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Methods for Economic Optimization of Reservoirs

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Methods for Economic Optimization of Reservoirs

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Dedication

To my family.
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Abstract

Methods for Economic Optimization of Reservoirs

Kyle Lane Smith, M.S.E.
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Supervisor: Larry W. Lake

Operators can improve a reservoir’s value by optimizing it in a more holistic manner, or over its entire life cycle. This thesis developed approaches to life cycle optimization, with emphasis on accessible technical and economic modeling techniques for production.

The challenges of life cycle optimization are properly scheduling the times at which the operator should switch from one recovery phase to the next, along with determining other field design parameters such as well spacing and injection pressures for waterflooding and enhanced oil recovery processes. To deliver the most value, the operator needs to produce from a reservoir the greatest quantity of oil, at a relatively low cost, reasonably soon, and ideally at a time when the oil price is high. This is quite a tall order, as these goals are often in conflict.

This thesis extended existing research regarding lifecycle optimization, first modeling production from a reservoir using an exponential decline model and assuming the oil price’s behavior can be approximated with mean-reverting processes. Implications of operating and capital costs potentially being correlated with the oil price were also
examined. Finally, a mean-reverting price model that forecasts the mean oil price as increasing and described by a logistic model was proposed to accommodate both recent price forecasts and economic reality.

As exponential decline models are more appropriate for characterizing existing production history rather than making *a priori* predictions, a geologic-parameter-based model was developed using a tank model for primary recovery and a model based on Koval theory and parameterizing a reservoir in terms of flow capacity and storage capacity for waterflooding and CO₂ flooding. This model was adapted from existing theory to account for situations where a waterflood has incompletely swept a reservoir at the start of CO₂ flooding. Analytical expressions were also derived for estimating injection rates into a formation parameterized by flow capacity and storage capacity.

The geologic-parameter-based model was combined with economic assumptions and optimized using a genetic algorithm. This optimization suggested an operator should switch from primary recovery to a CO₂ flood with a large WAG ratio relatively early in the reservoir’s life.
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Chapter 1: Introduction

1.1 Background

In the exploration and production (E&P) industry, operators often produce oil and gas from conventional reservoirs using primary recovery until they reach the economic limit of production, which is defined as the time when revenues no longer exceed the costs required to maintain operations. At this point, an operator frequently implements a waterflood, which yields substantial additional recovery. Once the economic limit is reached for waterflooding, he may consider switching to an enhanced oil recovery (EOR) technique such as CO₂ flooding or polymer flooding to yield even higher recoveries. Such timing for these recovery phases does not necessarily derive the most value out of a field.

The goal of many operators, both private and state-owned, is to maximize the value of their assets for the benefit of shareholders or citizens. Therefore it is important to consider how best to develop a reservoir throughout its life cycle, or all phases of production, to achieve this goal. Life cycle optimization of a reservoir, with an emphasis on accessible technical and economic modeling techniques for production, is the primary focus of this thesis. The end result is a geologic-parameter-based model that, when combined with economic models and assumptions, provides insights regarding economically optimal design parameters for solution gas drive reservoirs.

Primary recovery entails producing oil and gas through the reservoir’s natural drive mechanisms; reservoir pressure declines as oil production progresses (Walsh and Lake 2003). The ultimate recovery from this phase is generally only 12 to 15 percent of the original oil in place in the reservoir (Walsh and Lake 2003). Displacement techniques such as waterflooding and most enhanced oil recovery processes involve the injection of
fluids into a reservoir to displace oil left after primary recovery. Displacement methods are often viewed as tools to mitigate the decline in production from primary recovery, especially given the extra expense and technical expertise necessary to implement them well. By contrast, life cycle optimization views these methods as options to accelerate production that can be exercised to maximize the value of a reservoir.

For the remainder of this thesis, “secondary recovery” is taken to be synonymous with waterflooding while “tertiary recovery” refers to CO₂ flooding. This terminology reflects the assumed development choices of the operator and is not generally applicable to all reservoirs. Indeed, Lake (1989) notes enhanced oil recovery techniques have been used immediately after primary recovery, in which case they would be classified as secondary recovery.

1.2 GOALS AND ORGANIZATION OF THESIS

Given the comprehensive nature of topic considered here, insights and modeling techniques are drawn from a variety of sources in the economics and petroleum engineering literatures. Literature is reviewed for the subject examined in each chapter. Some of these existing models are enhanced through additional derivations to adapt them to the purposes of life cycle optimization.

Projects live and die based on expectations regarding their revenues and costs, hence the importance of understanding and modeling the behavior of oil prices and production costs. The economics of these factors, with an emphasis of how they vary across time and may be related, are considered in chapter two. Stochastic price models such as those that assume mean-reverting oil price behavior are reviewed and their
parameters are estimated using past price data. Some of these models are then adapted to account for price forecasts and economic realities that constrain oil prices.

The implications of these economic models on life cycle optimization using a decline curve approach to modeling oil production, as suggested by Parra Sanchez (2010), are explored in chapter three. Additional parameters such as taxes, royalties, and capital costs are applied to capture more economic factors that influence decisions regarding recovery phase switching times and valuation of a reservoir. This simplified approach yields some insights regarding the optimal time to switch from one recovery phase to the next for a particular reservoir. Also explored is how altering various economic modeling assumptions can alter the optimal switching times and the estimated value of the reservoir.

Monte Carlo simulation is a method used to estimate the potential behavior of a system characterized by a model when some or all of its input parameters are uncertain, but have known (or estimated) distributions that characterize their outcomes (Jensen et al. 2000). The distribution of outcomes that the system itself produces is determined first by sampling from each of these input parameter distributions for a specified number of iterations. Each iteration of the simulation therefore has a set of fixed input parameters that are used to calculate a predicted outcome of the system from the model of its behavior. The system outcomes generated by all the simulation iterations form a distribution that characterizes the system’s behavior when accounting for uncertainty regarding input parameters. Chapter three relies on Monte Carlo simulation to model the uncertainty of a reservoir’s value due to uncertainty in the oil price. It is also used to estimate the value of information regarding oil price behavior.

The goal of the rest of the thesis is to build a model of a reservoir’s production through its entire life cycle that is based on geologic and technical parameters. This
model is designed to be simple enough for implementation without the computational expense of numerical simulation software while still capturing relevant physical behavior of the system. The reservoir’s drive mechanism, which provides the energy necessary to bring fluids to the surface for primary recovery, is assumed to derive from solution gas. As a base case assumption, the reservoir is assumed to be developed by using a waterflood for secondary recovery followed by a CO$_2$ flood for tertiary recovery. Depending on the economics and geological parameters of the reservoir, its value may be maximized by skipping the waterflood and switching from primary recovery directly to the CO$_2$ flood.

A tank model for primary recovery based on one proposed by Walsh and Lake (2003) is discussed in chapter four. As reservoir fluid properties vary with pressure, this behavior is also modeled to capture changes throughout primary production. These estimations are also necessary to determine when the average reservoir pressure has reached the bubble point, which is assumed to be the time when the operator switches from primary recovery to waterflooding. This development decision prevents the evolution of a gas saturation throughout the reservoir that ultimately leads to substantial declines in oil production. Since fluid properties are assumed to vary throughout production, predictions are made using the tank model equations by finite differencing time and recalculating properties at each step.

Estimating production from displacement methods using simplified models based on fractional flow theory and Koval factors is considered in chapter five. The advantages of these models are that they can closely match field data, require relatively few input parameters, and are not as computationally expensive as numerical simulation. These characteristics make such models relatively easy to use for economic evaluation, particularly Monte Carlo simulation, which requires many calculations to find the
possible distribution of outcomes for the value of a reservoir. Chapter five also discusses algorithms for calculating key points on fractional flow diagrams.

Existing modeling techniques for displacement methods proposed by Mollaei (2011) assume that a reservoir has a uniform oil saturation when switching from waterflooding to CO₂ flooding. For life cycle optimization, this condition might not necessarily be true as it may be economically optimal to switch before the shock front formed by injected water has completely swept the reservoir. This incomplete sweep means that when the CO₂ flood is initiated, certain zones of the reservoir will still have the oil saturation that existed at the beginning of waterflooding. In chapter five, Mollaei’s model is enhanced by accounting for the possibility of incomplete waterflood sweep at the beginning of the CO₂ flood.

Mollaei’s model also specifies oil recovery in terms of dimensionless time, or the amount of fluid injected into a reservoir, measured as a proportion of the reservoir’s pore volume. Economic evaluations require the measurement of cash flows as a function of actual time. Chapter six is therefore covers the practice of estimating fluid injection rates for waterflooding and CO₂ flooding when an operator maintains a particular pressure difference between injector and producer wells in a reservoir. Formulae are derived to estimate injection rates into heterogeneous layered reservoirs in the context of Koval-theory-based recovery predictions. These derived expressions are based on existing analytical models for injection rate calculation into single homogeneous layers. Estimated injection rates are used to map recovery predictions specified in terms of dimensionless time to actual time.

The geologic-parameter-based model discussed in chapters four through six is combined with estimations of costs, taxes, and royalties in chapter seven to yield a characterization of the reservoir’s entire life cycle from both economic and technical
standpoints. This full model is used to estimate the value of a representative reservoir. Decision variables and constraints for the model parameters are chosen for the purposes of life cycle optimization, and commercial optimization software that employs genetic algorithms is used to maximize the value of the reservoir. Finally, reservoir parameters are altered to examine the sensitivity of the optimal solution to differences in key physical and economic assumptions. The optimal solution for this example reservoir suggests switching directly from primary recovery to CO$_2$ flooding, and using a large injection pressure and WAG ratio for the latter.
Chapter 2: Oil Prices, Costs, and Inflation

2.1 INTRODUCTION

In order to assess the economics of a reservoir, it is necessary to have some understanding of drivers that impact the revenues and costs of production. This chapter examines a few models of how these factors fluctuate over time; these models form a basis for estimating the values of reservoirs in subsequent chapters.

2.2 THE ECONOMICS OF OIL PRICES

For E&P companies, the oil price is a major source of uncertainty in the value of an upstream development project. Therefore it is important to consider fluctuations in oil prices when determining a project’s profitability. Oil prices have been alternately volatile and relatively stable over various periods since the commodity has been widely traded. Figure 2-1 illustrates the evolution in both real (corrected for inflation) and nominal oil prices over the last century and a half, along with relevant major events.
As with any commodity, oil’s price is determined by supply and demand. Demand is a function of variables such as world population, economic development, technology and efficiency, while supply is a function of OPEC cartel behavior, wars, myriad political factors, extraction technology, reservoir management practices, geological knowledge, and many other drivers. The relative importance of any factor in setting prices can vary across time and is not necessarily apparent, underscoring the
difficulty of modeling supply and demand. Moreover, even if one were to build a model that properly characterizes how supply and demand shift with changes in their determinants, one would still have to know how each determinant might change in the future to predict the price of crude oil.

Hamilton (2008) reviews a few economic models that could be used to analyze the behavior of crude oil prices, particularly around the large price swings of 2008. One is Hotelling’s notion of scarcity rent, which holds that a nonrenewable resource’s price will be above its marginal cost even in a perfectly competitive market\(^1\). For oil, according to this model, the expectation of declining reserves means that oil producers have an incentive to wait to sell some of their product under anticipated higher future prices. Although a scarcity rent model could be directionally correct in predicting oil prices over particular time scales, it does not explain sharp drops like the one that occurred towards the end of 2008. Another drawback to using this model is that it is difficult to estimate the magnitudes of the effects of future improvements in technology and new geological discoveries, which affect the scarcity of the resource.

Hamilton also cites commodity price speculation, strong world demand, time lags in and technical limits to raising output, and the influence of OPEC in setting the global oil price (2008). It is difficult to disentangle the relative contribution of each factor; they likely all play some role. He notes that, particularly over short times, the price elasticity of oil is relatively small. In other words, the demand for oil does not decrease

\(^1\) Basic economic theory states that in a perfectly competitive market for most goods, the price of a good is equal to the marginal cost of producing it. See Nicholson (2004) for further discussion (p. 296).
significantly in response to higher prices. As there seems to be relatively meager production growth and strong demand growth from emerging markets like China, India and the Middle East, he suggests that the scarcity rent could play a larger role in setting (higher) oil prices in the future.

Since the large oil price swings of 2008, prices have returned to relatively high levels by historical standards but have not yet surpassed their previous peaks. In response to elevated prices of the last decade and improvements in extraction technology, energy companies have been keener to develop unconventional resources such as shale oil and heavy oil. Indeed, in the United States, the use of hydraulic fracturing has opened vast oil and gas reserves and substantially boosted once-declining production. Today’s upstream projects are in increasingly remote locations, are much more challenging, and have entailed massive capital expenditures and some instances of significant cost overruns (Pfeifer and Chazan 2013). These high costs for new projects suggest it might be very difficult to increase production without higher oil prices in the future, barring substantial technological breakthroughs that could reduce the cost of extracting unconventional or difficult-to-reach conventional resources.

On the other hand, large increases in prices over a short time period could lead to economic recessions, bringing substantial drops in demand for oil, and over a longer period of time induce improvements in energy efficiency, shifts in land usage towards denser growth patterns and switching to substitutes for oil. The first effect is supported by historical correlation of upward oil price shocks and the onset of recessions, which generally then result in a drop in oil prices because of low demand from reduced
economic activity (Kilian 2008). Since oil is a significant input to economic output, a large rise in the oil price deters economic activity.

There is a substantial literature that examines the link between oil prices and the macroeconomy, although it is still not entirely clear whether there is causation or simply correlation (Kilian 2008). Oil price spikes may be a significant factor in precipitating recessions, but they might also simply serve as indicators of increased economic activity before recessions. In any case, because of the vagaries of the demand side of the equation, ever-increasing oil prices are not by any means guaranteed. Even under a scenario of constrained production, shifts in demand could ultimately lead to oil price fluctuations around a higher, but still stable, mean value.

2.3 A SIMPLE MEAN-REVERTING OIL PRICE MODEL

With all these caveats in mind about long-term trends in oil prices and the difficulty of forecasting them, it is still desirable to proceed with a model that at least approximates the behavior of oil prices. Ideally, such a model would also provide some insight on how to manage a reservoir under fluctuating oil prices.

Dixit and Pindyck suggest a mean-reverting process to model oil price behavior (1994). This model is a stochastic process, meaning the variable in question changes over time in at least a partly random fashion, that suggests that the price shifts randomly over shorter time periods but reverts back to a mean value over longer ones (p. 60). The definitions of “shorter” and “longer” time periods are specified by a coefficient that determines the speed of mean reversion. Their proposed theoretical justification for
using this model is that oil prices are impacted by transitory factors like wars and OPEC decisions but are ultimately grounded by longer-term production costs (p. 74). Alternatively, Laughton and Jacoby suggest “in case of a cartelized commodity like oil, the long-run profit-maximizing price sought by cartel managers” determines the mean value (1995 p.188).

Dixit and Pindyck describe the simplest version of a mean-reverting process as the Ornstein-Uhlenbeck process, which in continuous time is:

\[ dx = \kappa(\bar{x} - x)dt + \sigma dz; \quad dz = \epsilon_t \sqrt{dt} \]

Equation 2-1

where \( x \) is the natural logarithm of the oil price, \( \kappa \) is the speed with which \( x \) reverts back to its mean value, \( \bar{x} \), \( t \) represents time, \( \sigma \) is the volatility, and \( \epsilon_t \) is a normally distributed variable with a mean of zero and a standard deviation of one (Dixit and Pindyck pp. 63-75). Intuitively speaking, a large \( \kappa \) means that random deviations from \( \bar{x} \) are shorter in duration. A large \( \sigma \) means that these random deviations or shocks to the oil price are more substantial.

The mean-reverting process provided by Equation 2-1 is a variant of a Weiner process, which is also known as Brownian motion (Dixit and Pindyck 1994). It is a Markov process, which means that for modeling purposes, future values of the variable \( x \) only depend directly on its current value and not previous values or other information that we may currently have on the process. This is not to say that past oil price data cannot be used to estimate parameters for the mean-reverting process, but merely that when using
the process to simulate a future price path for oil, the next price after a time increment is a function of the current one and the process parameters. Dixit and Pindyck note that it is more natural to assume that any asset price has a lognormal distribution, as prices cannot be less than zero (p. 64). The variable $x$ is modeled instead of $S$, the actual spot price of crude oil, where $x = ln(S)$.

Given a current value for the natural log of spot oil price $x_o$, the expected value of the mean-reverting variable and the variance of its departure from the mean value $(x_t - \bar{x})$ at a future time $t$ are given by (Dixit and Pindyck 1994 pp. 74-75):

$$E[x_t] = \bar{x} + (x_o - \bar{x})e^{-kt}$$

Equation 2-2

$$V[x_t - \bar{x}] = \frac{\sigma^2}{2\kappa}(1 - e^{-2kt})$$

Equation 2-3

The Ornstein-Uhlenbeck process given by Equation 2-1 can be modeled in discrete time steps, which allows us to use actual oil price data to fit parameters (Dixit and Pindyck p. 76):

$$x_t - x_{t-1} = \bar{x}(1 - e^{-\kappa}) + (e^{-\kappa} - 1)x_{t-1} + \epsilon_t$$

Equation 2-4

$\epsilon_t$ is the stationary increment of $x_t - x_{t-1}$. In this case, the variance of $\epsilon_t$ is given by:

---

2 In theory, one could use many alternative positively skewed distributions to model prices, but the lognormal distribution is generally the starting point for price modeling. Another rationale for using this distribution could be to capture occurrences of extreme prices, as can be seen in Figure 2-1.
\[ \sigma_t^2 = \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t}) \]

Equation 2-5

Dixit and Pindyck note that Equation 2-4 is a first-order autoregressive process\(^3\) and that one could use past price data and run a linear regression of the change in the natural log of oil price \(x_t - x_{t-1}\) on the previous period price \(x_{t-1}\) as follows:

\[ x_t - x_{t-1} = a + bx_{t-1} + \epsilon_t \]

Equation 2-6

Following this procedure, the fitted parameters \(\hat{a}\) and \(\hat{b}\) from the regression in Equation 2-6 are used to obtain fitted parameters for the mean of the natural log of the oil price, the mean-reversion coefficient, and the volatility of the mean-reverting model\(^4\) (Dixit and Pindyck 1994 p. 76):

\[ \bar{x} = -\frac{\hat{a}}{\hat{b}} \quad \hat{\kappa} = -\ln(1 + \hat{b}) \quad \hat{\sigma} = \hat{\sigma}_e \sqrt{\frac{2\ln(1 + \hat{b})}{(1 + \hat{b})^2 - 1}} \]

Equation 2-7

where \(\hat{\sigma}_e\) is the standard error of the regression.

