturation, $S_{wi} = S_{wirr}$.

The important simulation data for this Case are presented in Table 5.1, Table 5.2, Table 5.3 and Fig 5.3 (a) and Fig 5.3 (b).

Results and Discussion:

The plot of the incremental well-bore pressure versus log (time) is shown in Fig 5.6. The figure compares three injection sequences:

1. Water injection for 0.25 day followed by surfactant injection for the remaining simulation time, Curve 1.
2. Water injection for 0.25 day followed by surfactant injection for 5 days followed by water injection for the remaining simulation time, Curve 2.
3. Water injection for 0.25 day followed by surfactant injection for 1 day followed by water injection for the remaining simulation time, Curve 3.

Curve 1 is the same curve which was discussed in Case 1. The slopes for different sections are marked on the curve. The early time slope for all the three curves is the same and is marked as ‘m1’. This slope can be calculated as:

\[
\text{Slope}_{\text{Early Time}} \frac{\partial p}{\partial \log t} = 419 - 323 = 96 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 41.69 \text{ psi / Cycle}
\]

The early-time slope is marked as slope ‘m1’ in Fig. 5.6.

For Curve 1 (which is the red curve in Fig. 5.6), the late-time slope is marked as equal to ‘m2’. This slope can be calculated as:
Slope_Late Time \( \frac{\partial p}{\partial \log t} = 388-367 = 21 \text{ psi / Cycle} \)

\( \frac{\partial p}{\partial \ln t} = 9.12 \text{ psi / Cycle} \)

As in the previous cases, the very-early time portion of the pressure-log (time) has been omitted from the figure. The early-time slope represents the fluid mobility conditions during the early water injection within 0.25 day for each of the three injection sequences. As for Case 1 and Case 2 the expected logarithmic derivative for this early injection period is:

\[ \frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-water}} = 70.716 \times 500 / (20 \times 200 \times 0.2 / 1) \]

\[ = 44.2 \text{ psi / Cycle} \]

From the pressure profile, the observed derivative value is 41.69 psi. To calculate the water saturation corresponding to the observed value of the logarithmic derivative, we first calculate the relative mobility to water after water-flood using the following formula:

\[ k_{rw - water} = \frac{q}{4\pi h \hat{k} \mu \frac{\partial p}{\partial \ln t}} = 70.716 \times 500 / (20 \times 200 \times 41.69 / 1) = 0.21 \]

Using Corey type relative permeability curves for calculating the water saturation corresponding to the relative permeability calculated above, the water saturation in the water-flooded region behind the front was found to be 0.75 resulting in residual oil saturation after water-flood to be calculated as 1 − 0.75 = 0.25.

The expected logarithmic derivative for the late-time portion on Curve 1 was calculated as:

\[ \frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-surf}} = 70.716 \times 500 / (20 \times 200 \times 1 / 1) = 8.84 \text{ psi / Cycle} \]
From the pressure profile, the observed derivative value = 9.12 psi / Cycle. To calculate the water saturation corresponding to the observed value of the logarithmic derivative, we first calculate the relative mobility to water after surfactant-flood using the following formula:

\[
k_{rw - surf} = \frac{q}{4\pi h \left( \frac{k}{\mu} \right) \frac{\partial p}{\partial \ln t}} = 70.716 \times 500 \div (20 \times 200 \times 9.12 \div 1) = 0.03
\]

Using straight line relative permeability curves for high capillary number displacement, the corresponding water saturation was calculated to be 0.97 which leaves the residual oil saturation to be calculated as \(1 - 0.93 = 0.03\) close to the simulator input of 0.00.

It is important to note the differences and similarities between the three curves which represent three different injection sequences. When Curve 1 is compared with Curve 2, as expected the curves are similar till almost 200 hrs (for almost 8 days) after which the two curves begin to separate. For Curve 2, the injection sequence was water for 0.25 days, followed by surfactant injection for 5 days followed by water injection. Once we switch from surfactant injection back to water injection, the region of increased water mobility due to decreased residual oil saturation is no more expanding and with continued water injection the region behind the fluid front is now mostly residual oil at low capillary number. Hence, the region behind the fluid front is becoming increasingly similar in properties to the region that developed behind the front during the initial water injection for 0.25 day. This is manifested in the fact that the late time slope of Curve 2 much after surfactant injection was stopped is the same as the early time slope developed during early water injection. On the figure, the two portions of Curve 2 have been shown to have equal slopes ‘m1’. Similar explanations hold for Curve 3. In generating Curve 3, surfactant injection was stopped after 1 day following which we switched to water
injection. Hence, the departure from Curve 1 occurs much earlier and the attainment of late time slope as equal to early time slope (m1) is also much earlier in time than Curve 2.

An important conclusion that can be drawn is that for estimating the residual oil saturation after a surfactant-flood, the surfactant should be injected for a sufficiently long period of time so as to develop a considerably large high water mobility region around the well bore. If this is not done, then the response of the increased mobility region will last only for a small duration of time that may be imperceptible as is the case for Curve 2 wherein surfactant was injected only for 1 day. For Curve 3, surfactant was injected for 5 days and consequently the increased mobility response was observed for almost 3 days after the surfactant injection was stopped.

As we have seen for the three cases discussed so far, residual oil saturation after surfactant-flood can be obtained from the pressure- log (time) response during the injection period. We can have a prolonged surfactant injection or can switch to water injection after sufficient volume of surfactant has been injected and study the pressure response to determine residual oil saturation. Next, we study the case of determining residual oil saturation in a reservoir that has been under surfactant-flooded for several years so that a big bank of increased water mobility exists at the start of the injection test.

**Case 4: Water injection in the presence of an initial water bank**

In this case, we assumed that a previously water-flooded reservoir has been under surfactant-flood such that a bank of increased water mobility (decreased residual oil saturation) existed for a radius of 500 ft from the well bore at the start of the injection test. Water was then injected in the reservoir and the pressure – log (time) curve studied for determining phase mobilities and residual oil saturations.