---

\(^3\) Such a process means that values of a time series variable (say, the oil price for a given week) are linearly related to the values lagged one period before each value.

\(^4\) The caret symbol here denotes that the variable has been determined from a regression using actual price data.
2.4 ACCOUNTING FOR INFLATION

Economies experience inflation and deflation, which in turn complicates the task of valuation. Inflation refers to an increase in an economy’s overall price level, which is a measure of average prices of goods across an economy. Conversely, deflation occurs when there is a decrease in an economy’s price level (Abel et al. 2008 pp. 6-7, 46-52). Figure 2-2 shows the evolution of the Consumer Price Index (CPI), one of the most commonly used measurements of the price level, over the last century. The CPI measures, for a point in time, the prices of a basket of goods that an average consumer purchases relative to that basket’s prices in a specified base period (Abel et al. 2008 p. 618). Prior to the mid-20th Century, the CPI was relatively constant over a long period, with short bouts of inflation generally being balanced by subsequent deflation (Abel et al. 2008 p. 7). Since the 1960s, inflation has become normal, resulting in a large increase in the price level.
Periods of inflation, which have been pervasive in the modern economic era, are said to erode the “purchasing power” of a dollar. This means that since prices on average have increased, one dollar at the end of the inflationary period generally buys fewer goods and services than it could at the beginning of the period. Fortunately, workers’ wages tend to rise over time to compensate for the effects of inflation, so it usually does not render everyone destitute. On the other hand, this is not to say that inflation cannot be harmful and that wages fully adjust to the rate of inflation.

Thus, to adjust for the effects of inflation and deflation, economists make a distinction between nominal and real values when studying economic variables. The
nominal value of an economic variable (for example, an interest rate or the spot price of oil) is its actual quoted market value at a given point in time. By contrast, the real value of the variable is the nominal value adjusted to the prices from a “base year” by using a price index to reflect changes in the general price level over time. Real variables allow one to make a better comparison of economic metrics across time, reflecting true movements in value rather than inflationary or deflationary changes. Many economists believe that economic decisions are based on real interest rates, not nominal ones (Williamson 2007 p. 20).

The CPI is an inflation measure that captures movements in the prices of goods consumers buy, whereas the implicit GDP deflator is a broader measure of inflation across the range of goods produced in an economy. The latter is a more appropriate measure for the purposes of this study because it also accounts for changes in investment good prices. Moreover, the CPI tends to overstate the magnitude of inflation since it does not account for changes in purchasing behavior because of changes in relative prices. Instead, it uses a fixed basket of goods to measure changes in the price level (Williamson 2007 p. 55). The GDP deflator also uses a geometric averaging technique known as chain-weighting to correct for distortions in measurement of inflation.

The nominal price of oil can be adjusted to its real value using the following expression (“Deflating Nominal Values to Real Values”):

\[
real \ oil \ price = \frac{nominal \ oil \ price}{price \ index \ (decimal \ form)}
\]

Equation 2-8
where the price index,\(^5\) which measures the price level and is traditionally given as 100 for an arbitrarily-chosen base year, would be 1 in decimal form. Provided a price index data series with an existing base year, it is possible to reindex it to a desired new base year simply by dividing all price index values in the data series by the existing price index for the chosen new base year ("Deflating Nominal Values to Real Values"). Table 2-1 illustrates this calculation for the implicit GDP deflator as provided by the U.S. Department of Commerce’s Bureau of Economic Analysis (BEA) ("Gross Domestic Product: Implicit Price Deflator"). By using Equation 2-8 and the reindexed GDP deflator in Table 2-1, one can calculate the (real) oil price in 2013 dollars.

Table 2-1: GDP deflator, 2010 to 2013, with reindexing

<table>
<thead>
<tr>
<th>Date</th>
<th>GDP Deflator</th>
<th>Original: Base Year = mid-2005</th>
<th>Reindexed: Base Year = 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/2010</td>
<td>110.216</td>
<td>94.657</td>
<td></td>
</tr>
<tr>
<td>4/1/2010</td>
<td>110.706</td>
<td>95.078</td>
<td></td>
</tr>
<tr>
<td>7/1/2010</td>
<td>111.238</td>
<td>95.535</td>
<td></td>
</tr>
<tr>
<td>10/1/2010</td>
<td>111.795</td>
<td>96.013</td>
<td></td>
</tr>
<tr>
<td>1/1/2011</td>
<td>112.372</td>
<td>96.509</td>
<td></td>
</tr>
<tr>
<td>4/1/2011</td>
<td>113.109</td>
<td>97.142</td>
<td></td>
</tr>
<tr>
<td>7/1/2011</td>
<td>113.95</td>
<td>97.864</td>
<td></td>
</tr>
<tr>
<td>10/1/2011</td>
<td>113.987</td>
<td>97.896</td>
<td></td>
</tr>
<tr>
<td>1/1/2012</td>
<td>114.599</td>
<td>98.421</td>
<td></td>
</tr>
<tr>
<td>4/1/2012</td>
<td>115.035</td>
<td>98.796</td>
<td></td>
</tr>
<tr>
<td>7/1/2012</td>
<td>115.81</td>
<td>99.462</td>
<td></td>
</tr>
<tr>
<td>10/1/2012</td>
<td>116.089</td>
<td>99.701</td>
<td></td>
</tr>
<tr>
<td>1/1/2013</td>
<td>116.437</td>
<td>100.000</td>
<td></td>
</tr>
</tbody>
</table>

\(^5\)The price index considered could be the CPI, GDP deflator, or any other suitable measure of the general price level.
Figure 2-3 provides the weekly West Texas Intermediate (WTI) spot oil price in both nominal dollars and 2013 dollars, corrected using the implicit GDP deflator from the BEA (EIA/Thompson Reuters). Since 2013 is chosen as the base year, there is a substantial gap between nominal and real oil prices for earlier years.

![Figure 2-3: Correcting oil prices for inflation (nominal price data from the EIA/Thompson Reuters)](image)

Since the GDP deflator is given on a quarterly basis and WTI price data is on a weekly basis, the GDP deflator on a weekly basis was inferred by assuming a simple linear interpolation between quarterly data points. As the most recent GDP deflator data point is for January 1, 2013, any weekly WTI values after this date were assumed to have a corresponding GDP deflator value for January 1, 2013.
2.5 Estimating Parameters for the Mean-Reverting Model

Using weekly oil price data for WTI crude from January 2000 onward and Equation 2-7, one can estimate the parameters for the mean-reverting model for both nominal and 2013 dollars. This exercise grounds estimates of the mean-reverting process’ parameters in actual data, although historical data do not necessarily characterize the nature of future price movements. Indeed, a forecaster could alternatively subjectively choose parameters for the model depending on beliefs regarding future market behavior. The aim here is to use parameters that reflect recent price data for the purposes of illustrating how assuming a mean-reverting model would potentially affect an operator’s decisions for a reservoir and the NPV that the operator ultimately realizes.

Table 2-2 illustrates the results of this estimation exercise; it contains fitted parameters for four price series covering two time periods. Each time period has a price series for both nominal and real prices. The parameters with gray shading were used for the simulated price data shown in Figure 2-4 and Figure 2-5.

---

6 A special thanks to Dr. Joe Hahn for providing a spreadsheet template to calculate parameters for the simple mean-reverting model.
Table 2-2: Fitting parameters to a simple mean-reverting model to four possible data series (comparing fits to nominal and real data)

<table>
<thead>
<tr>
<th></th>
<th>WTI - 2000 Onward</th>
<th>WTI - 2008 Onward</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fit to Nominal</td>
<td>Fit to Real (2013)</td>
</tr>
<tr>
<td></td>
<td>Prices</td>
<td>Prices</td>
</tr>
<tr>
<td>$x_\sigma$</td>
<td>4.56</td>
<td>4.56</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>4.634</td>
<td>4.53</td>
</tr>
<tr>
<td>$S_\sigma$</td>
<td>$95.84$</td>
<td>$95.84$</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>$82.89$</td>
<td>$79.15$</td>
</tr>
<tr>
<td>$\hat{a}$</td>
<td>0.0199</td>
<td>0.0261</td>
</tr>
<tr>
<td>$\hat{b}$</td>
<td>-0.0045</td>
<td>-0.0060</td>
</tr>
<tr>
<td>Observations</td>
<td>697</td>
<td>280</td>
</tr>
<tr>
<td>Standard Error</td>
<td>Weekly</td>
<td>Monthly</td>
</tr>
<tr>
<td>from Regression</td>
<td>4.40</td>
<td>9.17</td>
</tr>
<tr>
<td>($\sigma$, percent)</td>
<td>4.41</td>
<td>9.19</td>
</tr>
<tr>
<td>Volatility ($\sigma$,</td>
<td>Weekly</td>
<td>Monthly</td>
</tr>
<tr>
<td>percent)</td>
<td>4.83</td>
<td>10.06</td>
</tr>
<tr>
<td>Mean Reversion</td>
<td>0.0045</td>
<td>0.0195</td>
</tr>
<tr>
<td>Coefficient ($\bar{\kappa}$)</td>
<td>0.0178</td>
<td>0.0773</td>
</tr>
</tbody>
</table>

Data from the 1990s and earlier have been excluded from this analysis, as they are not likely to be relevant to current oil price behavior. Including data from these time periods would likely lead to a significant underestimate in the mean oil price for the model. For each of the four data series, there are parameters for both weekly and monthly data. Since the price data used in the regression is given on a weekly basis, to simulate potential future oil price paths using monthly time increments, it is necessary to scale some fitted parameters for the weekly data to obtain parameters appropriate for monthly time increments. This scaling is accomplished as follows (Fuleky 2012):

\[
\begin{align*}
\sigma_{\text{monthly}} &= (\sigma_{\text{weekly}}) \sqrt{\frac{52}{12}}, \\
\sigma_{\epsilon,\text{monthly}} &= (\sigma_{\epsilon,\text{weekly}}) \sqrt{\frac{52}{12}}, \\
\bar{\kappa}_{\text{monthly}} &= \frac{\bar{\kappa}_{\text{weekly}}}{12}
\end{align*}
\]

Equation 2-9
For the data series for 2000 onward, the mean reversion coefficient is higher for real prices than it is for nominal prices. This result is to be expected since removing the effects of inflation narrows the range over which the price fluctuates; this adjustment has the effect of making the oil price revert to the mean value more quickly for the real data. Adjusting for inflation has a similar but smaller impact for the fitted mean reversion coefficient for the 2008 onward data, which also makes sense since the effects of inflation are smaller over a shorter time period. Volatility is essentially identical for nominal and real prices for each time period, and it is a bit larger for the 2008 onward data versus the same value for the larger data set. The later result is consistent with the historically large swings in the oil price within the last five years. Finally, the estimated mean oil price varies, although for the larger data set (2000 onward), the estimated values for the real and nominal oil prices are still relatively close to each other.

For subsequent simulations of mean-reverting prices, the parameters for the real price data series for 2000 onward (shaded in gray in Table 2-2), will be used by default. Further modifications are made to adjust the fitted volatility and account for possible changes in the mean price.

2.6 Monte Carlo Simulation of Potential Mean-Reverting Price Realizations

Using the estimated parameters from the previous section, it is relatively straightforward to simulate a set of oil price realizations (a potential path of prices in the future) using Monte Carlo simulation (Dixit and Pindyck 1994). Rearranging Equation
2-4, using the expression of variance of $\epsilon_t$ and shifting the subscript by one time period yields an expression to simulate the natural log of the oil price at time $t+1$ given the value at time $t$:

$$x_{t+1} = x_t e^{-\bar{R} \Delta t} + \bar{x} \left(1 - e^{-\bar{R} \Delta t}\right) + \hat{\sigma} \sqrt{\frac{1-e^{-2\bar{R} \Delta t}}{2\bar{R}}} \epsilon$$

Equation 2-10

where the actual spot price at $t+1$ is given by $S_{t+1} = e^{X_{t+1}}$ and $\epsilon$ is a random variable with a mean of 0 and a standard deviation of 1. Starting with the current spot price, Equation 2-10 is used to generate a realization with monthly time steps for an arbitrary number of months. This process is repeated for multiple trials to produce several realizations and, since $\epsilon$ is a random variable, each is essentially a unique, independent path. On the other hand, while each path is different, individual paths still obey the stochastic mean-reverting process with the parameters estimated.

Figure 2-4 contains five simulated price paths to May 2023 using Equation 2-10 and the relevant monthly parameters for real WTI price data from 2000 onwards. It also plots the actual real price data up to May 10, 2013. Particularly noteworthy is the volatility the simulated series exhibit. The fitted volatility merely reflects price behavior from the last 13 years, particularly the steep run-up and subsequent crash in 2008, which might not reflect the volatility of the oil price going forward. For the purposes of this investigation, however, properly fitting values of the mean-reverting process is not nearly as important as the implications of the process and its parameters’ sizes on reservoir management.
Figure 2-4: Simulated price realizations for simple mean-reverting process (fitted volatility)

For comparison, Figure 2-5 contains five simulated price paths for a mean-reverting process with the same parameters as those used in Figure 2-4, with the exception of volatility, which is half the fitted value. This arbitrary change might reflect the judgment of a forecaster adapting information from historical data for expectations of the future, such as oil companies expanding investment in more expensive unconventional projects in response to higher global demand and prices. As expected, the oil price in the simulated price paths do not fluctuate nearly as much as they do in Figure 2-4, although they still exhibit substantial volatility when compared to pre-2004 data.
A simple mean-reverting model, which assumes the mean oil price is constant, is one possible method for modeling oil price behavior. Other options include mean-reverting models with short-term shocks from surprise events such as wars and supply disruptions approximated using price jumps that arrive according to Poisson processes (Dias and Rocha 2001) and two-factor models\(^7\) that incorporate both mean-reverting behavior and uncertainty regarding the mean price itself (Schwartz and Smith 2000). These models entail more complexity and therefore require the estimation of more

\(^7\) The term “two-factor” does not refer to the number of parameters in the model; there are seven.
parameters. Determining the jump size distribution for the former can be difficult because of the small sample size of past jump events; the alternative is to use market data from financial options to estimate an implied jump distribution (Dias 2005).

The two-factor model seems appealing given the behavior of oil prices in recent years, as it appears that the mean price has shifted to a higher equilibrium level. This model treats the mean (long-term) price as variable and the observed spot price as following a mean-reverting process around the long-term mean price. It also requires significantly more estimation effort and still might not yield results that reflect economic reality.

Hahn et al. forecast the long-term price of crude oil using this two-factor approach, which involves the use of both spot oil price and futures data to fit parameters (2010). Using a Kalman filter, which entails a recursive estimation procedure, they fit a two-factor model to the crude oil price and futures data for three points over the past decade. These three parameter estimations yield quite distinct forecasts for the long-term oil price and confidence envelopes representing the predicted 10th and 90th percentiles of forecast outcomes, as seen in Figure 2-6. The observed oil price is essentially within the confidence envelopes for the first two forecasts, but the last two project steeply increasing oil prices, which would be unlikely to last indefinitely. Indeed, Hahn et al. note the prices given by these forecasts would likely result in negative impacts on the global economy, reducing demand for oil, and the introduction of more supply because of more marginal fields becoming economic. These effects, which are not incorporated into the two-factor model, would most likely in turn cause the realized long-term price to be
below its predicted value. Hahn et al. then adjust their model by solely projecting forward the long-run component of the stochastic process, removing the short-term component by assuming some of the model parameters to be zero. They find that the resulting forecasted oil price trajectories are more plausible, albeit still on steeply-increasing tracks.

Figure 2-6: Three two-factor forecasts (from Hahn et al. 2010)

The decision regarding which type of stochastic model is most applicable for simulating the future oil price is clearly quite difficult. Hahn et al. follow Dixit and Pindyck’s (1994) suggestion to rely on both theoretical considerations and statistical tests to guide this choice. They apply the Dickey-Fuller test to two overlapping sets of oil prices, one from 1990 to 2012 and the other from 1990 to 2004 (Hahn et al. 2010). The latter is chosen as it represents a period before the recent run-up in oil prices and during which it is believed that a mean-reverting model is more applicable. The results of this
test, along with others, suggest that the oil price was likely mean-reverting from 1990 to 2004 but did not exhibit mean-reverting behavior for the longer 1990 to 2012 period. They observe this finding corroborates those of others who find evidence of mean-reverting behavior in oil prices before 2005. Hahn et al. conclude that a variable (i.e. not constant) mean oil price is likely a better assumption for modeling future oil prices.

On the other hand, it is worthwhile also to resort to theoretical considerations, such as “intuition regarding the operation of equilibrating mechanisms” when deciding an appropriate model for forecasting oil prices, as Dixit and Pindyck suggest (1994 p. 78). It is also quite possible that the crude oil price will exhibit mean-reverting behavior for at least a decade going forward, as it has previously following other tumultuous periods. The price increases of the last decade, combined with technological advances, have improved the economics of many reservoirs once considered marginal, such as unconventional resources. While the steeper decline rates of some wells used in shale formations suggest they will not be able to boost oil supplies indefinitely, the sheer quantity of available unconventional resources could hold the mean price to a relatively high but stable level for years, if not decades, to come. Furthermore, emerging economies such as China’s cannot grow at double-digit rates indefinitely. Thus here the base case assumption will be that a simple mean-reverting process applies. A modification to this base case is discussed in the next section.