The initial water saturation in the reservoir was assumed to be $S_{wi} = (1-S_{or-surf})$ for first 500 from the well bore and $S_{wi} = 1-S_{orw}$ for the rest of the reservoir. That is, for the
first 500 ft around the well bore the oil saturation is the residual oil saturation after surfactant-flood and for the rest of the reservoir the oil saturation is the residual oil saturation after water-flood.

This example explains how to interpret water injection pressure response to determine residual oil saturation for mature fields that have been previously water-flooded and are under tertiary recovery with surfactant-flooding process.

The important simulation data are presented in Table 5.1, Table 5.2, Table 5.3 and Fig 5.3 (a) and Fig 5.3 (b). Table 5.2 and Fig 5.3 (a) represent the rock properties that were assigned to the grid blocks that were beyond 500 ft from the well bore for low capillary number displacement and Table 5.3 and Fig 5.3 (b) represent the rock properties that were assigned to the grid blocks within 500 ft from the well bore which was assumed to be previously surfactant flooded.

**Results and Discussion**

The plot of the incremental well-bore pressure versus log (time) is attached at the end of this chapter, Fig 5.7. From the graph, two linear sections are evident. The slopes of these linear sections can be calculated as below:

\[
\frac{\partial p}{\partial \log t} = 116-95 = 21 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 9.12 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \log t} = 295 - 197 = 98 \text{ psi / Cycle}
\]

\[
\frac{\partial p}{\partial \ln t} = 42.56 \text{ psi / Cycle}
\]

The biggest difference that exists between the cases discussed so far and Case 4 is the presence of an initial water bank that extends some distance from the well-bore at the start of the injection test. In the earlier cases we were discussing the 2\(^{nd}\) and 3\(^{rd}\) segments.
of the pressure – log (time) curve as was discussed in Chapter 1, Fig. 1 from Woodward and Thamblynayagam\textsuperscript{1} but for Case 4 the two portions of the curve shown in Fig. 5.7 correspond to the 1\textsuperscript{st} and 2\textsuperscript{nd} segments. This is so because the very-late-time (3\textsuperscript{rd} segment) pressure response for Case 5 corresponds to water displacing oil at high capillary number and since the initial water bank is big (500 ft), the time taken to reach the very-late-time response is in the order of years for the injection rate used for the simulations (500 bwpd).

The 1\textsuperscript{st} segment of the pressure – log (time) curve reveals the fluid flow conditions in the water bank that extends 500 ft from the well-bore. These conditions are mobile water at residual oil saturation after surfactant-flood. The expected logarithmic derivative for the early injection period is:

\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h\lambda_w - surf} = 70.716 \times 500 / (20 \times 200 \times 1.00 / 1) = 8.84 \text{ psi / cycle}
\]

From the pressure profile, the calculated derivative value is 9.12 psi / cycle. Using the methods described for the earlier cases, residual oil saturation after surfactant flood was calculated to be 0.03 which is the oil saturation in the water bank that was formed after surfactant-flood.

The second portion of the curve corresponds to the fluid mobility in the zone that extends beyond the initial water-bank into the reservoir. The fluid flow conditions in this portion of the reservoir are mobile water at residual oil saturation after water flood. The expected logarithmic derivative for this period is:

\[
\frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h\lambda_w - water} = 70.716 \times 500 / (20 \times 200 \times 0.2 / 1) = 44.2 \text{ psi / cycle}
\]

From the pressure profile, the calculated derivative value = 43.43 psi / cycle. Using the methods described for the earlier cases, oil saturation in the region beyond the
water bank in the reservoir was found to be 0.25 which is the residual oil saturation after water-flood.

In the various cases discussed so far, we looked at several possible conditions that might prevail prior to taking up an injection test for surfactant evaluation in lowering residual oil saturation and we also looked at cases where we could utilize injection well test data to determine the residual oil saturation in a reservoir that has been previously surfactant-flooded. In the next section, we discuss the effects of some of the parameters such as initial water saturation, mobility ratio of displacement and injection of non-Newtonian fluid on the pressure – log (time) curve.

**THE EFFECT OF INITIAL WATER SATURATION**

Previously, Case 1 and Case 2 were compared for evaluating the effect of initial water saturation on the pressure – log (time) curve obtained during surfactant injection. It was found that the initial water saturation has no effect on the residual oil saturation obtained at the end of surfactant flood. Hence, the portion of the pressure – log (time) curve the slope of which indicates the mobility of the injected water in the surfactant flooded region behind the fluid front has the same slope regardless of the initial water saturation. The absolute pressures obtained, however, are different for varying initial water saturations as the total compressibility of the reservoir is varying with varying water saturation. Oil being more compressible than water will result in lower absolute pressures with decreasing initial water saturation.

**THE EFFECT OF MOBILITY RATIO ON THE PRESSURE – LOG (TIME) CURVE**

An important aspect of the methods discussed so far (except for Case 4) to evaluate residual oil saturations due to a surfactant is that the surfactant is injected in relatively small volume over a period of only a few days. The intent of such a test would
be to use the data collected over a short period of time to evaluate surfactant performance in lowering residual oil saturation. This data can be affected by viscous instability effects if the injection of fluids takes place under unfavorable mobility conditions. Polymers are added to the injected fluid to stabilize the displacement process in an actual full-field surfactant flood process. Polymer viscosity is dependent on shear-rate, polymer concentration and salinity which are not constant in a reservoir. As a result, the polymer viscosity varies with distance from the well bore and the mobility of the injected fluid is not constant behind the fluid front. Hence, the pressure-log (time) slope is cannot be used to calculate fluid mobility or residual saturations. This fact will be discussed in detail in a later section.

An important consequence of not adding polymer to the displacing fluid is that the displacement mobility ratios are unfavorable (M > 1). In this section we analyze the effect of unfavorable mobility ratios on the interpretation of the pressure-log (time) curve.

**Case 5 and Case 6: Water Injection: MR > 1, Surfactant Injection: MR > 1**

The end-point relative permeabilities and the viscosities of the displacing and the displaced fluid were chosen such that both the water displacement as well as surfactant displacement of oil was under unfavorable mobility conditions, MR > 1.

Water was injected for 6 hrs followed by surfactant injection. The initial water saturation in the reservoir was assumed to be the irreducible water saturation, $S_{wi} = S_{wirr}$.

*Miscellaneous data for Case 5 and Case 6*

Miscellaneous data that were used in the simulation runs for Case 5 and Case 6 are listed below. The fluid viscosities and end-point relative permeabilities were chosen to give an adverse mobility ratio of 5 and 10 respectively for Case 5 and Case 6 the surfactant flood sequence of the test.
\( k = 200 \text{ md} \)
\( Q = 500 \text{ bpd} \)
\( H = 20\text{ft} \)

*Fluid Data for Case 5 (MR = 5)*

\[ \mu_w = 1 \text{ cp} \]
\[ \mu_s = 1 \text{ cp} \]
\[ \mu_o = 5 \text{ cp} \]

*Fluid Data for Case 6 (MR = 10)*

\[ \mu_w = 1 \text{ cp} \]
\[ \mu_s = 1 \text{ cp} \]
\[ \mu_o = 10 \text{ cp} \]

Table 5.2 and Fig 5.3 (a) represent the rock properties and the relative permeability curves used for low capillary number displacement and Table 5.3 and Fig 5.3 (b) represent the same for high capillary number displacement.

*Mobility ratio calculations:*

For case 5:

For water-flood:

\[
MR = \frac{k_{rw - water}^\circ}{k_{row - water}^\circ} \frac{\mu_{water}}{\mu_{oil}} = \frac{0.2}{0.8} = 1.25
\]

\[ (5.10) \]

For surfactant-flood:

\[
MR = \frac{k_{rw - surf}^\circ}{k_{row - surf}^\circ} \frac{\mu_{water}}{\mu_{oil}} = \frac{1}{5} = 5
\]

\[ (5.11) \]
For case 6:

For water-flood:

\[
MR = \frac{\frac{k_{rw - \text{water}}}{\mu_{\text{water}}}}{\frac{k_{row - \text{water}}}{\mu_{\text{oil}}}} = \frac{0.2}{0.8 \times 10} = 2.5
\]  
(5.12)

For surfactant-polymer-flood:

\[
MR = \frac{\frac{k_{rw - \text{surf}}}{\mu_{\text{water}}}}{\frac{k_{row - \text{surf}}}{\mu_{\text{oil}}}} = \frac{1}{1 \times 10} = 10
\]  
(5.13)

**Results and Discussion**

The plot of the incremental well-bore pressure versus log (time) are shown in Fig 5.8 and Fig 5.9. From the graph, two linear sections are evident. The slopes of these linear sections can be calculated as below:

**Case 5 (MR = 5)**

**Slope-Early Time**
\[
\frac{\partial p}{\partial \log t} = \frac{532 - 429}{44.73} = 103 \text{ psi / Cycle}
\]

**Slope-Late Time**
\[
\frac{\partial p}{\partial \ln t} = \frac{431 - 410}{9.12} = 21 \text{ psi / Cycle}
\]

**Case 6 (MR = 10)**

**Slope-Early Time**
\[
\frac{\partial p}{\partial \log t} = \frac{897 - 796}{43.86} = 101 \text{ psi / Cycle}
\]

**Slope-Late Time**
\[
\frac{\partial p}{\partial \ln t} = 43.86 \text{ psi / Cycle}
\]
Slope_Late Time \[ \frac{\partial p}{\partial \log t} = 774 - 753 = 21 \text{ psi / Cycle} \]

\[ \frac{\partial p}{\partial \ln t} = 9.12 \text{ psi / Cycle} \]

Expected derivative values: At early time, water is displacing oil and the expected early time derivative response for both the cases can be calculated as:

\[ \frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-water}} = 70.716 * 500 / (20*200*0.2 / 1.0) \]

\[ = 44.2 \text{ psi / cycle} \]

At late time, surfactant is displacing oil and the expected late time derivative response for both the cases can be calculated as:

\[ \frac{\partial p}{\partial \ln t} = \frac{q}{4\pi h \hat{\lambda}_{w-surf}} = 70.716 * 500 / (20*200*1.0 / 1.0) = 8.84 \text{ psi / cycle} \]

So, for both the cases of MR = 5 and MR = 10, the observed derivative value is very close to the expected derivative value. Residual oil saturations were calculated following the procedure described as for earlier cases and were found to be very close to the values that were input to the simulator.

The plots in Fig 5.8 and 5.9 are similar to the ones we have discussed before. Each shows two segments from the pressure – log (time) relationship, the early time representing the fluid flow conditions during the time of water-flood and the late time during surfactant flood. These plots clearly show an improvement in the injectivity of water caused by the decrease in residual oil saturation post surfactant flood. As discussed for the previously discussed cases, the residual oil saturation was found using the logarithmic derivative values calculated above. Calculations of the residual oil saturation from the observed pressure derivative response match closely with the input to the simulator, as had been for all the earlier cases as well, suggesting that the method can be
used even for highly unfavorable mobility ratio displacement processes. It is important to note that we have not yet introduced the polymer in the displacement phase so that the viscosity of the displacing phase is essentially that of water leading to mobility ratios greater than one in case the oil viscosity is high. This leads to viscous instability in the porous media which can severely distort the pressure response at the well-bore. The effect of viscous fingering is difficult to model in reservoir simulators, and all the simulations run and discussed so far neglect such effects. Though it appears that meaningful interpretations of phase saturations are possible from the pressure response gathered during an injection test with highly unfavorable mobility ratios, in the actual scenario this may not be possible. If the oil is very viscous, polymer must be added to the displacing fluid so that the viscous instability effects can be overcome. Since polymer is a shear thinning fluid, it affects the pressure response at the well-bore very significantly and special analysis techniques are required for interpreting polymer injection well tests. These are discussed next.