Dixit and Pindyck also note “it is usually impossible to obtain analytical solutions for optimal investment when underlying stochastic variables are modeled as mean-reverting, so that one must instead use numerical solution techniques” (1994 p. 78). This
fact combined with the nuances of the models used to estimate oil recovery discussed in subsequent chapters mean that it unfortunately will be unrealistic to obtain analytical solutions for determining optimal reservoir design parameters such as switching times from one recovery method to the next. As a result, optimization tools from Excel software add-ins will be used to numerically solve for these optimal values in each case.

2.8 **Using a Logistic Function to Model the Long-Term Mean Oil Price**

The two most recent forecasts Hahn et al. provide in Figure 2-6 suggest mean oil prices could continually increase at a substantial rate in the future to historically high levels. If real oil prices were to increase indefinitely and operators expected this behavior would persist, at certain rates of price appreciation they would have an incentive to wait forever to produce from known reservoirs. This would create a feedback loop in which no oil is produced, causing the price to skyrocket. Of course, there are stabilizing mechanisms on the demand side that prevent this extreme scenario from happening; at certain price levels, demand for oil declines because increased economic activity becomes too costly and some consumers substitute oil for other sources of energy.

Therefore, for the purpose of evaluating reservoirs on longer time scales, a forecast like the final one depicted in Figure 2-6 must be modified to account for these stabilizing mechanisms. One possibility for approximating this behavior would be to assume a logistic growth model, which is used in applications such as modeling population growth (Tsoularis and Wallace 2002). This model assumes that a time-varying parameter, which here will be the mean oil price, exhibits exponential growth for
early times but slows as it reaches a carrying capacity. A general equation for this model is given by the following expression:

\[ p_{\text{mean}}(t) = \frac{g}{1+e^{-h(t-k)}} + m \]

Equation 2-11

This equation can be modified to reflect information regarding the fitted mean price for the mean-reverting model and expectations regarding future mean price growth. The parameters \( g, h, k, \) and \( m \) determine the carrying capacity, or maximum value that the mean price can attain, and how quickly it increases. The parameters are chosen so that the initial mean price is realistic based on recent data and future mean prices reflect a forecaster’s expectations regarding the trajectory of the mean price. For example, the most recent mean price forecast that Hahn et al. (2010) estimate could be used as a guide for the rate of increase that the price in Equation 2-11 exhibits, and the forecaster could choose parameters accordingly to ensure the mean price never exceeds a specified maximum mean price.

Here it is assumed that the initial mean price predicted by Equation 2-11 will be approximately the mean price fitted using regressions for the simple mean-reverting model with real price data from 2000 onward. The other parameters are chosen to reflect a possible scenario where the mean oil price steadily increases over the next two decades before approaching $150 per barrel for dates further in the future.\(^8\) $150 per barrel is chosen as a hypothetical “carrying capacity” value. One could speculate that since this is

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\(^8\) This trajectory does not reflect the third forecast provided by Hahn et al. in Figure 2-6, but rather a more moderate price appreciation over a longer time period.
The level around which crude oil traded at its peak before the last recession, it is the highest mean price the economy could handle before demand effects put downward pressure on the price. The chosen parameters yield the following equation for the mean oil price as a function of time $t$ measured in years since 2013:

$$p_{\text{mean}}(t) = \frac{75}{1 + e^{-0.3(t-10)}} + 75$$

Equation 2-12

The mean-reverting process is now modeled in the following manner. Half the fitted volatility is used for the model here:

$$x_{t+1} = x_t e^{-\bar{\kappa} \Delta t} + \bar{x} (1 - e^{-\bar{\kappa} \Delta t}) + \frac{1}{2} \bar{\sigma} \sqrt{\frac{(1-e^{-2\bar{\kappa} \Delta t})}{2\bar{\kappa}}} \varepsilon,$$

$$\bar{x} = \ln \left( p_{\text{mean}}(t) \right) = \ln \left( \frac{75}{1 + e^{-0.3(t-10)}} + 75 \right)$$

Equation 2-13

The historical WTI is plotted in Figure 2-7 along with five simulated realizations of the mean-reverting price based on the process of Equation 2-13. The projected mean price according to the logistic function as well as the base case assumption of a constant mean price are also included for reference.
2.9  OIL PRICES AND COSTS

2.9.1  Introduction

The capital and operating costs for developing a reservoir appear to be related to the price of crude oil, as can be seen in Figure 2-8, so it is worthwhile to consider capturing this behavior when modeling the value of a field. This chart plots IHS CERA’s Upstream Capital Cost Index (UCCI) and Upstream Operating Cost Index (UOCI) alongside the WTI crude oil price, all expressed as an index compared to their values in the year 2000. The UCCI “…tracks the costs of equipment, facilities, materials and personnel (both skilled and unskilled) used in the construction of a geographically
diversified portfolio of 28 onshore, offshore, pipeline and LNG projects” while the UOCI “…measures cost changes in the oil and gas field operations arena” (“IHS CERA UCCI and UOCI”). The increase in capital and operating costs beginning around 2004 followed by a drop in late 2008 and subsequent increase afterwards mirror the movements in the price of crude oil.

Figure 2-8: Nominal oil price (WTI) versus capital and operating costs

2.9.2 Economic Theory

One can conceptualize the markets for oilfield capital investment and services in terms of supply and demand, as shown in Figure 2-9, to help explain the links between costs and the oil price. The diagrams plot generic supply and demand curves Q^S and Q^D in a perfectly competitive market for oilfield services. A supply curve represents, at a particular point in time, the relationship between the quantity of a product or service
businesses in a market will provide for a given price of that product. As the price of the product increases, more businesses enter the market and existing businesses increase their production. This behavior explains the upward slope of the supply curve. A demand curve represents, at a particular point in time, the relationship between the quantity of a product or service buyers in a market will consume for a given price of that product. The downward slope of the demand curve is similarly intuitive; a higher price for a product means that buyers will consume less of that product. For the markets of oilfield services, $Q^S$ represents what providers of oilfield services are willing to supply while $Q^D$ represents what operators are willing to purchase.

The intersection of the supply and demand curves yields the equilibrium price and quantity produced/consumed for the product or service in question. Consider the curves $Q^D_1$ and $Q^S$ in Figure 2-9, which represent initial demand and supply curves for the market for oilfield services. If the price were greater than the equilibrium price $P_1$, there would be more oilfield services provided than operators would be willing to consume. The service providers would have to lower their prices to induce enough operators to consume their services.

Now consider the effects of an increase in the crude oil price; namely, the likely mechanisms that lead to increases in capital and operating costs. This oil price increase is not directly measured in Figure 2-9, which models the market for oilfield services, but is instead reflected by shifts in the curves as a result of behavior changes by market participants. Operators are now willing to increase investment by expanding current projects and starting others that were previously unprofitable. This boost in activity spurs
a shift in the demand curve from \( Q_D^1 \) to \( Q_D^2 \) and the equilibrium price and quantity consumed shift from \( P_1 \) to \( P_2 \) and \( Q_1 \) to \( Q_2 \). Figure 2-9 illustrates this change.

Figure 2-9: Illustration of supply and demand

2.9.3 Correlation of Costs with Oil Prices

The previous discussion on the economic theory behind the crude oil price and costs of extraction is certainly simplified, but provides some basis for assuming that there is could be correlation between the oil price and costs. Figures 2-10 and 2-11 plot the real UCCI and UOCI versus real WTI prices. For each plot, the data are further subdivided into three overlapping groups: all data (2000 to present), data up to and including the first quarter of 2008 (labeled “Early Data”), and data after the first quarter
of 2008 (labeled “Late Data”). This division was chosen to highlight possible trends before and after the large oil price swings in 2008. CERA provides the UCCI on an annual basis for 2000 through 2004 and then at least for two quarters of each year for 2005 onward. They provide the UOCI on an annual basis for 2000 through 2007 and then at least for three quarters of each year for 2008 onward. The WTI and the cost indices have been adjusted for general inflation to 2013 values using the GDP deflator. See Appendix A for the full data. Finally, each plot contains linear regressions for the full data set, the early data, and the late data; the parameters for these appear in Tables 2-3 and 2-4.
Figure 2-10: Correlation of capital costs (UCCI) with WTI (real 2013 values)

Figure 2-11: Correlation of operating costs (UOCI) with WTI (real 2013 values)
Upon inspection of these figures, one notices a few potential trends. There are definitely positive relationships between costs and the oil price, as were also evident in Figure 2-8. Operating costs appear to be more strongly correlated with the oil price than are capital costs, but this is likely at least in part because the capital cost data is more granular than the operating cost data. Indeed, the early data points that appear far off-trend for capital costs are those for Q1 2005, Q3 2005 and Q1 2006; for this period operating costs are only reported on an annual basis. Based on the capital costs, it appears that the increase in costs lagged the increase in the oil price over this short period. A time series regression might provide some interesting insight into the dynamics of oil prices and costs; unfortunately this is not feasible with the incomplete data available from CERA.

For both capital and operating costs, there appear to be distinct relationships for the early data and late data. This apparent shift in the relationship between costs and the oil price is probably partially because of the increased granularity of the data for the later time period. The extreme volatility in the oil price from Q3 2008 to Q2 2009 is also likely responsible for the shift; indeed, the three outliers on the left and one on the right for the “Late Data” series on both plots are from this time period. Overall, operators likely did not immediately respond to such rapid changes in the oil price over this period with drastic cutbacks in spending on capital and operating costs, rightfully viewing the price shifts as transitory phenomena. As a result, costs remained “sticky” at elevated levels over a wide range of prices. The data in Figures 2-12 and 2-13 support operators’
muted response to large price swings in 2008. As shown in Figure 2-12, U.S. production appears to be relatively insensitive to the dramatic change in the oil price.

Figure 2-12: U.S. crude oil production and WTI prices, 2008-2009

While Figure 2-13 shows that relatively low oil prices in 2009 did lead to a decline in capital spending by E&P companies, the drop in capital expenditure (CAPEX) was proportionally much smaller than the fall in the oil price from its 2008 peak.
An interpretation of the stickiness of costs at higher levels is that the nature of oilfield investment has fundamentally changed along with higher oil prices. In other words, the cost structure of the E&P business has experienced a step change as “easier” oil has become scarce and new projects are more expensive and technically challenging. It is entirely possible that the increase in costs is correlated with a shift in the long-term mean oil price and overall investment is driven based on market-wide expectations of the long-term oil price. In the future, costs could stabilize along with the long-term oil price, as evidenced by past data on drilling costs discussed below. The step change in the overall level of the oil price also means that there is higher demand for capital investment and operating necessities, as more projects are economically viable.

The supply and demand effects are illustrated in Figure 2-14. Higher costs because of the greater difficulty of projects undertaken are reflected in the shift of the
supply curve upwards. Increased demand for investment from operators in response to higher oil prices shifts the demand curve up. Both shifts cause an increase in the price of oilfield services. Assuming another step change that shifts the supply curve substantially does not occur, further fluctuations in the oil price lead to shifts in demand for oilfield services and the now the price of oilfield services changes in the future around a higher mean level near \( P_2 \). This reasoning could explain the different correlations that apply for the late data versus the early data, as depicted in Figures 2-10 and 2-11.

Figure 2-14: Supply and demand shifts

Overall, it appears that the level of costs is associated with the trajectory of the long-term oil-price, which appears to have increased from 2000 to present. While the oil
price experienced wide fluctuations from 2007 to 2009, costs remained relatively stable 
over this period at higher levels than they were pre-2004. This observation suggests that 
the correlations associated with later price data might be more representative of how 
costs might be related to the oil price in the future.

Although in reality the relationships between the real oil price and real costs are 
not exactly simple and linear, it is worthwhile to check how well linear regressions fit the 
data as first approximations. While this technique might not adequately capture the 
complexities of the economics and does not prove causation, the regressions can 
potentially provide guidelines for the degree to which costs are correlated with the oil 
price. For the real capital and operating cost indices in Figures 2-10 and 2-11, linear 
regressions of the following forms were used on each of the three sets of data:

\[
CAPEX_{index,real} = a(p_{real,WTI}) + b
\]

Equation 2-14

\[
OPEX_{index,real} = b(p_{real,WTI}) + c
\]

Equation 2-15

Table 2-3 and Table 2-4 contain the fitted values of these regressions as well as 
the coefficient of determination (R\(^2\)) values, which indicate the strength of fit. An R\(^2\) 
value closer to 1 indicates that the regression explains the variance in the data well, while 
a value closer to 0 indicates that it does not.\(^9\) The slopes \(a\) and \(b\) indicate the degree to 
which costs change as the oil price changes. Clearly there is shift from the early data to

\(^{9}\) This assumes a Gaussian distribution is expected for errors.
the late data where costs fluctuate less than the oil price and remain at relatively high. On
the other hand, the fit is poorer for the late data, particularly for capital costs. Overall,
these regressions rely on little data and do not necessarily represent statistically
significant fits. Moreover, as future trends in costs may differ from past ones, a
forecaster could interpret past data as a starting point for projecting what might occur in
the future.

Table 2-3: Regressions for capital costs

<table>
<thead>
<tr>
<th>Data Set</th>
<th>a</th>
<th>b</th>
<th>R²</th>
<th>Data Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Data</td>
<td>1.32</td>
<td>98.759</td>
<td>0.601</td>
<td>28</td>
</tr>
<tr>
<td>Early Data</td>
<td>1.4134</td>
<td>75.531</td>
<td>0.755</td>
<td>12</td>
</tr>
<tr>
<td>Late Data</td>
<td>0.2403</td>
<td>203.59</td>
<td>0.243</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 2-4: Regressions for operating costs

<table>
<thead>
<tr>
<th>Data Set</th>
<th>b</th>
<th>c</th>
<th>R²</th>
<th>Data Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Data</td>
<td>0.7448</td>
<td>117.02</td>
<td>0.757</td>
<td>26</td>
</tr>
<tr>
<td>Early Data</td>
<td>0.979</td>
<td>96.868</td>
<td>0.962</td>
<td>9</td>
</tr>
<tr>
<td>Late Data</td>
<td>0.2267</td>
<td>165.12</td>
<td>0.641</td>
<td>17</td>
</tr>
</tbody>
</table>

2.9.4 Correlation of Drilling Costs with Oil Prices

Drilling costs constitute a substantial portion of the capital investment for oil and
gas fields, so it is worthwhile to consider how they may be related to the crude price.
There are also relatively consistent data available to examine these particular costs, so
they may help corroborate previous observations from the CERA indices.
The Bureau of Labor Statistics (BLS) publishes Producer Price Indices (PPIs) that track the change in the prices that businesses in particular industries receive for the products and services they sell. PPIs are calculated using a modified Laspeyres Index, which is described by the following expression:

$$Index_t = \frac{\sum q_o p_i}{\sum q_o p_o} (100)$$

Equation 2-16

where $Q$ and $P$ are the quantity and price of the good produced and the subscripts $o$ and $i$ represent a “base” period and the period for which an index is being measured (“PPI FAQ”). The sums in the numerator in the denominator of Equation 2-16 represent weighted averages that aggregate the prices of the goods and services sold in the industry in the base period and period $i$.

One particular PPI with data in monthly time increments measures costs for drilling for oil and gas wells. Figure 2-15 illustrates this index, adjusted for inflation using the GDP deflator, along with the WTI oil price. Both are expressed in terms of 2013 dollars. On inspection, it is clear that there is some general positive correlation between the oil price and the drilling PPI.
Figure 2-15: Nominal oil price (WTI) versus capital and operating costs

Figure 2-16 expresses these data in a format more conducive to examining this possible correlation. The data markers are shaded according to the time periods for which they represent; lighter markers correspond to more recent dates. There are two clusters of data on this plot that correspond to distinct drilling cost regimes. Note the similarities to the data in Figures 2-10 and 2-11. The low-cost regime corresponding to older data is associated with lower variation in the oil price and there is no clear correlation of costs with the oil price. By contrast, the high-cost regime is associated with more volatile oil prices, which explains the greater scatter in the data in the upper region of the plot. It also appears here that if one assumes the long-term oil price has shifted from the low-cost regime to the high-cost one, there is a correlation between the long-term oil price and the cost level. Within each cluster of data and over shorter
timeframes, however, it is entirely possible that drilling costs are essentially uncorrelated with oil prices. Indeed, it is possible that costs could remain comparatively stable along with the mean oil price in the future, as they did during the price spike associated with the Persian Gulf War in the 1990s and the drastic swings of 2008.

Figure 2-16: Correlation of drilling cost PPI with WTI (real 2013 values)

Figure 2-17 replots the data in Figure 2-16, except now dividing it into the same overlapping periods used for Figures 2-10 and 2-11. There is also an additional series added for data from the 1980s and 1990s to compare older data present in this figure that is not available for the series in the previous figures. The trends here are similar for the “Early Data” versus the “Late Data”, although it is now apparent that the trend exhibited in the regression on the early data captures the upward movement in the long-term oil
price. Particularly noteworthy is the negative relationship between the oil price and drilling costs in the data from the 1980s and 1990s. This behavior, which is in contrast to the positive relationships found for other time periods, illustrates the difficulty of characterizing cost behavior.