**POLYMER INJECTION BASED WELL-TEST INTERPRETATION**

**Introduction**

Polymers are widely used to increase the water phase viscosity so as to provide favorable mobility ratio during displacement of viscous oils. A small fraction of polymer mixed in water can increase the water phase viscosity ten-fold or more. Polymers are frequently added to water for displacing oil in a polymer flooding process or used as the mobility improvement additive in a surfactant / alkali-surfactant flood. In each case, the respective displacement technique is called the polymer-flood, surfactant-polymer-flood (SP) or the alkali-surfactant-polymer flood (ASP). Polymer viscosity depends on a wide range of parameters such as salinity, polymer concentration, and shear rate in the water
phase. For the purpose of this study, the shear rate dependence of polymer viscosity was modeled in polymer-surfactant flood simulations in STARS. Special techniques from the literature\cite{20, 21, 22} were utilized for the interpretation of well-bore pressure response for a shear thinning fluid and these are discussed in the following sections.

**Polymer Rheology**

Polymer solutions are non-Newtonian fluids and the relationship between shear stress and shear rate is given by the following expression:

\[ \tau = H \gamma^n \]  \hspace{1cm} (5.14)

Where:

- \( \tau \) = Shear Stress (Pa)
- \( H \) = Consistency (Power Law parameter) Pa-s\(^n\)
- \( \gamma \) = Shear rate (s-1)
- \( n \) = Flow behavior index (Power law parameter)

For a Newtonian fluid, the viscosity is given by \( \mu = \frac{\tau}{\gamma} \) and is constant.

Combining the two expressions for Newtonian and non-Newtonian fluids, one can define “apparent viscosity”. The apparent viscosity can be expressed as below:

\[ \mu_{app} = H \gamma^{n-1} \]  \hspace{1cm} (5.15)

For a Newtonian fluid, \( n = 1 \) and the viscosity is constant and equal to \( H \). For pseudoplastic (shear thinning) fluids \( 0 < n < 1 \) and a plot of log (viscosity) vs. log (shear rate) yields a straight line (Fig.5.10) with a negative slope. If \( n > 1 \), the fluid shows dilatancy that is its viscosity increases with shear rate and hence is a shear-thickening fluid. As shown in Fig 5.10, the viscosity of the polymer solution follows a power law relationship over a range of shear rates. Below the lower bound of this range, the polymer viscosity is constant and is equal to the viscosity of the ‘unstirred’ solution at zero shear.
rate. Above the upper bound of the range, the polymer viscosity is constant and is equal to the viscosity of the polymer carrying phase without the polymer.

**Effect of Polymer Injection on Well-Bore Pressure Response**

Since polymer viscosity is dependent on shear rate, it is varying as a function of the radial distance from the well bore. Near the well-bore the shear rates are very high and the polymer viscosity is equal to the water phase viscosity without the polymer. At large distances from the well-bore within the polymer flooded zone, the shear rates are low and the polymer viscosity is constant and equal to the water phase viscosity with polymer at zero shear rates. In between these two regions of very-high and very-low shear rates, the shear rates are continuously varying and so is the polymer viscosity. In such a case, the mobility of the injected fluid is not constant in the reservoir due to continuously changing viscosity, therefore, the classical well test interpretation techniques like the straight line analysis on a semi-log paper fail to give conclusive results. Note that in all the cases discussed so far, the straight line analysis yielded conclusive results in the form of constant slope regions on the pressure – log (time) curve from which the fluid mobilities could be calculated. When such a plot of well-bore pressure vs. log (time) is made for the pressure recorded during polymer injection, no straight line portions are seen. Hence, we cannot associate a constant fluid mobility to the polymer-flooded zone which renders the calculation of residual saturations using phase mobilities calculated from conventional techniques inappropriate for polymer floods.

**Pressure Analysis Technique**

Ikoku et al. have presented an analysis technique which could be used to determine permeability from the pressure data obtained during polymer injection. They derived a diffusivity equation for the flow of non-Newtonian fluid in the reservoir and
sought analytical solutions to the diffusivity equation based on which an interpretation
technique could be established. The steps involved in the interpretation technique by
Ikoku et al. have been summarized below:

**Step 1**

This step involves the plotting of the well-bore pressure or differential well-bore
pressure (Pascals) vs. time (minutes). This would give a straight line the slope of which
can be calculated from the graph. The slope from this graph is related to the flow
behavior index (n) as follows:

\[
m = \frac{1 - n}{3 - n}, n = \frac{1 - 3m}{1 - m}
\]  

(5.16)

Where ‘m’ is the dimensionless slope obtained from the plot. It is important to
note that the laboratory measured value of ‘n’ may be different from what is obtained
from this plot. In all the calculations involving the flow behavior index, the above
calculated value of ‘n’ is used.

**Step 2**

From the value of ‘m’ as obtained from Step 1, a plot of ‘\(\Delta P_{wf} vs. t^m\)’ is made on
Cartesian graph paper. Here ‘Pwf’ is the well-bore pressure obtained during polymer
injection. A straight line should be obtained in this step the slope of which can be
calculated from the straight line equation:

\[y = m_{nn}x + c\]

where ‘\(m_{nn}\)’ is the slope and ‘c’ is the intercept.