![Figure 2-17: Division of drilling cost correlation into subperiods](image)

The regressions depicted in Figure 2-16 take the following form:

\[ PPI_{\text{real}} = f(p_{\text{real,WTI}}) + k \]

Equation 2-17

The fitted parameters appear below in Table 2-5. Unlike previous regressions, these are based on more data.
Table 2-5: Regressions for drilling costs

<table>
<thead>
<tr>
<th>Data Set</th>
<th>j</th>
<th>k</th>
<th>R²</th>
<th>Data Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980s and 1990s</td>
<td>-1.6135</td>
<td>207.51</td>
<td>0.2102</td>
<td>168</td>
</tr>
<tr>
<td>All Data</td>
<td>3.172</td>
<td>81.006</td>
<td>0.7545</td>
<td>329</td>
</tr>
<tr>
<td>Early Data</td>
<td>3.6949</td>
<td>59.007</td>
<td>0.6903</td>
<td>267</td>
</tr>
<tr>
<td>Late Data</td>
<td>0.3788</td>
<td>338.49</td>
<td>0.0847</td>
<td>62</td>
</tr>
</tbody>
</table>

Clearly, the relationship between the crude oil price and its costs of extraction is complex and varies depending on the time period examined. Evidence discussed above suggests that costs are associated with long-term oil prices, but there are more nuances that are not captured by this observation.

Augustine et al. (2006) examine aggregate well cost data from the American Petroleum Institute’s Joint Association Survey (JAS) to construct the MIT Depth Dependent Drilling (MITDD) Index. The index is divided based on the depth intervals of wells drilled, and is available for dates from 1972 to 2003. From their data and information on historical rig counts, they conclude that oil prices, rig counts, and the non-linearity of well costs with increasing depth strongly influence drilling costs, consistent with basic supply-and-demand arguments. They also note that shallow well cost indices have increased the most since the 1980s, as most rigs built since then have been for intermediate and deep wells. Augustine et al. (2006) also find that drilling costs declined in real terms from the 1980s through the 1990s because of technology improvements.

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10 An update to the index for more recent data is currently awaiting publication, per email communication with Chad Augustine and Maciej Lukawski.
These findings suggest that a more sophisticated analysis is required to study drilling costs’ relationship with oil prices. A forecaster would need to examine trends in drilling rig construction and availability for those suitable for the depth of the reservoir under consideration, as well as drilling technology improvements. Forecasts would project how this supply side may evolve in conjunction with demand for rigs in the required depth interval. Unfortunately, as is the case with making forecasts for oil prices based on assumptions regarding supply and demand, these determinants of costs are essentially impossible to predict for longer timeframes.

In the next chapter, costs will be modeled using two separate assumptions: no correlation with the oil price and correlation based on the regressions on the full data sets used in figures above. These two extreme cases will be used to examine differences in the ranges of NPV distributions that might result from Monte Carlo simulation.
Chapter 3: Reservoir Life-Cycle Optimization Using Decline Curve Models

3.1 INTRODUCTION

Forecasting the rate and amount that an oil and gas that a reservoir can produce is no simple task, as there are many complex phenomena for which to account and almost always limited information on key variables that influence these phenomena. It is often far simpler to extrapolate past production data using decline curve analysis to estimate future production for economic evaluations. According to Walsh and Lake (2003), there is a semitheoretical basis for using decline curves. Tank models based on physical intuitions regarding a reservoir’s depletion can predict exponential and hyperbolic decline behavior, depending on assumptions regarding the heterogeneity of the structure of the reservoir. Therefore this type of model can be used as a tool to approximate the production behavior of an actual reservoir, and is particularly useful for situations when production data exhibiting a declining rate are readily available.

Parra Sanchez (2010) examines the problem of optimizing the economic value of a reservoir over its entire life cycle, focusing on the time spent on each recovery method. She approximates past production of the Lost Soldier Tensleep reservoir in Bairoil, Wyoming by history matching its production data to an exponential decline model. Using this model and applying a set of economic assumptions, she then applies a series of tools to determine what the optimal switching times from primary recovery to waterflooding and then to CO\textsubscript{2} flooding should have been if the operator wanted to maximize the net present value (NPV) of the reservoir assuming it were beginning production today. One of her significant findings is that the operator would likely realize a substantial increase in NPV of the reservoir if it were to switch to waterflooding and CO\textsubscript{2} flooding far earlier than is common practice in the E&P industry today.
Models entail simplifications of real-world behavior for them to be more tractable for analysis. Parra Sanchez begins her analysis by optimizing a “deterministic case” where she assumes that model parameters describing production estimated by fitting the exponential decline model to existing production data are valid for hypothetical scenarios where the switching times have been altered. In reality, these parameters may not necessarily be the same if an operator had decided to switch recovery methods at different times. Moreover, an optimization of an actual reservoir requires estimates of the exponential decline model parameters before development. To approach this problem of maximizing the NPV of an undeveloped reservoir with uncertain production characteristics, she uses a decision analysis framework that employs three-point discretizations\(^\text{11}\) to approximate potential realizations of the exponential decline model parameters. She applies this framework to create decision trees that model the switch from primary recovery to secondary recovery.

Parra Sanchez (2010) also assumes that the crude oil price and development cost structure remain constant through the life of a project. This chapter extends her analysis by examining the implications of a mean-reverting oil price, both with and without correlated costs, on the optimal switching times predicted using the exponential decline model. It also adds additional assumptions regarding capital costs, taxes and royalties that alter the conclusions regarding optimal switching decisions. While extra effort is made here to model the economics of reservoir development, geologic and technical uncertainty is not captured; it is assumed that the operator knows these parameters \textit{a priori} and maximizes the reservoir’s value accordingly. The model discussed in subsequent chapters is better suited to accounting for these technical uncertainties.

\(^{11}\) In decision analysis, a three-point discretization is an approximation of a continuous distribution by assuming three possible outcomes, each associated with predefined probabilities. See Bickel et al. (2011) for further explanation of this process.
The decision analysis framework that Parra Sanchez uses to maximize the NPV of a reservoir \textit{a priori} requires an operator to estimate the ultimate recovery efficiencies and time constants for primary, secondary, and tertiary recovery. These estimates are not trivial; they involve complex relationships with many interacting design parameters in addition to switching times that influence the profitability of a project. Consequently, chapters four through six will address methods for predicting the production of three phases of recovery using models based on physical parameters and that account for other design choices in addition to switching times.

3.2 \textbf{THE EXPONENTIAL DECLINE MODEL}

3.2.1 \textbf{Introduction}

Parra Sanchez (2010) models the recovery efficiency $E_R$ for a reservoir, measured as the proportion of original oil in place (OOIP) produced, as a function of the time $t$ since the beginning of production. The details of this exponential decline model as applied to the Lost Soldier Tensleep reservoir are reviewed here. This reservoir was developed using primary recovery followed by two waterflooding phases, a peripheral flood and a pattern flood, and finally a CO$_2$ flood. The subscripts “1”, “2” and “3” correspond to primary, secondary and tertiary recovery; secondary recovery is accordingly divided into two separate phases denoted by “per” and “pat”.

Brokmeyer et al. (1996) estimate theoretical ultimate recovery efficiencies $E_{R1}^\infty$, $E_{R2\text{per}}^\infty$, $E_{R2\text{pat}}^\infty$, $E_{R3}^\infty$, corresponding to the four phases of production for the Lost Soldier Tensleep reservoir. These recovery efficiencies are shown in Figure 3-1 below.
Parra Sanchez (2010) fits the time constant parameters $\tau_1$, $\tau_{2\text{per}}$, $\tau_{2\text{pat}}$, and $\tau_3$ to the production history of the reservoir. All values of time are measured in years for the decline curve model.

### 3.2.2 Primary Recovery

Parra Sanchez models the recovery efficiency of primary production using the following equation:

$$E_{R1}(t) = E_{R1}^0 + (E_{R1}^\infty - E_{R1}^0) \left(1 - e^{-\frac{t}{\tau_1}}\right), t < t_{Life1}$$

Equation 3-1

where

- $E_{R1}^0 = 0$ is the recovery efficiency at the beginning of primary recovery (time 0)
- $E_{R1}^\infty$ is the theoretical ultimate recovery efficiency for primary recovery
- $\tau_1$ is the time constant for primary recovery
- $t_{Life1}$ is the amount of time spent on primary recovery
Equation 3-1 essentially states that the recovery efficiency increases at a decreasing rate as time progresses, governed by the time constant $\tau_1$, until it approaches the theoretical ultimate recovery efficiency for primary. The term “exponential decline” is used because the rate of production declines exponentially over time.

### 3.2.3 Peripheral Waterflood

The recovery efficiency for the next stage of production is defined in a similar fashion as it is for primary recovery:

$$E_{R2per}(t) = E_{R1}(t_{Life1}) + \left( E_{R2per}^\prime - E_{R1}(t_{Life1}) \right) \left( 1 - e^{-\frac{(t-t_{Life1})}{\tau_{2per}}} \right),$$

$$t_{Life1} \leq t < t_{Life2per} + t_{Life1}$$

Equation 3-2

where

- $E_{R1}(t_{Life1})$ is the recovery efficiency at the end of primary production
- $\tau_{2per}$ is the time constant for peripheral waterflooding
- $t_{Life2per}$ is the amount of time spent on peripheral waterflooding
- $E_{R2per}^\prime$ is an adjusted theoretical ultimate recovery efficiency for peripheral waterflooding, given by

$$E_{R2per}^\prime = E_{R1}(t_{Life1}) + \left( E_{R2per}^\infty - E_{R1}^\infty \right)$$

Equation 3-3
Equation 3-3 implies that switching earlier in primary production results in a penalty in the form of a reduced theoretical ultimate recovery efficiency for the peripheral waterflood. The incremental gain in theoretical ultimate recovery $E_{R2}^{\infty} - E_{R1}^{\infty}$ still remains the same regardless of the value of the primary production time $t_{Life1}$, though.

3.2.4 Pattern Waterflood

Based on Parra Sanchez’s model, the recovery efficiency of the pattern waterflood is defined as follows:

$$E_{R2pat}(t) = E_{R2per}(t_{Life1} + t_{Life2per}) + \left( E_{R2pat}^{\infty} - E_{R2per}(t_{Life1} + t_{Life2per}) \right) \left( 1 - e^{-\frac{|t-t_{Life2per}-t_{Life1}|}{\tau_{pat}}} \right),$$

$t_{Life2per} + t_{Life1} \leq t < t_{Life2pat} + t_{Life2per} + t_{Life1}$

Equation 3-4

where

- $E_{R2per}(t_{Life1} + t_{Life2per})$ is the recovery efficiency at the end of peripheral waterflooding\(^\text{12}\)
- $\tau_{pat}$ is the time constant for pattern waterflooding
- $t_{Life2pat}$ is the amount of time spent in pattern waterflooding

\(^\text{12}\) Parra Sanchez (2010) writes this variable as $E_{R2}(t_{Life2})$ for the general waterflooding case, but it is written here as $E_{R2}(t_{Life1} + t_{Life2})$ instead to be consistent with the fact that the recovery efficiencies are defined as functions of total production time $t$. 

55
• $E_{R2pat}^\infty \; \prime$ is an adjusted theoretical ultimate recovery efficiency for peripheral waterflooding, given by

$$E_{R2pat}^\infty \; \prime = E_{R2per}(t_{life1} + t_{life2per}) + \left(E_{R2pat}^\infty - E_{R2per}^\infty\right)$$

Equation 3-5

Equation 3-5 also implies a reduced theoretical ultimate recovery for pattern waterflooding if the operator chooses to switch from peripheral waterflooding before the theoretical ultimate recovery for this stage is reached.

3.2.5 Tertiary Recovery (CO$_2$ Flooding)

Finally, the recovery efficiency for tertiary production is defined as follows:

$$E_{R3}(t) = E_{R2pat}(t_{life1} + t_{life2per} + t_{life2pat}) + \left(E_{R2pat}^\infty \; \prime - E_{R2pat}(t_{life1} + t_{life2per} + t_{life2pat})\right)\left(1 - e^{-\frac{(t-t_{life2pat}-t_{life2per}-t_{life1})}{\tau_3}}\right),$$

$t_{life2pat} + t_{life2per} + t_{life1} \leq t < t_{life3} + t_{life2pat} + t_{life2per} + t_{life1}$

Equation 3-6

where

• $E_{R2pat}(t_{life1} + t_{life2per} + t_{life2pat})$ is the recovery efficiency at the end of pattern waterflooding

• $\tau_3$ is the time constant for tertiary recovery

• $t_{life3}$ is the amount of time spent in tertiary recovery
• $E_{R3}^{\infty'}$ is an adjusted theoretical ultimate recovery efficiency for tertiary recovery, given by

$$E_{R3}^{\infty'} = E_{R2pat}(t_{life1} + t_{life2per} + t_{life2pat}) + (E_{R3}^{\infty} - E_{R2pat}^{\infty})$$

Equation 3-7

3.2.6 Example Calculation of Recovery Efficiency Using Fitted Parameters

Table 3-1 provides the fitted parameters from Parra Sanchez (2010) and Brokmeyer et al. (1996) that Parra Sanchez uses in her exponential decline model for the Lost Soldier Tensleep reservoir. An example calculation of the recovery efficiency using these parameters appears in Figure 3-2, which closely follows the actual production history shown in Figure 3-1.

Table 3-1: Fitted Lost Soldier Tensleep parameters used in exponential decline model

<table>
<thead>
<tr>
<th>Recovery Phase (Subscript)</th>
<th>Time Spent on Method as of 1996 ($t_{life}$: years)</th>
<th>Time Constant ($\tau$: years)</th>
<th>Fitted Theoretical Ultimate Recovery Efficiency from Brokmeyer et al. ($E_R^{\infty}$: %OOIP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Recovery (1)</td>
<td>24</td>
<td>10</td>
<td>19.9</td>
</tr>
<tr>
<td>Peripheral Waterflooding (2per)</td>
<td>13</td>
<td>5</td>
<td>35.6</td>
</tr>
<tr>
<td>Pattern Waterflooding (2pat)</td>
<td>10</td>
<td>5</td>
<td>44.3</td>
</tr>
<tr>
<td>Tertiary Recovery (3)</td>
<td>8</td>
<td>7</td>
<td>54.2</td>
</tr>
</tbody>
</table>
Figure 3-2: Calculated Lost Soldier Tensleep recovery efficiency from exponential decline model

3.3 **ECONOMIC ASSUMPTIONS AND EVALUATION**

Parra Sanchez (2010) chooses NPV as the economic metric by which to judge the value of a project, noting that alternative measures such as internal rate of return (IRR) can provide misleading results. In particular, Parra Sanchez cites conclusions from Barua et al. (1986) that IRR can rank a project that produces cash flows very quickly as better than one that has a NPV that is many times greater. NPV already accounts for the time value of money and therefore treats cash flows far into the future as less valuable today, so this characteristic of IRR is definitely undesirable. NPV will also be used here to evaluate projects and as an objective function to maximize.

NPV is defined as the sum of the discounted cash flows from a project:
\[ NPV = \sum_{t=0}^{t_{\text{project}}} \frac{CF(t)}{(1+R)^t} \]

Equation 3-8

where \( CF(t) \) is the cash flow the project produces at time \( t \), including capital expenditures and expressed in real 2013 dollars, \( t_{\text{project}} \) is the lifespan of the project, and \( R \) is the real discount rate. Parra Sanchez (2010) takes the latter to be 7 percent, based on a 10 percent cost of capital and a historic rate of inflation of 2.8 percent; for consistency this value will also be used here. Times are measured in increments of one year.

The real cash flow, which can be negative if costs exceed revenues, depends on the type of recovery method employed at \( t \). In general, cash flows here are taken to be revenues minus costs, taxes and royalties, as these are assumed to have direct economic relevance for the project:

\[ CF(t) = Revenues(t) \times (1 - Roy_{\text{total}})(1 - T_{\text{sev}}) - Revenues(t-1) \times (T_{\text{adval}}) - Costs(t) \]

Equation 3-9

\( Roy_{\text{total}} \) is the overall royalty rate for the project, \( T_{\text{sev}} \) is the severance tax rate, and \( T_{\text{adval}} \) is the ad valorem tax rate. Wyoming has a 16.67 percent royalty on state land mineral leases on top of the 12.5 percent federal mineral royalty plus a 5.25 percent override royalty\(^\text{13}\) (Cook 2012). Cook assumes an additional 18.75 percent private royalty on top of these state and federal ones, based on information regarding a lease in the state, which brings the overall royalty burden \( Roy_{\text{total}} \) to 53.17 percent (2012). The

\(^{13}\) Cook states that most oil in Wyoming is produced on federal land.
state also levies a 6 percent severance tax net of royalties on production statewide (Cook 2012). Cook notes that ad valorem taxes range from 5.9 to 7.2 percent and are assessed on 100 percent of the value of a prior year’s production; here it will be assumed that $T_{adv} \text{ is 6.5 percent (2012).}$

Revenues and costs are modeled as follows:

\[
Revenues(t) = (OOIP)[E_R(t) - E_R(t - 1)](P_{WTI} - D)
\]

Equation 3-10

\[
Costs(t) = CAPEX(t) + OPEX(t)
\]

Equation 3-11

where $OOIP$ is the original oil in place, which is 240 million STB for the Lost Soldier Tensleep reservoir. The recovery efficiency function $E_R$ depends on the type of recovery method employed at $t$, as described in the previous section. $P_{WTI}$ is the oil price at $t$, modeled in this chapter using the simple mean-reverting process discussed in the previous chapter with parameters outlined in Table 2-2 and shaded in gray. These parameters are fitted to the real weekly WTI data from 2000 onward; some of the relevant parameters have been scaled below in Table 3-2 to annual values since the exponential decline model is evaluated in increments of one year.\(^\text{14}\) Finally, $D$ represents the discount at which Wyoming oil trades versus WTI,

\(^\text{14}\) The initial and mean oil prices, two other parameters necessary for simulation, remain the same regardless of the time step for simulated prices.
taken to be $14.54 based on Cook’s findings (2012). For modeling purposes, it is assumed that this discount remains constant even as the oil price fluctuates.

Table 3-2: Fitted mean-reverting model parameters for 2000 onward weekly real WTI data – scaled for annual simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Error from Regression ($\hat{\sigma}_e$)</td>
<td>0.3174</td>
</tr>
<tr>
<td>Volatility ($\hat{\sigma}$)</td>
<td>0.3183</td>
</tr>
<tr>
<td>Mean Reversion Coefficient ($\hat{\kappa}$)</td>
<td>0.3112</td>
</tr>
</tbody>
</table>

$\text{CAPEX}(t)$ and $\text{OPEX}(t)$ are the capital and operating costs of the project for year $t$, which depend on the phase of recovery. Parra Sanchez (2010) assumes that operating costs vary depending on the stage of recovery but that these values do not fluctuate over the life of the field; these costs are given in Table 3-3. Here, the two limiting cases of operating and capital costs being (a) uncorrelated with the oil price and (b) positively correlated with the oil price based on long-term data discussed in the previous chapter are examined. The costs in Table 3-3 are converted to current dollars using the CERA Upstream Operating Cost Index, assuming the cost figures in Table 3-3 correspond to Q2 2010. Operating costs are assumed to include the expense of injecting and producing water and CO$_2$ for later recovery phases, which is reflected in larger expenses for these later recovery phases. Unfortunately, this model does not directly link operating costs to the quantity of fluid injected, but instead approximates this link by assuming operating costs on a per barrel of produced oil basis. By contrast, the evaluation of the geologic-

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15 This is an average discount that applies to Wyoming, not a specific contractual amount for any particular project.
16 The latest available cost index figure is for Q3 2012, which is assumed to represent 2013 dollars.
parameter-based model in chapter seven does account for the fluid injection and production quantities for secondary and tertiary recovery.

Table 3-3: Operating cost assumptions for Lost Soldier Tensleep

<table>
<thead>
<tr>
<th>Recovery Phase (Subscript)</th>
<th>OPEX ($/bbl oil produced – 2010 dollars)</th>
<th>OPEX ($/bbl oil produced – 2013 dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Recovery (1)</td>
<td>3</td>
<td>3.37</td>
</tr>
<tr>
<td>Peripheral Waterflooding (2per)</td>
<td>7</td>
<td>7.85</td>
</tr>
<tr>
<td>Pattern Waterflooding (2pat)</td>
<td>8</td>
<td>8.98</td>
</tr>
<tr>
<td>Tertiary Recovery (3)</td>
<td>12</td>
<td>13.46</td>
</tr>
</tbody>
</table>

Capital costs are separated into drilling and completion costs, modeled using a regression in the Energy Information Administration’s (EIA’s) Oil and Gas Supply Module of the National Energy Modeling System, and equipment costs, modeled using data from a 2009 EIA cost study. The drilling cost regression is based on 2004 through 2007 data from the American Petroleum Institute’s Joint Association Survey of Drilling Costs and accounts for the nonlinear relationship of cost with depth as well as completion costs. The EIA estimates regressions for each region of the U.S.; the regression for the Rocky Mountain region is:

\[ Drilling\ Cost = \beta_0 + \beta_2 Depth^2 + \beta_3 Depth^3 \]

\[ \beta_0 = 85843.776 \]

\[ \beta_2 = 0.024046279/ft^2 \]

\[ \beta_3 = 3.11588 \times 10^{-6}/ft^3 \]

Equation 3-12\(^{17}\)

\(^{17}\) The coefficient for \(Depth^3\) is omitted from their regression because it was not found to be statistically significant.
Brokmeyer et al. (1996) note that the average depth of the sandstone in the reservoir is 5000 feet; assuming this depth and using the average value of the drilling PPI from 2004 to 2007 to adjust Equation 3-12’s predictions to current dollars yields a cost of $1,447,139 per well.\(^{18}\) The number of wells drilled for each recovery phase is not entirely clear, so a few assumptions have been made. Watson (2010) notes that 250 wellbores have been drilled into the reservoir by 2010.\(^{19}\) Here it is assumed that all wells are drilled at the beginning of a recovery phase, and the 250 wells have been allocated to each phase as described in Table 3-5. This schedule is frontloaded, and once pattern waterflooding has begun, infill drilling has more than doubled the number of wells in operation to convert existing wells to injectors and drill some new producers. For tertiary recovery, relatively few (20) additional wells are drilled.

For each phase of recovery, capital costs other than drilling are incurred as equipment such as rods, flowlines, storage tanks, manifolds, and separators are necessary to bring oil and gas from wellbore to market and handle produced fluids. For secondary recovery, upgrades are required to convert producers to injectors. The EIA estimated costs for representative 10-well leases in various regions of the U.S. by contacting the supply, service and contracting companies in each region to determine prices of equipment in June 2009. Costs were determined based on the producing depth of the wells in a representative lease and phase of recovery. Primary recovery costs are available for the Rocky Mountain region, but secondary recovery costs are only available

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\(^{18}\) This is $289.43 on a per foot basis.

\(^{19}\) Brokmeyer et al. (1996) reference a well numbered “206”, so this well count seems plausible.
for the West Texas region. Table 3-4 illustrates the costs relevant to the Lost Soldier Tensleep reservoir. Here tubing costs for primary recovery have been excluded from the EIA figures, as they are assumed to be included in the drilling and completion costs. The EIA also includes drilling and completion costs for secondary recovery, which have been removed here to avoid double counting.

Table 3-4: Non-drilling capital costs, excluding CO₂ plant (from 2010 EIA study)

<table>
<thead>
<tr>
<th>Producing Depth (ft.)</th>
<th>4000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs per 10 Wells (2009 Dollars)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Primary Recovery</strong></td>
<td>$1,507,000</td>
<td>$2,250,400</td>
</tr>
<tr>
<td>(Rocky Mountain Region figures)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Secondary Recovery</strong></td>
<td>$1,931,800</td>
<td>$3,462,100</td>
</tr>
<tr>
<td>(West Texas figures)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Recovery</strong></td>
<td></td>
</tr>
<tr>
<td>(Rocky Mountain Region figures)</td>
<td></td>
</tr>
<tr>
<td>$1,690,780</td>
<td>$189,930</td>
</tr>
<tr>
<td><strong>Secondary Recovery</strong></td>
<td></td>
</tr>
<tr>
<td>(West Texas figures)</td>
<td></td>
</tr>
<tr>
<td>$2,167,385</td>
<td>$259,662</td>
</tr>
</tbody>
</table>

Costs have been converted to current dollars in the lower half of Table 3-4 using the UCCI. Assuming a linear relationship between cost and producing depth from 4000 ft. to 8000 ft., these cost figures have been converted to a cost per well for the depth interval relevant to the Lost Soldier Tensleep reservoir.
To estimate non-drilling capital costs for each phase of recovery, a few assumptions have been made regarding the number of wells in operation, which are also outlined in Table 3-5. The number of wells in operation increases as primary recovery progresses to each of the two stages of secondary recovery. Brokmeyer et al. (1996) indicate the CO\textsubscript{2} flood uses about 90 wells, so this is assumed to be the number of wells in operation for tertiary recovery. The per-well non-drilling capital cost for primary recovery is incurred for every well drilled in a particular phase, as basic equipment included in the primary recovery figures such as flowlines must be purchased regardless of the phase of recovery for which a well comes online.\textsuperscript{20} Secondary recovery non-drilling capital costs apply to existing wells plus those drilled for the switch from primary to peripheral waterflooding, and for the additional wells drilled for pattern waterflooding. Finally, the secondary recovery per well non-drilling capital costs are assumed to be representative of the costs to upgrade the 90 wells and their associated facilities used for tertiary recovery. These cost assumptions exclude the cost of the CO\textsubscript{2} recycling plant, which is discussed in the next paragraph.

\textsuperscript{20}\textsuperscript{20} For example, the 20 wells drilled for tertiary recovery will still incur the primary recovery non-drilling capital costs.
Table 3-5: Capital cost assumptions, excluding CO$_2$ plant

<table>
<thead>
<tr>
<th>Recovery Phase (Subscript)</th>
<th>Wells Drilled</th>
<th>Cost of Drilling (2013 dollars)</th>
<th>Wells Used for Phase of Production</th>
<th>Non-Drilling Capital Costs (Excluding CO$_2$ Plant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Recovery (1)</td>
<td>100</td>
<td>$144,713,979</td>
<td>100</td>
<td>$18,992,951</td>
</tr>
<tr>
<td>Peripheral Waterflooding (2per)</td>
<td>65</td>
<td>$94,064,086</td>
<td>165</td>
<td>$55,189,580</td>
</tr>
<tr>
<td>Pattern Waterflooding (2pat)</td>
<td>65</td>
<td>$94,064,086</td>
<td>230</td>
<td>$29,223,421</td>
</tr>
<tr>
<td>Tertiary Recovery (3)</td>
<td>20</td>
<td>$28,942,796</td>
<td>90</td>
<td>$27,168,133</td>
</tr>
<tr>
<td><strong>Total Wells Drilled</strong></td>
<td><strong>250</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The capital cost of a plant to separate CO$_2$ from the produced fluids for reinjection is substantial. Advanced Resources International conducted a study for the Department of Energy in 2006 for which they assessed CO$_2$ EOR in the Rocky Mountain region. They assume that a CO$_2$ recycling plant costs about $700,000 for each MMscf$^{21}$ per day of capacity in February 2006. Brokmeyer et al. (1996) indicate CO$_2$ plant capacity for the Lost Soldier Tensleep reservoir is 110 MMscf/day, but that this is shared with another adjacent reservoir, so the capacity is multiplied by 0.5 here to reflect this distribution of the plant’s capacity between the two fields. The plant cost for the field is converted to current dollars using the UCCI and multiplied by this capacity estimate in Table 3-6 to yield an estimate for the CO$_2$ plant cost.

$^{21}$ 1 MMscf = $10^6$ scf
Table 3-6: CO₂ plant cost estimates

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$700,000</td>
<td>55</td>
<td>1.55</td>
<td>$59,831,081</td>
</tr>
</tbody>
</table>

Finally, Table 3-7 summarizes the total capital costs incurred at the beginning of each recovery phase, expressed in current dollars.

Table 3-7: Summary of capital costs for each recovery phase (2013 dollars)

<table>
<thead>
<tr>
<th>Recovery Phase (Subscript)</th>
<th>Total CAPEX ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Recovery (1)</td>
<td>163,706,930</td>
</tr>
<tr>
<td>Peripheral Waterflooding (2per)</td>
<td>149,253,666</td>
</tr>
<tr>
<td>Pattern Waterflooding (2pat)</td>
<td>123,287,508</td>
</tr>
<tr>
<td>Tertiary Recovery (3)</td>
<td>115,942,010</td>
</tr>
</tbody>
</table>

3.4 Maximizing NPV

To gain insight on how best to develop a reservoir from an economic perspective, it is assumed that the operator’s goal is to maximize NPV as measured over the entire life of the reservoir, as Parra Sanchez does (2010). This objective function is subject to the constraints of the model as defined above, as well as the following constraints on the length of each recovery phase and overall life of the project:
\[ t_{project} = t_{Life_1} + t_{Life_2} + t_{Life_3} = 150 \text{ years} \]

\[ t_{Life_1} \geq 0 \]
\[ t_{Life_2} \geq 0 \]
\[ t_{Life_3} \geq 0 \]

Equation 3-13

The maximum life of the project is assumed to be 150 years as production is essentially nonexistent by this point and, because of the effects of discounting, is worth very little in present value terms. Here the decision variables are \( t_{Life_1}, t_{Life_2}, \) and \( t_{Life_3} \); \( t_{Life_3} \) is then fixed by the constraints above. The decision variables are restricted to integer values.

@Risk, an add-in software package for Excel, contains a full suite of tools for optimization under uncertainty and is used here both for Monte Carlo simulation of oil prices and maximization of NPV. This software package contains the OptQuest optimization tool, which combines Tabu Search, Neural Networks, Scatter Search, and Linear/Integer Programming to find the best solution to a problem.

As Parra Sanchez (2010) evaluates the exponential decline model assuming a constant oil price and non-fluctuating costs, a similar case is evaluated first here to have a basis for comparison for subsequent scenarios where mean-reverting price models are considered. Parra Sanchez assumes an oil price of $55 per barrel, whereas here it is
$64.61 per barrel. This price is the mean WTI oil price from the fitted mean-reverting parameters ($79.15), adjusted for the market’s discount of Wyoming crude.\textsuperscript{22}

When examining the results of the optimized model, it is worthwhile to consider a plot of the cumulative discounted cash flow (CDCF) and the recovery efficiency versus time. The former essentially measures the amount of NPV accrued by a particular point in time and is defined for any time $t^*$ as follows:

$$CDCF(t^*) = \sum_{t=0}^{t^*} \frac{CF(t)}{(1+R)^t}$$

Equation 3-14

Figure 3-3 illustrates the recovery efficiency and CDCF versus time when oil prices are assumed to be constant, taxes, royalties and other costs are modeled as described above, and NPV is maximized using @Risk. For comparison, the recovery efficiency resulting from operators’ actual field decisions is also included. The CDCF function exhibits a “sawtooth” shape where it steadily increases over time with drops when capital costs are incurred at the beginning of each recovery phase. The switch time for the transition from primary to peripheral waterflooding is important as most of the NPV of the field is realized in these phases. While this observation applies to this model applied to this particular reservoir, it is not necessarily generally applicable to other reservoirs, as discussed in chapter seven.

\textsuperscript{22} Optimizing NPV with the cost, tax, and royalty assumptions used here with a $55 per barrel oil price suggests that CO\textsubscript{2} flooding is uneconomic. On the other hand, this optimization does not account for the fact that the lower oil price would likely be associated with lower costs.
Given the costs, taxes and royalties an operator would face in the hypothetical scenario where the Lost Soldier Tensleep reservoir were recently discovered and facing imminent development, this analysis suggests that, to maximize NPV, it would be worthwhile to switch recovery methods earlier in the life of the field than the operators did. Of course, the field’s operators faced fluctuating oil prices and costs over the life of the field, so it is difficult to judge the quality of decisions based on a relatively simple deterministic optimization and constant parameters.

According to this model, the maximum NPV is not associated with the maximum recovery efficiency attainable for the field. This result seems a bit counterintuitive, but is easily explained by the time value of money. Sacrificing some ultimate recovery efficiency is justified because it allows the operator to produce oil quickly, realizing cash flows that can be reinvested. This strategy yields more value than does one where the operator produces slowly and achieves the largest possible recovery efficiency.
Table 3-8 compiles the optimal decision variables and maximized NPV when the model is optimized according to the assumptions described above and the results from the comparable deterministic optimization from Parra Sanchez’s thesis. It also lists the actual decisions made for the field for reference. Parra Sanchez’s analysis suggests that the operator should switch recovery methods relatively quickly; accounting for taxes, royalties, and capital costs as described above alters the results of the deterministic optimization to a significant degree. On the other hand, the results from optimizing the model here are still directionally consistent with Parra Sanchez’s, suggesting that earlier switching times maximize NPV. Indeed, the exponential decline model used for both analyses effectively imposes a penalty for switching early in the form of reduced ultimate recovery efficiencies for each phase of production, so the model likely underestimates the
oil recoverable by switching early. Finally, the divergence in the optimal NPVs in Table 3-8 is because of the cost, tax and royalty assumptions used for the model here.

Table 3-8: Comparison of optimal decision variables and NPV²³

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model Including Taxes, Royalties, Capital Costs and Current Oil Price</th>
<th>Parra Sanchez’s Deterministic Life cycle Optimization</th>
<th>Parra Sanchez’s “Myopic” Optimization²⁴</th>
<th>Actual Decisions for the Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{Life1}$</td>
<td>14</td>
<td>9</td>
<td>34</td>
<td>24</td>
</tr>
<tr>
<td>$t_{Life2per}$</td>
<td>13</td>
<td>7</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>$t_{Life2pat}$</td>
<td>14</td>
<td>7</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>Optimal NPV</td>
<td>$500,898,601</td>
<td>$2,057,034,993</td>
<td>$1,550,243,893</td>
<td></td>
</tr>
</tbody>
</table>

³⁵ MEAN-REVERTING OIL PRICE – CONSTANT MEAN

³⁵.¹ Uncorrelated Costs, Imperfect Information

The crude oil price is guaranteed to fluctuate over the life of a project, so the implications of assuming a mean-reverting oil price are considered here. For now, costs are assumed to remain constant, which is consistent the notion that costs could be linked to the mean, or long-term oil price rather than the more transient oscillations of the spot price. The simple mean-reverting model used here assumes that $\bar{x}$, the log of the mean price, is constant. The process parameters for the mean-reverting model are also assumed

²³ $t_{Life3}$ is not listed here as it is assumed to be the remaining economic life of the field in each case.

²⁴ This optimization assumes that the operator maximizes the NPV of each recovery phase individually and produces for each until the economic limit is reached.
to remain constant over the life of the field. In reality, these process parameters will likely change over multi-decade time scales like those considered here. Other assumptions regarding the exponential decline model, costs, taxes, royalties, and constraints as discussed above remain the same as they were for the scenario above where the adjusted Wyoming oil price was constant at $64.61 per barrel.

The operator’s optimization problem now has an uncertain element from the oil price, which is now a stochastic variable. For the constant oil price case, he maximizes a single value of NPV for the field. With a stochastic oil price, a single set of chosen decision variables defining the length of each recovery phase will now yield an infinite number of potential NPV outcomes for the field; these outcomes can be characterized by a statistical distribution. Now changing the decision variables alters the character of this distribution of NPV outcomes rather than a single NPV of the field. If the operator is to maximize the value of the field by selecting decision variables at the outset, he chooses ones that maximize the mean of the distribution of NPV outcomes.

@Risk is used to generate a Monte Carlo simulation with 500 iterations of the mean-reverting price process. Each realization of this process yields a time series of possible future oil prices, for example those illustrated in Figure 2-4. For each time series, the NPV of the reservoir for its entire life cycle is computed according to the methods discussed earlier in this chapter, except now with a time-varying oil price rather than a constant one.²⁵ This simulation therefore results in a distribution of NPV outcomes.