**Step 3**

The effective mobility to the polymer solution can be calculated as:

\[
\lambda_{eff} = \frac{k}{\mu_{eff}} = [m_{nn}(1 - n)\Gamma(\frac{2}{3 - n})]^2 \left(\frac{q}{2\pi\mu h}\right)^\frac{1+n}{2} \left[\frac{(3 - n)^2}{n\phi c_1}\right]^\frac{1-n}{2}
\]  

(5.17)

Where:

\[\Gamma = \text{Gamma operator}\]
\[ \phi = \text{Porosity} \]
\[ c_t = \text{Total Compressibility (1/Pa)} \]
\[ k = \text{Permeability (m}^2\text{)} \]
\[ h = \text{Reservoir thickness (m)} \]
\[ q = \text{Injection rate (m}^3\text{s}^{-1}\text{)} \]

**Step 4**

If the consistency of the fluid is known, it can be used to calculate the permeability using the following equation:

\[
k = (\frac{q}{2\pi h})(1-n)\Gamma\left(\frac{2}{3-n}\right)\left\{\frac{H}{12}(9 + \frac{3}{n})(150)^{\frac{1-n}{2}}(3-n)^{2(1-n)}\right\}^{\frac{1}{1+n}}
\]

Where:

\[ H = \text{Consistency, Pa-sn} \]

**CASE STUDIES INVOLVING POLYMER INJECTION**

In this section we discuss some of the simulation results from polymer injection studies done using STARS simulator from CMG. First we discuss the case of polymer injection at low rates so that the shear rates produced by the injection of fluid are small and die out as the distance from the well increases. In such a case, after only a few feet away from the well, the polymer viscosity is constant and the aqueous phase viscosity is equal to that when polymer is mixed at zero shear rates. This leads to a possibility of conventional well test interpretation techniques from a semilog plot of well-bore pressure and log (time). Then we discuss the cases of polymer injection at high flow rates such that there is considerable shear rate even at longer distances from the well-bore so that the polymer viscosity is not constant even at those distances. This precludes the use of
conventional well test interpretation methods and special analysis methods like that developed by Ikoku et al\textsuperscript{20,21} have been discussed.

**Case 7: Polymer Injection at Small Injection Rate.**

Water-polymer solution is injected at low rate for 1 day to establish baseline mobility value. This is followed by surfactant-polymer injection at low injection rate and the resulting pressure – log (time) plot is analyzed. The initial water saturation in the reservoir was assumed to be $S_{wi} = S_{wir}$ This example helps establish the fact that the limitation of the shear rate dependence of polymer solutions which otherwise precludes the use of conventional well test analysis methods can be overcome by injecting polymer solutions at small rates with small shear rate values in the reservoir.

The important simulation data are presented in Table 5.2, Table 5.3, Table 5.4 and Fig 5.3 (a) and Fig 5.3 (b).

**Polymer Rheology:**

The polymer rheology is described for a non-Newtonian fluid where the apparent viscosity is given by the power-law relationship with shear rate.

$$\mu_{app} = H \gamma^{n-1}$$

$H =$ Consistency $= 75 \text{ Pa-s}^n$

$n = 0.5$

A log-log plot of apparent viscosity vs. shear rate is shown in Fig 5.10. Given the power-law relationship, the log-log plot yields a straight line.

**Results and Discussion:**

Fig 5.11 shows a plot of apparent viscosity vs. distance for continuous polymer and surfactant-polymer injection. Due to low injection rates, the viscosity of the polymer solution becomes constant after a short distance from the well. The shear rates beyond the
point of constant viscosity correspond to the early constant viscosity region of the apparent viscosity – shear rate plot shown in Fig 5.10.

The effect of constant viscosity being attained in the reservoir some distance from the well-bore results in the constant slope period in a pressure – log (time) curve as shown in Fig 5.12. In Fig. 5.12, pressure response for the case of initial polymer injection followed by surfactant-polymer injection is shown bounded between two cases of constant polymer and constant surfactant-polymer injection cases. It is important to note that for both the continuous injection cases, the slope of the pressure – log (time) curve becomes constant at late time. This slope was calculated from the graph and the obtained values are given below compared with the expected values for the respective slopes.

**For Polymer Injection**

*Expected derivative value:* The polymer doesn’t reduce the residual oil saturation and so the endpoint relative permeability to water is 0.2. The value of the expected derivative can be calculated as:

\[
\frac{\partial p}{\partial \log t} = \frac{q}{4\pi h} \frac{\hat{\lambda}_w}{\lambda_{polymer}} = \frac{162.6 \times 100 \text{ bpd} \times 1 \text{ RB}}{8.5 \times 200 \text{ md} \times 20 \text{ ft}} = 172.76 \text{ psi / cycle}
\]

*Observed derivative value:* The derivative or the slope of the curve was calculated using the straight line technique and also by taking two points on the straight line portion of the curve and using the following formula

\[
\text{Slope-Late Time} = \frac{\Delta P_{wf}(t_2) - \Delta P_{wf}(t_1)}{\log(t_2) - \log(t_1)}
\]

\[= 171.67 \text{ psi / log cycle}\]
For Surfactant-Polymer Injection

*Expected derivative value:* The surfactant reduces the residual oil saturation and the endpoint relative permeability to water is 1.0. The value of the expected derivative can be calculated as:

\[ \frac{\partial p}{\partial \log t} = \frac{q}{4\pi h \lambda_{w-surf+polymer}} = \frac{162.6 \times 100 bpd \times 1 RB / STB \times 8.5 cp}{200 md \times 20 ft \times 1.0} \]

= 34.55 psi / log cycle

*Observed derivative value:* The derivative or the slope of the curve was calculated using the straight line technique and also by taking two points on the straight line portion of the curve and using the following formula

\[ \text{Slope}_{\text{Late Time}} = \frac{\partial p}{\partial \log t} = \frac{\Delta P_{\text{wf}}(t_2) - \Delta P_{\text{sf}}(t_1)}{\log(t_2) - \log(t_1)} \]

= 35.25 psi / log cycle

As shown above, the expected and observed derivative values closely match. Also, when these values were used to calculate the oil saturation by the method discussed earlier for several cases, the respective values were found to be 0.27 and 0.01 which are in very good agreement with the values used for the simulation, 0.25 for polymer flood and 0.0 for surfactant polymer flood.

Thus, we conclude that even though the viscosity of the polymer solution is shear rate dependent and varies with distance from the well-bore, if the polymer is injected at small injection rates, the viscosity of the polymer solution is constant after a small distance from the well–bore as the shear rate drops below the critical shear rate under which the viscosity of the polymer solution is constant and independent of shear rate. The fact that viscosity becomes constant after some distance from the injection well results in the mobility of the displacing fluid becoming constant beyond that distance. Hence, when analyzed on a pressure vs. log (time) plot, we see a constant slope section at late-time
corresponding to constant viscosity and constant mobility regions in the reservoir. The observed derivative values could then be used to calculate relative permeabilities and those can be converted to oil saturation values using appropriate relative permeability curves.