---
²⁵ A “NPV outcome” here is taken to be the sum of the discounted cash flows from the project over 150 years. That is, for any price realization, the operator produces in each recovery phase for a predetermined
outcomes for the reservoir; each is associated with a particular price realization of the mean-reverting process. The optimization engine is therefore set to maximize the mean of this distribution of NPV outcomes by adjusting the $t_{life}$ decision variables, as the operator would like to maximize the expected NPV of the field.

Figure 3-4 plots the distribution of NPV outcomes that result from a Monte Carlo simulation using 10,000 iterations when the optimal value of each $t_{life}$ variable is chosen, along with relevant summary statistics for the NPV outcome distribution. This histogram measures the relative frequencies on the vertical axis of each range of NPV outcome resulting from the Monte Carlo simulation. Each bar on the graph represents a “bucket” containing a particular number of NPV outcomes from the simulation.

amount of time governed by his choice of the $t_{life}$ variables at the outset of production. Tertiary recovery is long enough for the project to last 150 years in total. Since OPEX is calculated on a per-barrel basis and is a fraction of the mean oil price, each barrel of oil produced yields additional revenue unless the oil price is very low. Once production has essentially ceased, the project generates neither revenues nor costs. Indeed, the operator has no way of knowing exactly when the maximum NPV is reached because of the uncertain nature of the oil price, so even if the maximum NPV has been passed, he still has an incentive to produce if doing so is profitable.

26 That is, the values that yield a maximum mean of the NPV distribution. Note the optimal values of the $t_{life}$ variables are found with a 500-iteration simulation; performing the same optimization with 10,000 iterations was found to yield the same values but required substantially more computation time.
The distribution of NPV outcomes appears lognormal, which is consistent with the fact that the simple mean-reverting process models a logarithm of the oil price. This behavior, which is entirely due to uncertainty in the oil price, leads to the model predicting larger upside than downside potential for the oil price, even though the mean is assumed to be constant. As a result, both the mean and median NPVs are larger when the oil price is mean-reverting than the NPV when it is assumed constant at the mean price. The realization with the largest NPV, $1.9 billion, is almost four times the maximized NPV from the constant oil price case discussed above, $501 million.

None of the price realizations results in a NPV less than zero. This is not to say such an outcome is impossible; rather, it appears to be very unlikely despite a relatively large volatility for the oil price. Moreover, downward swings in the oil price can destroy...
the economics of switching to a higher-cost recovery phase after the capital costs of doing so have been invested. A case where this occurs is illustrated in Figure 3-5, which shows a price realization along with the CDCF of the project, assuming the operator uses the $t_{life}$ values obtained from the optimization above. In this case, the oil price falls substantially below the long-term mean around year 20 of the project, and, because of large CAPEX payments for late phases of recovery, the CDCF of the reservoir never fully recovers to its highest value of $578$ million in year 25 of production, just when the operator switches to pattern waterflooding. If the operator wished to obtain the maximum possible NPV from the reservoir for this price realization, he probably would have continued with peripheral waterflooding and ceased production afterwards. On the other hand, by continuing production, the NPV outcome of the project is $563$ million, a 2.6 percent decrease from the maximum NPV. While the NPV outcome of the reservoir is a suboptimal result, it is not substantially different from the maximum possible NPV.
Table 3-9 lists the time spent on each recovery phase such that the mean of the NPV distribution is maximized. These times are not significantly different than those for the constant oil price case. This intuitively makes sense because the oil price is modeled as mean-reverting with a constant log mean. If this model matches reality, the effects of high prices from one realization will cancel with those of lower prices from another. Therefore assuming a constant log mean, that the process parameters for a mean-reverting model will hold for the actual oil price on the scale of decades, and imperfect information regarding the actual realization of the oil price, an operator who faces a decision from the outset of production when to switch recovery methods nearly
maximizes the mean of his NPV outcomes for a reservoir by assuming the oil price will be constant at its long-term mean value. This is not to say that he cannot improve the NPV outcome for a particular price realization by adjusting these $t_{Life}$ values, though, as shown in the next section for the case of perfect information. Moreover, in reality an operator faces a more dynamic problem where decisions regarding when to switch recovery phases are made in the presence of evolving information regarding the oil price realization. Such a problem suggests the use of dynamic programming or real options to model the operator’s decisions. Ideally, such an approach should be applied to a more complete model of a reservoir that incorporates price, geologic, and engineering uncertainty, such as the one discussed in subsequent chapters, so it is not pursued here.

Table 3-9: Time spent on recovery phases for maximized mean NPV (years) – uncorrelated costs and imperfect information

<table>
<thead>
<tr>
<th>$t_{Life1}$</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{Life2per}$</td>
<td>12</td>
</tr>
<tr>
<td>$t_{Life2pat}$</td>
<td>12</td>
</tr>
<tr>
<td>$t_{Life3}$</td>
<td>113</td>
</tr>
<tr>
<td><strong>Mean NPV</strong></td>
<td>$641,152,369</td>
</tr>
</tbody>
</table>

3.5.2 Uncorrelated Costs, Perfect Information

The expected value of perfect information (EVPI) is defined as the maximum amount a decision maker is willing to pay for complete and accurate knowledge of how events relevant to an economic decision will evolve in the future (Birge and Louveaux...
This metric is useful here for measuring the expected additional value an operator could derive from the reservoir by knowing the oil price realization *a priori*.

In the simplified problem described here where the physical parameters of the reservoir, costs, and mean-reverting process parameters are known but the exact realization of the oil price is uncertain, the EVPI at the outset of reservoir development is approximated as the difference between the expected NPV outcome when the operator of the reservoir has perfect information on the oil price realization *a priori* and optimizes NPV outcomes accordingly minus the expected (mean) NPV found in the optimization above. In the latter case, only the parameters regarding the mean-reverting process are known for the future oil price. For simplicity, it is assumed that these parameters are valid indefinitely for both the perfect and imperfect information cases.

\[
EVPI_o = E\left[Opt(NPV_{\text{perfect info}})\right] - E\left[Opt(NPV_{\text{imperfect info}})\right]
\]

Equation 3-15

where \(Opt(NPV_{\text{perfect info}})\) and \(Opt(NPV_{\text{imperfect info}})\) signify that the operator uses the information he has to choose optimal \(t_{Life}\) values. For the case of perfect information, the operator chooses optimal \(t_{Life}\) values idiosyncratic to each price realization, while for imperfect information he chooses one set of values that apply to all realizations.

\[E\left[Opt(NPV_{\text{perfect info}})\right]\] can be determined using another Monte Carlo simulation. First, a set of 100 realizations of the mean-reverting price path is generated. For each realization, @Risk’s optimization tools are used to deterministically optimize
the NPV by choosing appropriate $t_{life}$ values, assuming that the operator knows the particular price realization. Each realization then has a maximized NPV value, yielding a distribution of NPV values for the set of price realizations. The mean of this distribution of NPV values is $E[Opt(NPV_{PerfectInfo})]$. This calculation process is automated using VBA macros in Excel; for each realization, @Risk was instructed to use 1000 iterations to solve for the optimal $t_{life}$ values.

Figure 3-6 illustrates the distribution of NPV outcomes for the 100-realization Monte Carlo simulation assuming the operator has perfect information as described above. This distribution also appears to be roughly lognormal. For comparison, the NPV that would have been realized if the operator had imperfect information and instead chosen the $t_{life}$ values given in Table 3-9 for each of the 100 realizations is also calculated. The results of these calculations are summarized in Figure 3-7, which essentially shows 100 realizations from the scenario depicted by Figure 3-4. Having perfect information clearly shifts the distribution of NPV outcomes to the right, as expected. The estimated EVPI is $88,499,709$, the improvement in the mean NPV by having perfect information. This expected value derived from having perfect information of the oil price represents roughly a 13 percent increase in expected NPV over the imperfect information case. While this increase is not an insubstantial amount of value, it is surprisingly small given the large volatility assumed for the price.
Figure 3-6: Distribution of maximized NPVs with perfect information (uncorrelated costs)

Figure 3-7: Distribution of NPVs with imperfect information and optimal $t_{life}$ values (uncorrelated costs)
Since each of the 100 price realizations in the Monte Carlo simulation used to calculate $E[Opt(\text{NPV}_{\text{perfectInfo}})]$ has different optimal $t_{life}$ values, it is worthwhile to examine the distribution of this parameter. Figure 3-8 illustrates this distribution; this decision variable is examined here because of its relative importance to NPV. The average $t_{life1}$ is 13.8 years, which is relatively close to the optimal $t_{life1}$ of 13 years in the case of imperfect information on price. On the other hand, there is substantial variation in the optimal $t_{life1}$ with perfect information, owing to the large variations in the mean-reverting oil price. The price parameters correspond to those for the simulated realizations in Figure 2-4, except here with annual time steps.

![Figure 3-8: Distribution of optimal $t_{life1}$ values for Monte Carlo simulation with perfect information and uncorrelated costs](image)

Figure 3-8: Distribution of optimal $t_{life1}$ values for Monte Carlo simulation with perfect information and uncorrelated costs
The calculations described in this section are repeated and summarized in analogous tables and figures in Appendix B for the case of half the fitted volatility.\textsuperscript{27} The distribution of optimal $t_{lifef1}$ values with perfect information is unsurprisingly narrower when there is less price volatility. Moreover, the EVPI in this case is $32,732,406$, which represents only a 6 percent increase from $E[Opt(NPV_{imperfectInfo})]$. With less volatility in the oil price, there is less NPV to be gained by adjusting each $t_{lifef}$ variable from its optimal value as determined for the imperfect information case. If one sees this volatility as a more realistic estimate for characterizing how the oil price will behave in the future, these results suggest the gain in value that could be achieved by knowing the oil price realization in advance is relatively small.

Figure 3-9 illustrates how the optimal $t_{lifef}$ values with perfect information are usually associated with subsequent time periods where the oil price is well above average, which is no surprise. Intuitively, an operator with perfect information should schedule the boost in production obtained from waterflooding so that the majority of increased production occurs when oil prices are higher. For each of the 100 realizations of the Monte Carlo simulation with perfect information, the average oil price for the 10 years from the optimal value of $t_{lifef1}$ onward is calculated. The average oil price for the 10 years from year 13 onward\textsuperscript{28} is also calculated. The difference between these two average oil prices is taken and the distribution is tabulated in Figure 3-9. The average

\textsuperscript{27} For time steps of one year, this corresponds to $\frac{1}{2}\bar{\sigma} = 15.92\%$

\textsuperscript{28} This was chosen as a benchmark as year 13 is the optimal $t_{lifef1}$ for the imperfect information case, what the operator would have chosen by default if he did not know how prices would ultimately evolve.
difference is about $19, and the difference is greater than zero for 80 percent of the realizations from the Monte Carlo simulation. While this result displayed in this plot is intuitive, it unfortunately underscores the difficulty of optimizing the switching times for a reservoir even when the oil price is assumed to be mean-reverting and one can perfectly characterize the future production of the reservoir for multiple recovery methods. Choosing the optimal switching times appears to require predicting future oil prices. On the other hand, an operator could possibly improve upon the NPV obtained simply from choosing the optimal $t_{Life}$ values from the imperfect information case. A more dynamic strategy that responds to evolving information on the oil price could capture some of the EVPI as approximated by the simplified definition in this section.

![Bar chart showing frequency of differences in average oil prices](image)

**Figure 3-9:** Average oil prices for 10 years after optimal of $t_{Life1}$ with perfect information minus average oil prices for 10 years after optimal of $t_{Life1}$ for imperfect information (uncorrelated costs)
3.5.3 Correlated Costs

The analysis of the previous section is repeated here, except now assuming that costs are correlated with the oil price according to the following regressions for capital and operating costs, as discussed in the previous chapter. These regressions correspond to all CERA index data from 2000 to present:

\[
\begin{align*}
CAPEX_{index, real} &= 1.32(p_{real,WTI}) + 98.759 \\
OPEX_{index, real} &= 0.7448(p_{real,WTI}) + 117.02
\end{align*}
\]

Equation 3-16

The operating and capital cost indices are first calculated at the initial time using the current WTI price\(^{29}\). For any time \( t \) in a particular realization, the operating and capital cost indices are calculated using the realized WTI price from the Monte Carlo simulation. The capital and operating costs for time \( t \) are then scaled by dividing the calculated operating and capital cost indices at \( t \) by those for the initial time. The resulting quotients are multiplied by their respective costs determined in current dollars earlier in this chapter to yield scaled costs that are correlated to the oil price\(^{30}\).

\(@Risk\) is again used to generate a Monte Carlo simulation with 500 iterations of the mean-reverting price process. The software is used to maximize the mean of NPV outcomes associated with these realizations by adjusting the \( t_{life} \) decision variables. Figure 3-10 plots the distribution of NPV outcomes that result from a 10,000-iteration

\(^{29}\) Here, the current price is for May 10, 2013.

\(^{30}\) The scaling used here would reflect real shifts in costs as the regressions from the previous chapter were performed on real values of the WTI and cost indices.
Monte Carlo simulation when the optimal $t_{life}$ values are chosen, along with relevant summary statistics for the NPV distribution.

With correlated costs, the distribution of NPV outcomes exhibits roughly a lognormal shape, as was also seen in Figure 3-4 for the uncorrelated cost case. The standard deviation of the distribution is smaller here, which is to be expected as a positive correlation of costs with the oil price narrows the range of outcomes of the project’s profitability. Any increase in the oil price now increases both the revenues and costs associated with the project, and conversely with a decrease in the oil price.

![Figure 3-10: Distribution of NPV outcomes with mean-reverting oil price and correlated costs](image)

Table 3-10 outlines the operator’s optimal choices for the length of each recovery phase at the beginning of the project, assuming correlated costs, imperfect information.

---

31 That is, the values that yield a maximum mean of the NPV distribution.
regarding the ultimate price realization, and certainty regarding the production characteristics of the reservoir. The $t_{life}$ variables for the first three phases here are each a year shorter than they were in the case of uncorrelated costs, which are not substantial differences.

Table 3-10: Time spent on recovery phases for maximized mean NPV (years) – correlated costs and imperfect information

<table>
<thead>
<tr>
<th>$t_{Life1}$</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{Life2per}$</td>
<td>11</td>
</tr>
<tr>
<td>$t_{Life2pat}$</td>
<td>11</td>
</tr>
<tr>
<td>$t_{Life3}$</td>
<td>116</td>
</tr>
<tr>
<td>Mean NPV</td>
<td>$653,095,773$</td>
</tr>
</tbody>
</table>

The operator’s optimal decisions with perfect information regarding prices are also examined here using a 100-realization Monte Carlo simulation. Figure 3-11 illustrates the distribution of NPV outcomes for this simulation with correlated costs. This distribution also appears to be roughly lognormal. The NPV that would have been realized if the operator had imperfect information and instead chosen the $t_{life}$ values given in Table 3-10 for each of the 100 realizations is also calculated, and the results of these calculations appear in Figure 3-12. As is also the case with uncorrelated costs, perfect information shifts the NPV distribution to the right and increases the expected NPV. The EVPI is now estimated as $61,181,870$, which means that having perfect information yields about a $9$ percent increase in expected NPV over $E[Opt(NPV_{imperfectinfo})]$. This increase is substantially lower than it was for the case
of uncorrelated costs. These trends suggest that if one were to assume correlated costs and half the fitted volatility for the mean-reverting price model, the EVPI would be quite small. Both reducing the volatility of the oil price and increasing the degree to which costs are positively correlated with the price constrain the degree to which the project’s profitability can vary in the Monte Carlo simulation.

Figure 3-11: Distribution of maximized NPVs with perfect information (correlated costs)
Figure 3-12: Distribution of NPVs with imperfect information and optimal $t_{life}$ values (correlated costs)

Figure 3-13 outlines the distribution of optimal $t_{life}$ values from the Monte Carlo simulation with perfect information; the average is 13.2 years with a standard deviation of 5.46 years. This distribution is narrower than the one depicted for the uncorrelated costs case in Figure 3-8, which has a standard deviation of 6.12 years for $t_{life}$. On the other hand, there is still substantial variation in the optimal $t_{life}$ with perfect information with correlated costs.
The computation summarized in Figure 3-9 is also performed here for the correlated costs case; the results appear in Figure 3-14. The average difference is about $15, and the difference is greater than zero for 82 percent of the realizations from the Monte Carlo simulation. These results corroborate the notion that achieving the optimal NPV of a reservoir with a mean-reverting oil price is an exercise in knowing the future before it occurs. This is not to say that one could not improve the NPV realized by simply choosing the optimal $t_{life}$ values from the imperfect information case, but merely that the true optimum is clearly unobtainable.

Figure 3-13: Distribution of $t_{life}$ values for Monte Carlo simulation with perfect information (correlated costs)
Finally, it is worthwhile to compare how an operator’s decision with perfect information might differ for the correlated costs case versus the uncorrelated costs case, as this provides some insight regarding whether the cost correlation might have a substantial impact on the decisions of when to switch recovery methods. To do this comparison, the same 100 realizations of the evolution of the oil price for the Monte Carlo simulation with perfect information in the previous section are used for the case of correlated costs. The NPV for each price realization, now assuming correlated costs, is maximized by adjusting the $t_{life}$ values. For each realization, the optimal $t_{life}$ values from the uncorrelated costs case are compared to those for the correlated costs case; namely, each $t_{life}$ for correlated costs is subtracted from the corresponding $t_{life}$ for
uncorrelated costs. This computation yields a distribution of changes in $t_{life}$, which is plotted in Figure 3-15.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure315.png}
\caption{Distribution of changes in $t_{life1}$ ($t_{life1}$ for correlated costs minus $t_{life1}$ for uncorrelated costs)}
\end{figure}

In 86 percent of the realizations, the optimal value of $t_{life1}$ changed by less than 2 years by assuming costs positively correlated with the oil price instead of costs uncorrelated with the oil price. This result suggests that the assumption of positive correlation of costs usually does not have a substantial impact on the optimal value of $t_{life1}$. The assumption does impact NPV, though.
3.6 MEAN-REVERTING OIL PRICE – VARIABLE MEAN

The possibility that the mean oil price increases through the life of the reservoir, with the mean price’s trajectory approximated by a logistic function, is considered here. The formulation discussed in section 2.7 of the previous chapter is used to evaluate the implications of this price trajectory for maximizing NPV.

Here costs are correlated with the escalating long-term mean price. The calculation procedure to scale costs is the same as it was in the previous section except now the regressions in Equation 3-16 are functions of $p_{mean}$:

$$CAPEX_{index\_real} = 1.32(p_{mean}) + 98.759$$

$$OPEX_{index\_real} = 0.7448(p_{mean}) + 117.02$$

Equation 3-17

This formulation is more consistent with evidence discussed in the previous chapter that costs seem to be correlated with the long-term oil price rather than the spot price.

@Risk is used to generate a Monte Carlo simulation with 500 iterations of the mean-reverting price process assuming a mean price modeled by Equation 2-12. The optimization engine is again set to maximize the mean NPV of this distribution by adjusting the $t_{life}$ decision variables. Figure 3-16 plots the distribution of NPV outcomes that result from a 10,000-iteration Monte Carlo simulation when the optimal $t_{life}$ values are chosen, along with relevant summary statistics for the NPV distribution. Table 3-11 lists the optimal $t_{life}$ variables with imperfect information, which are still quite similar to those from the case where the mean oil price is assumed to be constant.
and costs are uncorrelated with the oil price.\textsuperscript{32} Primary recovery is longer while the other phases are shorter.

![Figure 3-16: Distribution of NPV outcomes with mean-reverting oil price and correlated costs – logistic model for mean price](image)

Table 3-11: Time spent on recovery phases for maximized mean NPV (years) – logistic model for mean price

<table>
<thead>
<tr>
<th>$t_{\text{Life1}}$</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{Life2per}}$</td>
<td>10</td>
</tr>
<tr>
<td>$t_{\text{Life2pat}}$</td>
<td>9</td>
</tr>
<tr>
<td>$t_{\text{Life3}}$</td>
<td>116</td>
</tr>
<tr>
<td>Mean NPV</td>
<td>$853,453,978$</td>
</tr>
</tbody>
</table>

\textsuperscript{32} Although the operator does not have information on the exact realization of the oil price here, it is assumed that he knows the logistic long-term trend of the mean price.
To examine the logic behind the optimal $t_{lfe}$ variables, consider Figure 3-17, which plots the mean oil price, modeled logistically, as well as the recovery efficiency schedules assuming optimal $t_{lfe}$ values and imperfect information for the “base case” where the mean oil price is assumed to be constant and the case where it is assumed to follow the logistic model. Versus the base case, the optimizing operator facing an escalating mean oil price delays switching to peripheral waterflooding by a few years to schedule this boost in production for years in which the oil price is expected to be higher.\textsuperscript{33} Once the higher prices are expected to steady, the operator accelerates production by switching to tertiary recovery earlier.

\textsuperscript{33} All of these models implicitly assume that the operator has to begin production at year 0. Indeed, given the option to wait to develop a lease and facing an escalating oil price according to Equation 3-18, the operator might wait for the higher price regime.
For comparison, now consider a more dramatic escalation of the mean price:

\[ p_{mean}(t) = \frac{125}{1 + e^{-0.6(t-5)}} + 75 \]

Equation 3-18

This formulation models the mean price increasing to a maximum of $200 per barrel in real terms within about a decade. Table 3-12 lists the optimal \( t_{life} \) values from a Monte Carlo simulation that uses Equation 3-18 to model the mean price. Versus the case where the mean oil price is constant, the optimal choice for the operator with imperfect information appears to entail somewhat accelerated production for all recovery phases. This can be seen in Figure 3-18, which replicates Figure 3-17 for this case of more dramatic price escalation.

Table 3-12: Time spent on recovery phases for maximized mean NPV (years) – logistic mean, dramatic price increase

| \( t_{Life1} \) | 12 |
| \( t_{Life2per} \) | 10 |
| \( t_{Life2pat} \) | 9 |
| \( t_{Life3} \) | 119 |
| Mean NPV | $1,318,000 |
3.7 CONCLUSION

Table 3-13 summarizes some of the findings from previous sections. All of these cases include capital costs, taxes, and royalties. The expected NPV varies significantly depending on the economic assumptions used. Differences in the $t_{Life}$ values are less pronounced, but are more substantial for the cases where the mean oil price is modeled using a logistic equation.
Table 3-13: Summary of optimal $t_{Life}$ values and expected NPVs for imperfect information cases

<table>
<thead>
<tr>
<th></th>
<th>Constant Oil Price</th>
<th>Constant Mean Oil Price, Mean-Reverting Model</th>
<th>Variable Mean Oil Price, Mean-Reverting Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Uncorrelated costs</td>
<td>Uncorrelated costs, half fitted volatility</td>
</tr>
<tr>
<td>$t_{Life1}$</td>
<td>14</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>$t_{Life2per}$</td>
<td>13</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>$t_{Life2pat}$</td>
<td>14</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>$t_{Life3}$</td>
<td>109</td>
<td>113</td>
<td>110</td>
</tr>
<tr>
<td>Mean NPV</td>
<td>$500,898,601$</td>
<td>$641,152,369$</td>
<td>$572,575,163$</td>
</tr>
</tbody>
</table>

The cases discussed in this chapter suggest the following:

- Accounting for taxes, royalties, and capital costs somewhat alters the conclusions of Parra Sanchez’s life cycle optimization for the Lost Soldier Tensleep reservoir. Including these factors mean that an operator who faces a constant oil price and deterministically optimizes the reservoir’s NPV would still accelerate production versus operator’s actual choices, but not quite as aggressively.

- When the oil price is assumed to be mean-reverting, the expected value of perfect information regarding the oil price appears to be relatively small compared to the expected NPV of the field when the operator simply chooses recovery phase lengths that are essentially very close to what is optimum for when the oil price is assumed constant. This EVPI is estimated to be at most 6 to 13 percent of the optimum expected NPV obtained without knowledge of the price realizations, depending on assumptions regarding the oil price model. The lower end of this

$^{34}$ Costs are correlated to the long-term mean and half the fitted volatility is used for the logistic model cases.
range corresponds to a more realistic, but still relatively large, estimate of price volatility.

- The EVPI is smaller when the price is assumed to be less volatile and when costs are assumed to be correlated with the price. These results suggest that the amount of value to be gained by managing the length of a reservoir’s recovery phases around the fluctuations of the oil price is relatively small. The greater the volatility of the oil price, the more value that could be gained. The operator could potentially improve his decisions and realize more NPV by using a more sophisticated approach, such as dynamic programming, to manage price uncertainty. Another alternative would be to purchase futures to hedge the oil price risk. These results assume that a suitable oil price model can be chosen to characterize the future price for multiple decades; actual price data from the past have proven to fit to multiple model formulations depending on the time period considered.

- When costs are assumed to be correlated to the mean-reverting oil price with a constant mean as formulated here, the optimizing operator with imperfect information on the price realization switches recovery phases slightly earlier versus the similar case where costs are uncorrelated with the oil price and the oil price is mean-reverting. The assumption of cost correlation does not appear to have much impact on the decision at outset of when to switch recovery methods, assuming imperfect information on the oil price, but it does impact the NPV the operator realizes.

- Maximizing the NPV of a reservoir is a balance of producing close to the present to account for time value of money and producing while the price is high. Choosing the recovery phase lengths that maximize NPV under the assumption of
a mean-reverting oil price appears to require knowledge of when oil prices will remain elevated for an extended period in the future. If the long-term mean oil price is expected to increase, the operator might want to delay switching to secondary and/or tertiary recovery by a few years\textsuperscript{35} versus what would be optimal assuming a constant oil price.

The observations discussed in this chapter could provide some directional guidance regarding maximizing the value of a reservoir over its life cycle, but absolute numbers or the magnitudes of changes associated with recovery phase lengths are certainly not generalizable.

\textsuperscript{35} The sizes of these delays depend on the expected rate of price appreciation, the assumption that it is bounded and the expected upper limit, and the operator’s discount rate.
Chapter 4: Primary Recovery

4.1 INTRODUCTION

Primary recovery, generally the beginning of multiple phases of production of oil and gas from a conventional reservoir, is the use of a reservoir’s natural energy to bring hydrocarbons to the surface. This stage of production often contributes a large proportion of the overall value of the reservoir’s production. Owing to the time value of money, which holds that a dollar today is worth more than a dollar in the future (ignoring the effects of inflation), one tends to value the cash flows from earlier stages of production more highly than those from later ones.\(^{36}\)

For life cycle optimization, it is imperative to use models to forecast reservoir performance in primary recovery \(a\ priori\) that are grounded in physical characteristics of reservoirs and simple enough to be computationally practical for methods such as genetic algorithms and Monte Carlo simulation. To this end, a tank model is used here to estimate oil production through primary recovery. The approach discussed here and in subsequent chapters differs from that of the previous chapter, as it involves developing a more physically realistic method of predicting flow performance.

There are many subsurface mechanisms that can provide energy for primary production, such as solution gas drive, compaction drive, water drive, and gas cap drive. See Walsh and Lake (2003) for a discussion of the various reservoir drive mechanisms and methods for diagnosing them. It is important to diagnose the reservoir drive mechanism, as for a reservoir with water drive, the natural energy present could justify producing using primary recovery for a longer period of time than what is economically optimal for one exhibiting solution gas drive.

\(^{36}\) See the discussion of discounting and net present value in the previous chapter
The model discussed here assumes an undersaturated solution gas drive reservoir, which means the initial reservoir pressure is always above the bubble point. This is a type of black oil reservoir, which Walsh and Lake (2003) define as forming a maximum of two hydrocarbon phases at reservoir conditions and not producing significant quantities of condensate.

As an undersaturated solution gas drive reservoir undergoes primary production and its pressure depletes, the fluid in the reservoir approaches the bubble point, whereupon solution gas begins to form (Walsh and Lake 2003). At first, this gas appears as a small saturation that is an immobile phase. As the reservoir pressure declines further, the gas phase becomes mobile, which reduces the relative permeability to oil, leads to a higher amount of gas production and results in a decline in oil production.

Given this quick decline in production, it is worthwhile to consider switching from primary recovery to waterflooding relatively soon after commencing primary production. Craig (1971) discusses the advantages of waterflooding a reservoir at the bubble point pressure, noting that reservoir oil viscosity is at a minimum at this pressure (p. 94). This characteristic can yield a favorable mobility ratio for the displacement, improving recovery. Craig also notes that starting at waterflood at the bubble point means that there is no delay in increased production due to a “fill up” period where injected water dissolves trapped gas by raising the reservoir pressure.

Therefore to mitigate the adverse relative permeability effects and related decline in oil production that occur below the bubble point in solution gas drive reservoirs, it is advisable to maintain the reservoir pressure above the bubble point. Consequently, here it is assumed that average reservoir pressure is maintained above the bubble point by switching from primary recovery to waterflooding, or secondary recovery. This assumption also simplifies the prediction of oil recovery, as it avoids the issues of
accounting for complex phenomena such as the evolution of gas saturation in the reservoir and using three-phase relative permeability models. These effects are better modeled through numerical simulation than a tank model.

In assuming the switch to waterflooding should occur at the bubble point, the switching decision is based on a technical factor rather than an economic one. On the other hand, given that producing from a solution gas drive reservoir below the bubble point in primary recovery is associated with steep declines in oil production, this technical factor implicitly accounts for the economics of production. Finally, since the decision of when to switch from primary to secondary recovery is linked to the bubble point, it is important to model the behavior of the reservoir fluids. While these calculations imply the reservoir fluid is a mixture of oil and gas, for the purposes of economic modeling in later chapters it is assumed that the value of the reservoir is derived from oil production. This simplification is justified by the relatively low current price of natural gas.

4.2 Reservoir Fluid Properties

Predicting the bubble point pressure and other relevant fluid properties like oil formation volume factor, oil compressibility and oil viscosity as a function of pressure can be accomplished through a laboratory PVT analysis of reservoir fluids. Such an analysis is appropriate when modeling an actual reservoir where these data are available. Another option is to use correlations from past studies of reservoir fluids, which, for simplicity, will be used in the model here to estimate the PVT properties of a hypothetical reservoir’s fluids.
The Vasquez-Beggs correlations constitute a commonly-used method to estimate the properties of reservoir fluids (McCain 1990). Vasquez and Beggs (1980) examined 600 laboratory samples of fluids from around the world and conducted regression analyses on thousands of measurements. They organized their set of correlations into two groups: one for oil with an API gravity greater than 30° and another for oil gravities less than 30°.

Vasquez and Beggs’ (1980) correlation to estimate the dissolved gas ratio in scf/STB as a function of pressure is as follows:

$$R_s = C_1 \gamma_{gs} p^{C_2} \exp \left[ C_3 \left( \frac{\gamma_o}{T+460} \right) \right]$$

Equation 4-1

where $\gamma_{gs}$ is the gas gravity (assuming air takes a value of one) that would result from separator conditions of 100 psig, $T$ is the reservoir temperature in °F, $\gamma_o$ is the oil API gravity, and $p$ is the fluid pressure in psi. The coefficients are provided in Table 4-1 and are defined such that $R_s$ is in scf/STB.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$\gamma_o \leq 30^\circ$</th>
<th>$\gamma_o &gt; 30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>0.0362</td>
<td>0.0178</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.0937</td>
<td>1.1870</td>
</tr>
<tr>
<td>$C_3$</td>
<td>25.7240</td>
<td>23.9310</td>
</tr>
</tbody>
</table>

Table 4-1: Vasquez-Beggs coefficients for dissolved gas ratio (from Vasquez and Beggs 1980)

In a solution gas drive reservoir, the dissolved gas ratio is constant above the bubble point (McCain 1990). Equation 4-1 applies for values at or below the bubble point.
point. Assuming the reservoir is initially undersaturated, the initial produced gas-oil ratio $R_p$ that is observed when production begins is equal to the dissolved gas-oil ratio $R_s$ at the bubble point. Solving Equation 4-1 for pressure and substituting $R_p$ for $R_s$ yields the following expression to predict the bubble point pressure:

$$p_b = \left[ \frac{R_p}{c_1 y_{gs} \exp\left[c_3 \left(\frac{y_o}{y_{gs}}\right)\right]} \right]^{\frac{1}{c_2}}$$

Equation 4-2

For reservoir fluid at or below the bubble point pressure, Vasquez and Beggs provide the following correlation to predict the oil formation volume factor:

$$B_o = 1 + C_1 R_s + C_2 (T - 60) \left(\frac{y_o}{y_{gs}}\right) + C_3 R_s (T - 60) \left(\frac{y_o}{y_{gs}}\right)$$

Equation 4-3

with variables defined as they are in Equation 4-1 with the exception of the coefficients, which take the following values and are defined such that $B_o$ is in RB/STB

Table 4-2: Vasquez-Beggs coefficients for oil formation volume factor (from Vasquez and Beggs 1980)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$\gamma_o \leq 30^\circ$</th>
<th>$\gamma_o &gt; 30^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$4.677 \times 10^{-4}$</td>
<td>$4.670 \times 10^{-4}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$1.751 \times 10^{-5}$</td>
<td>$1.100 \times 10^{-5}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$-1.811 \times 10^{-8}$</td>
<td>$1.337 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
Since the reservoir is assumed to be undersaturated, the initial produced gas-oil ratio $R_p$ can be substituted for the dissolved gas-oil ratio $R_s$ to find the oil formation volume factor at the bubble point, $B_{ob}$.

Vasquez and Beggs use the following relationship to estimate the oil formation volume factor for reservoir pressures $p$ above the bubble point pressure $p_b$:

$$ B_o = B_{ob} \exp \left| c_o (p - p_b) \right| $$

Equation 4-4

They estimate the oil compressibility $c_o$ in psi$^{-1}$ using a linear regression model as follows:

$$ c_o = \frac{a_1 + a_2 R_s + a_3 T + a_4 Y_o + a_5 Y_o}{a_6 p} $$

Equation 4-5

with the following coefficients, which apply for API gravities both above and below 30 degrees and are defined so that $c_o$ is in psi$^{-1}$:

Table 4-3: Vasquez-Beggs coefficients for oil compressibility (from Vasquez and Beggs 1980)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>$-1433.0$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>5.0 STB/scf</td>
</tr>
<tr>
<td>$a_3$</td>
<td>17.2 °F$^{-1}$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$-1180.0$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>12.61 °API$^{-1}$</td>
</tr>
<tr>
<td>$a_6$</td>
<td>$10^{-5}$</td>
</tr>
</tbody>
</table>
Beggs and Robinson (1975) also conducted an analysis of 2,073 live oil observations to develop correlations for predicting oil viscosity as a function of pressure. They estimate the dead oil viscosity in centipoise as:

\[ \mu_{oD} = 10^X - 1; \quad X = yT^{-1.163}; \quad y = 10^Z; \quad Z = 3.0324 - 0.02023\gamma_o \]

Equation 4-6

Where \( T \) is the fluid temperature in °F and \( \gamma_o \) is the oil API gravity. The live oil viscosity (oil containing dissolved gas) in centipoise for pressures below the bubble point is then:

\[ \mu_o = A\mu_{oD}^B; \quad A = 10.715(R_s + 100)^{-0.515}; \quad B = 5.44(R_s + 150)^{-0.338} \]

Equation 4-7

Where \( R_s \) is the dissolved gas-oil ratio in scf/STB. Using the bubble point value for \( R_s \) (equal to \( R_p \), the initial produced gas-oil ratio), it is possible to estimate the oil viscosity at the bubble point, \( \mu_{oB} \). Finally, Vasquez and Beggs (1980) provide yet another correlation for estimating the live oil viscosity in centipoise for pressures above the bubble point:

\[ \mu_o = \mu_{oB} \left(\frac{p}{p_b}\right)^m; \quad m = C_1p^c \exp(C_3 + C_4p) \]

Equation 4-8

with the coefficients as follows defined such that viscosity is in centipoise:
Table 4-4: Vasquez-Beggs coefficients for oil viscosity (from Vasquez and Beggs 1980)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>2.6</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.187</td>
</tr>
<tr>
<td>$C_3$</td>
<td>-11.513</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$-8.98 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Assuming a relatively limited set of properties for a hypothetical reservoir fluid, which are provided in the top half of Table 4-5, the correlations provided by Vasquez et al. are used to estimate the fluid properties given in the bottom half of the table. Finally, the correlations are also used to compute the reservoir fluid properties over a range of pressures; the results of these calculations appear in Figures 4-1 and 4-2.