In the next two cases, we take a look at polymer and surfactant polymer injection at high injection rates such that the polymer viscosity is continuously varying in the reservoir over long distances from the well-bore and thereby no straight line portions are observed in the pressure – log (time) plot. For such cases we discuss the analysis methods suggested by Ikoku et al.\textsuperscript{20,21} as presented earlier.

**Case 8 and Case 9: Polymer and Surfactant-Polymer Injection at Large Injection Rates**

Here we discuss the polymer injection in Case 8 and surfactant-polymer injection in Case 9 at considerably high injection rates such that the shear rates are high even at long distances from the reservoir and special analysis methods are required to interpret the pressure log (time) data.

For both the cases, water-polymer solution is injected at high rates of 5000 bpd into a reservoir with differing initial conditions as indicated below.

For Case 8 the reservoir is assumed to be initially flooded with water-polymer solution such that a big bank of water-polymer solution at residual oil saturation after water-flood exists at the start of the injection period. For Case 9 the reservoir is assumed to be initially flooded with surfactant-polymer solution so that a big bank of surfactant-polymer at residual oil saturation after surfactant flood exists at the start of the injection period. When polymer solution is injected at high rates into the reservoir, the polymer viscosity increases continuously as the distance from the well-bore increases and does not become constant even at long distances from the well-bore. These examples demonstrate
the technique utilized for interpreting injection well tests using shear thinning fluids like polymer.

The important simulation data are presented in Table 5.2, Table 5.3, Table 5.5 and Fig 5.3 (a) and Fig 5.3 (b).

**Polymer Rheology:** The polymer rheology is described as for a non-Newtonian fluid where the apparent viscosity is given by the power-law relationship with shear rate.

\[ \mu_{app} = H \gamma^{n-1} \]

\( H = \text{Consistency} = 75 \text{ Pa-s}^n \)

\( n = 0.5 \)

A log-log plot of apparent viscosity vs. shear rate is shown in Fig 5.8. Given the power-law relationship, the log-log plot yields a straight line.

**Results and Discussion:**

*Case 8: Polymer Injection*

Fig 5.13 shows the three plots obtained from the results output file of the simulator. Fig 5.13 (a) gives a plot of the differential well-bore pressure with time. Fig 5.13 (b) gives a log-log plot of differential well-bore pressure with time. This plot is used to calculate ‘n’ from the slope of the curve ‘m’. From the graph, ‘m’ = 0.1736.

From this value of ‘m’, the flow behavior index ‘n’ is calculated:

\[ n = \frac{(1 - 3 \times 0.1736)}{(1 - 0.1736)} = 0.580 \]

It is important to note that this value of ‘n’ may be different from what is obtained from the laboratory experiments, which in this case is equal to 0.5.

Fig 5.13 (c) is a plot of the differential well-bore pressure vs. \( t^n \). From the graph, the slope of the curve is \( m_{in} = 5E+6 \text{ Pascals / Sec}^m \).
Using a value of \(H = 15 \text{ cp}.\cdot\text{sec}^{n-1} = 0.015 \text{ Pa}\cdot\text{Sec}^n\) the permeability was calculated as approx. 100 md using equation 5.18.

As the authors Ikoku et al.\(^{20,21}\) have indicated that both the flow behavior index ‘\(n\)’ and the consistency factor may be different from those calculated in the laboratory. The value of ‘\(n\)’ is determined from the characteristic plots as discussed. The value of the constancy can be estimated to match observable results. In this case, a constancy of 15 Pa-sec\(^n\) gave the calculated permeability to the polymer as 100 md which is what was input to the simulator. The value of the constant \(H\) was chosen so as to obtain a permeability of 100 md which is the water (polymer) permeability at residual oil saturation after water (polymer) flood. (\(k_{\text{polymer}} = 0.2\cdot500\) md = 100 md)

**Case 9: Surfactant-Polymer injection**

Fig 5.14 shows the three plots obtained from the results output file of the simulator. Fig 5.14 (a) gives a plot of the differential well-bore pressure with time. Fig 5.14 (b) gives a log-log plot of differential well-bore pressure with time. This plot is used to calculate ‘\(n\)’ from the slope of the curve ‘\(m\)’. From the graph, ‘\(m\)’= 0.1625.

From this value of ‘\(m\)’, the flow behavior index ‘\(n\)’ is calculated:

\[
n = \frac{(1-3\cdot1.625)}{(1-1.625)} = 0.612
\]

Fig 5.14 (c) is a plot of the differential well-bore pressure vs. \(t^m\). From the graph, the slope of the curve is ‘\(m_{\text{lin}}\)’ = 2E+6 Pascals / Sec\(^m\).

Next, we need to calculate the permeability to surfactant-polymer solution from the observed data. Using a value of \(H = 15 \text{ cp}.\cdot\text{sec}^{n-1} = 0.015 \text{ Pa}\cdot\text{Sec}^n\) the permeability was calculated as approx. 575 md using equation 5.18. The value of \(H\) was taken from Case 8 where it was used to match the permeability in case of water-flood (polymer-flood).
The calculated value of permeability is higher than the absolute permeability entered to the simulator. This could be for the fact that we used the value of H that was calculated to match the polymer injection in Case 8. Discounting these effects, the calculated value of permeability suggests relative permeability to the surfactant-polymer solution as 1.00 which gives the residual oil saturation after surfactant flood as 0.0 which is what was input to the simulator.

We saw that the well-bore pressure vs. log (time) for injection of non-Newtonian fluid does not yield a straight line which does not allow the use of conventional straight line semilog analysis for well test interpretation. Special plots and techniques are used to evaluate such injection well tests. It is possible to compare and contrast the mobility of the injected fluid when polymer is injected mixed with water and when mixed with surfactant using these special analysis techniques. The calculated value of fluid mobility could also be converted to fluid saturation values using appropriate relative permeability curves. If polymer solutions could be injected at small injection rates, late time pressure response on a semilog graph again yields a straight line which can be used for conventional well test interpretation.
CHAPTER SUMMARY

In this chapter, we studied the application of injection well testing in determining residual oil saturation after a surfactant flood. We began by analyzing simple cases of water-surfactant injection and later discussed the effects of a few factors such as initial water saturation and the mobility ratio on the pressure response and associated interpretation. Later, we learned how non-Newtonian fluid behavior can impact the pressure response in an injection well test and analyzed a few cases using special analysis techniques. Injection well testing methods for determining residual oil saturation have been shown to yield good results for several problems varying in nature and complexity.
TABLE 5.1: Simulation Data for Cases 1, 2, 3, 4

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness</td>
<td>H</td>
<td>ft</td>
<td>20</td>
</tr>
<tr>
<td>Reservoir Permeability</td>
<td>k</td>
<td>md</td>
<td>200</td>
</tr>
<tr>
<td>Injection Rate</td>
<td>Q</td>
<td>BPD</td>
<td>500</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>µ_w</td>
<td>cp</td>
<td>1</td>
</tr>
<tr>
<td>Surfactant Viscosity</td>
<td>µ_s</td>
<td>cp</td>
<td>1</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>µ_o</td>
<td>cp</td>
<td>3</td>
</tr>
<tr>
<td>Skin</td>
<td>S</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>C_w</td>
<td>Psi⁻¹</td>
<td>3E-6</td>
</tr>
<tr>
<td>Oil Compressibility</td>
<td>C_o</td>
<td>Psi⁻¹</td>
<td>9E-6</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>C_r</td>
<td>Psi⁻¹</td>
<td>5E-6</td>
</tr>
</tbody>
</table>
TABLE 5.2: Rock Properties Data for Water Flood (Low Capillary Number Displacement)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-point relative permeability to water in water flood</td>
<td>$k_{rw}^{w - water}$</td>
<td>0.2</td>
</tr>
<tr>
<td>End-point relative permeability to oil in water flood</td>
<td>$k_{row}^{o - water}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Saturation exponent for water</td>
<td>$n_w$</td>
<td>2</td>
</tr>
<tr>
<td>Saturation exponent for oil</td>
<td>$n_o$</td>
<td>2</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>$S_{wirr}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Residual oil saturation</td>
<td>$S_{orw}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>
### TABLE 5.3: Rock Properties Data for Surfactant Flood (High Capillary Number Displacement)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>End-point relative permeability to water in surfactant flood</td>
<td>$k_{rw}^o$ – surf</td>
<td>1</td>
</tr>
<tr>
<td>End-point relative permeability to oil in surfactant flood</td>
<td>$k_{row}^o$ – surf</td>
<td>1</td>
</tr>
<tr>
<td>Saturation exponent for water</td>
<td>$n_w$</td>
<td>1</td>
</tr>
<tr>
<td>Saturation exponent for oil</td>
<td>$n_o$</td>
<td>1</td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>$S_{wirr}$</td>
<td>0</td>
</tr>
<tr>
<td>Residual oil saturation</td>
<td>$S_{orw}$</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE 5.4: Simulation Data for Case 7

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness</td>
<td>H</td>
<td>ft</td>
<td>20</td>
</tr>
<tr>
<td>Reservoir Permeability</td>
<td>k</td>
<td>md</td>
<td>200</td>
</tr>
<tr>
<td>Injection Rate</td>
<td>Q</td>
<td>BPD</td>
<td>100</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>$\mu_w$</td>
<td>cp</td>
<td>1</td>
</tr>
<tr>
<td>Surfactant-Polymer Viscosity</td>
<td>$\mu_p$</td>
<td>cp</td>
<td>8.5</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>$\mu_o$</td>
<td>cp</td>
<td>5</td>
</tr>
<tr>
<td>Skin</td>
<td>S</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>$C_w$</td>
<td>Psi$^{-1}$</td>
<td>3E-6</td>
</tr>
<tr>
<td>Oil Compressibility</td>
<td>$C_o$</td>
<td>Psi$^{-1}$</td>
<td>9E-6</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>$C_r$</td>
<td>Psi$^{-1}$</td>
<td>5E-6</td>
</tr>
</tbody>
</table>
### TABLE 5.5: Simulation Data for Case 8 and Case 9

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness</td>
<td>H</td>
<td>ft</td>
<td>20</td>
</tr>
<tr>
<td>Reservoir Permeability</td>
<td>k</td>
<td>md</td>
<td>500</td>
</tr>
<tr>
<td>Injection Rate</td>
<td>Q</td>
<td>BPD</td>
<td>5000</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>$\mu_w$</td>
<td>cp</td>
<td>1</td>
</tr>
<tr>
<td>Surfactant-Polymer Viscosity</td>
<td>$\mu_s$</td>
<td>cp</td>
<td>10</td>
</tr>
<tr>
<td>Oil Viscosity</td>
<td>$\mu_o$</td>
<td>cp</td>
<td>5</td>
</tr>
<tr>
<td>Skin</td>
<td>S</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>$C_w$</td>
<td>Psi^{-1}</td>
<td>3E-6</td>
</tr>
<tr>
<td>Oil Compressibility</td>
<td>$C_o$</td>
<td>Psi^{-1}</td>
<td>9E-6</td>
</tr>
<tr>
<td>Rock Compressibility</td>
<td>$C_r$</td>
<td>Psi^{-1}</td>
<td>5E-6</td>
</tr>
</tbody>
</table>
Fig 5.1: Capillary desaturation curves (CDC) for oleic phase, 5.1(a) and aqueous phase, 5.1(b). At low capillary number, the residual saturations for the oleic and aqueous phases are 0.25 and 0.2 respectively. At high capillary number the residual saturation for each phase is zero.
Fig 5.2: Relative permeability curves from Pope et. al\textsuperscript{21} model for high capillary number Fig 5.2 (a) on Log-Log graph and 5.2 (b) on cartesian graph paper: the relative permeability curves become straight lines with zero residual phase saturations at high capillary number
Fig 5.3: Relative permeability curves for low capillary number 5.3 (a) and high capillary number 5.3 (b) for input to STARS. The relative permeability curves at low capillary number are Corey type curves and at high capillary number are straight lines with zero residual phase saturations.
Fig 5.4: DP (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with Swi = Swc
Fig 5.5: DP (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with Swi = 1-Sorw
Fig 5.6: DP (psi) vs. Log (time-hrs) for 3 different injection sequences: Water - Surf, Water -1 D Surf - Water, Water-5 D Surf - Water
Fig 5.7: DP (psi) vs. Log (time-hrs) for the case of water injection in reservoir with an initial bank of water resulting from surfactant flood.
Fig 5.8: DP (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with MR = 5
Fig 5.9: DP (psi) vs. Log (time-hrs) for the case of water injection for 0.25 days followed by surfactant injection with MR = 10
Fig 5.10: Polymer rheology, Power-law relationship between apparent viscosity and shear rate

\[ \text{Apparent Viscosity} = 75 \times \text{Shear Rate}^{(-0.5)} \]
Fig 5.11: Viscosity vs. Distance plot at low injection rate: polymer and surfactant-polymer cases at different times
Fig 5.12: DP (psi) vs. Log (time-hrs) for polymer injection for 0.25 days followed by surfactant-polymer injection, the bounding curves are for continuous polymer and surfactant-polymer injection.
Fig 5.13: Top (Cartesian plot of $\Delta P_{wf}$ vs. Time); Left (Log-Log plot of $\Delta P_{wf}$ vs. Time); Right (Cartesian plot of $(\Delta P_{wf} \text{ vs. } t^m)$ for polymer flood

$$y = 1E + 08 x^{0.1736}$$

$R^2 = 0.9998$

$$y = 5E + 06 x - 781081$$

$R^2 = 1$
Fig 5.14: Top (Cartesian plot of ΔPwf vs. Time); Left (Log-Log plot of ΔPwf vs. Time); Right (Cartesian plot of (ΔPwf vs. \( t^{m} \)) for surfactant-polymer flood

5.13 (a)

5.13 (b)

5.13 (c)
Chapter 6: Summary and Conclusions

Injection well-testing has important industry applications. It can be used for reservoir appraisal in determining parameters such as reservoir permeability and thickness. It is shown in this thesis that it can also be used for determining residual oil saturation after a waterflood or a surfactant flood (or any other EOR application).

Numerical simulations were performed for water injection in a homogenous reservoir with radial flow geometry. The results suggest that the pressure-time data collected during the process of water injection can be analyzed to calculate the mobility of the fluid in the flooded region behind the fluid front. These mobility values can then be used to calculate reservoir permeability using appropriate relative permeability curves. Numerous simulation runs were made under differing reservoir conditions and injection sequences - varying initial oil saturation, presence of an initial water bank and variable rate injection. In all these simulation studies, it was observed that the appropriate portions of the pressure-time curve (late time except in the case of an initial water bank) have in them information about the reservoir and fluid conditions. These segments from the pressure-time curve can then be utilized to calculate fluid mobility. The calculated fluid mobility values were in good agreement with the input values to the simulator corroborating the observation that such analysis methods can be useful in reservoir appraisal.

Injection well testing can also be used to determine residual oil saturation. As was noted in the case of water injection, the pressure derivative analysis can help us evaluate the fluid mobility in the flooded region. The same fact was utilized for pressure transient analysis in the case of surfactant injection. The derivative analysis showed that it was possible to obtain the fluid mobility in the surfactant flooded region with a fair degree of
resolution. These fluid mobilities could then be converted to fluid saturation values using a trapping number dependent relative permeability model. Numerous simulation runs were made under varying conditions of initial oil saturation, presence of an initial water bank, continuous or intermittent surfactant injection, – to name a few. On analyzing the results generated in these simulation runs, we showed that it is possible to estimate the fluid mobility in the surfactant invaded region. Using a trapping number dependent relative permeability model these mobility values were converted to residual oil saturation values that closely matched the input values to the simulator. These estimates of oil/water saturation had good resolution and definition in the different transient pressure data sets.

Pressure transients generated during surfactant-polymer injection were also studied. The surfactant-polymer solution has shear thinning properties which implies that the viscosity of the injected fluid is a function of the shear rate and varies with radial distance from the wellbore. Hence, the mobility of the injected fluid in the surfactant flooded region is not constant and the straight line derivative analysis cannot be directly applied to find the fluid mobility. However, it was observed that if the surfactant-polymer solution is injected at low rates, the fluid viscosity becomes constant beyond a short distance from the wellbore. As a result, at low injection rates the late time derivative analysis could be used to find the fluid mobility which is constant for most part of the flooded region. From this mobility value, a residual oil saturation could be calculated. In contrast when the surfactant-polymer solution is injected at high rates, the viscosity changes continuously with the radial distance and is not constant even at large distances from the wellbore. Therefore even at late time, the pressure derivative does not become constant and special analysis techniques are required to interpret the pressure behavior. Such techniques have been discussed in this thesis and specific case examples have been
outlined. It was observed that an improvement in the mobility of the injected fluid upon surfactant injection is manifested in the pressure transient analysis. This improvement in mobility of the fluid is brought about by the decreased oil saturation which results in a higher water relative permeability. Reasonable estimates of the decrease in residual oil saturation were possible by comparing the fluid mobility for surfactant-polymer injection with the fluid mobility for only water or only polymer injection.
References


14. Maute, R. E.: “Determination of Residual Oil Saturation with the Borehole Gravity Meter”, SPE 13703


35. Agarwal, R. G.: “A New Method to Account for Producing Time Effects when Drawdown Type Curves are Used to Analyze Pressure Buildup and other Tests Data, SPE 9289, 1980.


42. Medhat M. Kamal and Yan Pan: “Use of Transient data to Calculate Absolute Permeability and Average Fluid Saturations”, SPE 113903 2008.