Table 4-5: Using correlations to estimate reservoir fluid properties

<table>
<thead>
<tr>
<th>Known Fluid Properties</th>
<th>Calculated Fluid Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Reservoir Pressure</strong></td>
<td><strong>Predicted Bubble Point Pressure</strong></td>
</tr>
<tr>
<td>$p_i$ 4000 psi</td>
<td>$p_b$ 1060 psia</td>
</tr>
<tr>
<td><strong>Reservoir Temperature</strong></td>
<td><strong>Initial Oil Formation Volume Factor</strong></td>
</tr>
<tr>
<td>$T$ 150 ºF</td>
<td>$B_{oi}$ 1.24 rb/STB</td>
</tr>
<tr>
<td><strong>Oil Gravity</strong></td>
<td><strong>Predicted FVF at Bubble Point</strong></td>
</tr>
<tr>
<td>$\gamma_o$ 40 ºAPI</td>
<td>$B_{ob}$ 1.262 rb/STB</td>
</tr>
<tr>
<td><strong>Gas Gravity at Separator Conditions of 100 psig</strong></td>
<td><strong>Dead Oil Viscosity</strong></td>
</tr>
<tr>
<td>$\gamma_{gs}$ 1.50</td>
<td>$\mu_{oD}$ 2.108 cp</td>
</tr>
<tr>
<td><strong>Initial Solution Gas-Oil Ratio</strong></td>
<td><strong>Oil Viscosity at Bubble Point</strong></td>
</tr>
<tr>
<td>$R_p$ 500 scf/STB</td>
<td>$\mu_{oB}$ 0.626 cp</td>
</tr>
<tr>
<td></td>
<td><strong>Oil Compressibility at Bubble Point Pressure</strong></td>
</tr>
<tr>
<td></td>
<td>$c_o$ 2.25E-05 psi$^{-1}$</td>
</tr>
</tbody>
</table>
Figure 4-1: Oil formation volume factor and compressibility versus pressure

Figure 4-2: Dissolved gas-oil ratio and oil viscosity versus pressure
One can easily see that reservoir fluid properties relevant to calculating oil flow rates from a reservoir can vary substantially as the pressure declines. This result underscores the need to model these fluid property changes as primary recovery progresses. The correlations described above provide a relatively quick method to estimate reservoir fluid properties as a function of pressure in a spreadsheet-based tank model.

4.3 TANK MODELING

4.3.1 Introduction

Basic tank models assume that a reservoir is a homogeneous block; properties such as porosity, permeability, and fluid saturations are assumed to be constant at a given time (Walsh and Lake 2003). These models are in contrast to finite difference models, which subdivide the reservoir into many discrete blocks to model heterogeneity of rock properties and fluid saturations. Walsh and Lake (2003) note that such models can add unnecessary complexity in many instances (p. 291). Here, a multilayer tank model that assumes continuous parallel layers, each with a different permeability, is used to estimate oil production from primary recovery. Each layer is effectively a separate tank with homogeneous properties. Since crossflow between layers is assumed, they effectively act like a single layer. This model relies on material balance equations for oil production rate prediction; these equations are discussed below.
**4.3.2 Material Balance Theory**

Walsh and Lake (2003) use macroscopic mass balances for tank modeling because they are generally applicable for characterizing fluid flow into and out of the boundaries of a reservoir at any time during primary recovery (Walsh and Lake p. 141). They provide the following macroscopic rate equations to characterize the flow of stock-tank oil, surface gas, and stock-tank water:

\[
V_b \frac{d\left[\bar{\varphi}\left(\frac{\bar{S}_o}{B_o} + \frac{\bar{S}_g B_w}{B_g}\right)\right]}{dt} + \bar{q}_{osc} = 0 \quad \text{(stock-tank oil)}
\]

Equation 4-9

\[
V_b \frac{d\left[\bar{\varphi}\left(\frac{\bar{S}_g}{B_g} + \frac{\bar{S}_o B_w}{B_o}\right)\right]}{dt} + \bar{q}_{gsc} = 0 \quad \text{(surface gas)}
\]

Equation 4-10

\[
V_b \frac{d\left[\bar{\varphi}\left(\frac{\bar{S}_w}{B_w}\right)\right]}{dt} + \bar{q}_{wsc} = 0 \quad \text{(stock-tank water)}
\]

Equation 4-11

where \(V_b\) is the bulk volume of the reservoir, \(\bar{\varphi}\) is its average porosity, \(\bar{S}_o, \bar{S}_g,\) and \(\bar{S}_w\) are the average oil, gas and water saturations in the reservoir, \(B_o, B_g,\) and \(B_w\), are the average formation volume factors for oil, gas and water, \(\bar{R}_v\) is the average volatilized oil-gas ratio, \(\bar{R}_o\) is the average dissolved gas-oil ratio, and \(\bar{q}_{osc}\) is the net oil production rate from the reservoir, measured at surface conditions, with \(\bar{q}_{gsc}\) and \(\bar{q}_{wsc}\) are analogously defined for gas and water. According to Walsh and Lake, assuming undersaturated oil and a nonflowing water phase in the reservoir, these equations reduce to the following:
\[ V_b \frac{d\left[ \frac{S_o}{B_o} \right]}{dt} = -q_{osc} \]

Equation 4-12

\[ V_b \frac{d\left[ \frac{S_w}{B_w} \right]}{dt} = 0 \]

Equation 4-13

Walsh and Lake (2003) expand these equations and combine them to arrive at the following relation, which applies for all times of the reservoir’s primary production:

\[ V_p c_t \frac{d\bar{\rho}}{B_o \frac{dt}{dt}} = -q_{osc} \]

Equation 4-14

where \( V_p \) is the pore volume, which is the product of the bulk volume and the average reservoir porosity, and the total compressibility \( c_t \) is a saturation-weighted average of the formation, oil, and water compressibilities:

\[ c_t = c_f + \bar{S}_o c_o + \bar{S}_w c_w \]

Equation 4-15

Walsh and Lake also show that Equation 4-14 holds for heterogeneous multilayer systems with good communication between layers (pp. 318-320). This condition is called vertical equilibrium and occurs when \( \left( \frac{L}{h} \right) \left( \frac{k_r}{k_h} \right) > 10 \), where \( L \) is the interwell distance, \( h \) is the height of the reservoir, and \( k_v \) and \( k_h \) are the vertical and horizontal permeabilities. With good crossflow between layers, pressures are the same in each layer and the
crossflow between adjacent layers allow for simplification of each layer’s rate equation given by Equation 4-14 to a single rate equation for the entire reservoir. This result assumes that the total compressibility is the same for each layer.

4.3.3 Tank Model Theory

Walsh and Lake (2003) outline a method to predict oil production rates using a compressible tank model that relies on several assumptions (pp. 292-293):

1. There is a maximum of three fluids present in the system: stock-tank oil, surface gas and stock-tank water.
2. There is a maximum of two fluid phases at reservoir conditions: oleic and aqueous (i.e. the reservoir is at undersaturated conditions).
3. Stock-tank water does not partition into the oleic phase.
4. Stock-tank oil does not partition into the aqueous phase.
5. The aqueous phase is immobile.
6. The surface oil-producing rate from a well $i$ is related to the average reservoir pressure through the following deliverability relation:

$$q_{osc,i} = \frac{f_1(\bar{p} - p_{wf})}{b_o}$$

Equation 4-16

where $\bar{p}$ is the average reservoir pressure, $p_{wf}$ is the flowing bottomhole producing pressure for a well, and $f$ is the productivity index for a well. The productivity index and bottomhole pressure are assumed constant for each well but may differ between wells.\(^{37}\)

7. The reservoir is isothermal.

In their model, the field’s overall oil production rate is simply the sum of the rates of the individual wells. Assuming that all wells begin production at the same time, and have the same surrounding average reservoir pressure, bottomhole flowing pressure, and productivity index (implying even spacing and the same drainage area for each well), the

---

\(^{37}\) Here, it is assumed that all wells are operated with the same bottomhole pressure and have the same productivity index.
field production rate at standard conditions \( q_{osc} \) is equal to the product of the number of wells \( N_w \) in the field and the production rate of each well:

\[
q_{osc} = \sum_{l=1}^{N_w} q_{osc,l} = \frac{N_w f (\bar{\rho} - \rho_{wf})}{B_o}
\]

Equation 4-17

For multilayer reservoirs, Walsh and Lake assume the productivity index \( J \) for a single homogenous layer is replaced by a composite productivity index \( J_e \) that applies to all layers. For an individual well, the productivity \( J_e \) is defined as:

\[
J_e = \frac{0.00708 k H}{\mu_o \left( \frac{1}{2} \ln \frac{A}{r_w^2 C_A} + 5.75 + s \right)}
\]

Equation 4-18

where the thickness-weighted average permeability in millidarcies\(^{38}\) is defined as follows with \( k_l \) as the permeability of layer \( l \), \( h_l \) as the thickness of layer \( l \) in feet, \( H \) as the total thickness of the reservoir in feet and \( N_L \) layers in the reservoir:

\[
\bar{k} = \frac{\sum_{l=1}^{N_L} (kh)_l}{\sum_{l=1}^{N_L} h_l} = \frac{\sum_{l=1}^{N_L} (kh)_l}{H}
\]

Equation 4-19

\( \mu_o \) is the oil viscosity in centipoise, \( A \) is the drainage area for the well in acres, \( r_w \) is the wellbore radius in feet, \( C_A \) is the shape factor, and \( s \) is the skin factor. For a multilayer

\(^{38}\) Technically this is the relative permeability to oil – the endpoint relative permeability multiplied by the absolute permeability.
reservoir with crossflow, the overall field producing rate at standard conditions is given by:

\[ q_{osc} = \frac{N_{we}(\bar{p}-p_{wf})}{\bar{B}_o} \]

Equation 4-20

For Equation 4-16, Equation 4-17, Equation 4-18, and Equation 4-20, \( \mu_o \) and \( \bar{B}_o \) are evaluated at the mean pressure, which is defined by the following (Walsh and Lake p. 208):

\[ p_m = \frac{\bar{p}+p_{wf}}{2} \]

Equation 4-21

By contrast, \( \bar{B}_o \) is evaluated at the average reservoir pressure \( \bar{p} \) for Equation 4-14.

Thus Equations 4-14 and 4-20 both describe the oil production rate: one for the flow out of a tank and the other in terms of flow rate at the well. These rates should be the same, so Walsh and Lake equate these two expressions for the oil production rate and integrate to solve for an exponential decline model for reservoir pressure and oil production rate during the decline phase of primary production. They observe that this theoretical model of production behavior is consistent with empirical observations of exponential decline in many reservoirs.

While this model provides a useful theoretical basis for reservoir performance, it assumes that the compressibility, pore volume, and average oil saturation of the system are constant. It also assumes that the oil formation volume factor measured in the two
expressions for $q_{osc}$ is calculated at the same reservoir pressure value. To the extent that fluid properties vary with pressure, actual reservoir performance will likely deviate from the predictions of the exponential decline model.

Therefore to provide a more detailed characterization of a reservoir’s performance than the exponential decline model outlined by Walsh and Lake, a few modifications are suggested here to account for changes in compressibilities, pore volume, and oil saturation through primary recovery. These modifications also extend the tank model’s applicability to the infinite acting and pseudosteady state periods of primary recovery.

4.3.4 Tank Model Modifications

The modified tank model retains the basic list of seven assumptions above that Walsh and Lake employ for their tank model. A multilayer system with crossflow is also assumed. Rather than derive an analytical solution that would allow one to directly calculate oil production at a desired time, this model entails a finite differencing of time that relies on iterative calculation of the tank’s reservoir pressure, saturations and fluid properties using Euler’s Method to estimate a schedule of oil production versus time. This modified tank model is easily implemented with a modest amount of computation in a spreadsheet.

The modified model assumes that an operator produces each well in the field at a constant oil rate of $q_{o,max}$, measured at standard conditions, during the infinite acting and pseudosteady state periods. This rate could be the maximum oil rate allowed per well due to legal or technical constraints; in chapter seven it is taken to be a decision variable.
The overall field initial oil production rate at standard conditions \( q_{osci} \) is simply the product of the number of wells and \( q_{o, max} \):

\[
q_{osci} = N_w q_{o, max}
\]

Equation 4-22

Equation 4-14 can be rearranged to find the incremental change in average reservoir pressure at the beginning of primary recovery for a small time step \( dt \) while the field produces oil at the rate given by \( q_{osci} \). This change in average pressure is the departure from the initial reservoir pressure:

\[
d\bar{p} = -q_{osci} \frac{B_o}{\nu_{pc}} dt
\]

Equation 4-23

Oil production over this small time step is simply the product of the oil production rate and the time step:

\[
dN_p = q_{osci} dt
\]

Equation 4-24

At the new average reservoir pressure, the average oil FVF, oil viscosity, pore volume, oil compressibility, and average water and oil saturations have changed. The average oil FVF, oil viscosity, oil compressibilities are therefore recalculated at the new pressure using the correlations from Vasquez et al. as described above. Formation compressibility is assumed to be constant. Water compressibility is small and generally
does not vary much with pressure, so it is also taken to be constant (Walsh and Lake p. 124).

As the reservoir pressure declines, the pore volume shrinks. The definition for formation compressibility can be rearranged to determine the change in pore volume that occurs due to the drop in average reservoir pressure calculated above in Equation 4-23:

\[ c_f = \frac{1}{V_p} \frac{\partial V_p}{\partial p} \]

Equation 4-25

\[ dV_p = c_f V_p d\bar{p} \]

Equation 4-26

where the total differential is taken because the volume change is assumed to occur isothermally.

In reality, the pore volume change will be unevenly distributed throughout the reservoir, as points closer to wells will experience more substantial pressure declines than will those further into the reservoir. The effects of compaction are approximated here by using the change in average reservoir pressure. This approximation for pore volume reduction should capture reservoir behavior more accurately than a model that assumes pore volume remains constant.

The reduction in pore volume and production of oil has changed the average oil and water saturations in the reservoir. The cumulative oil produced \( N_p \), measured at standard conditions, is the sum of calculated oil produced \( dN_p \) for all previous time steps.
The remaining oil in the reservoir, measured at standard conditions, is simply the difference between the original oil in place and the oil produced. This quantity is multiplied by the oil FVF and divided by the pore volume to yield the average oil saturation in the reservoir:

$$\bar{S}_{o,new} = \frac{(001P_{STB} - N_p)\beta_o}{V_p}$$

Equation 4-27

Since it is assumed that the reservoir contains two phases, the average water saturation is simply the following:

$$\bar{S}_{w,new} = 1 - \bar{S}_{o,new}$$

Equation 4-28

Using these new saturations and oil compressibility, the total compressibility is recalculated for the new reservoir pressure:

$$c_{t,new} = c_f + \bar{S}_{o,new}c_{o,new} + \bar{S}_{w,new}c_w$$

Equation 4-29

The next iteration begins by starting again at Equation 4-23 and calculating the next change in average pressure over another time step using the new values for each variable. This procedure is repeated until the average reservoir pressure reaches the bubble point.
For each iteration in the procedure described above, it is necessary to check whether the reservoir has reached the end of the constant rate period, which brings the onset of depletion flow. This transition to the decline period of production for the field occurs when the wells in the reservoir have reached the minimum bottomhole pressure $p_{wf,min}$, an operational parameter for each well. At this point, the reservoir is no longer capable of producing oil at a constant rate of $q_{osci}$. Instead of calculating $p_{wf}$ for each iteration, Equation 4-20 is used to determine the oil production rate the wells would be capable of delivering at the current average reservoir pressure if their bottomhole pressures were $p_{wf,min}$. We check to see if this “open flow” rate (that is, not choked back to $q_{osci}$) exceeds $q_{osci}$:

$$
q_{osci,of} = \frac{N_w l e (\bar{p} - p_{wf})}{\bar{B}_o} > q_{osci}
$$

Equation 4-30

If the inequality above does not hold, then the reservoir has entered its decline period and Equation 4-23 and Equation 4-24 become:

$$
d\bar{p} = -q_{osci,of} \frac{\bar{B}_o}{\nu_p c_t} dt
$$

Equation 4-31

$$
dN_p = q_{osci,of} dt
$$

Equation 4-32
The rest of the procedure outlined above for recalculating pore volume, compressibilities, and reservoir fluid properties remains the same when the reservoir has reached the decline period.

Figure 4-3 summarizes the process used to estimate the solution gas drive reservoir’s performance with the modified tank model.
Figure 4-3: Process diagram for modeling primary recovery

**Reservoir is in decline period**
- Calculate $d\bar{p}$, $dN_p$ using Equation 4-31 and Equation 4-32
  - $d\bar{p} = -q_{osc,of}\frac{B_o}{V_p c_t}dt$
  - $dN_p = q_{osc,of}dt$
- Recalculate reservoir fluid properties at new average reservoir pressure using correlations provided by Vasquez, Beggs, and Robinson
- Calculate reduction in pore volume $dV_p$ due to change in average reservoir pressure $d\bar{p}$ (Equation 4-26)
- Recalculate oil and water saturations and total compressibility using Equation 4-27, Equation 4-28, and Equation 4-29

**Reservoir still produces at constant rate**
- Calculate $d\bar{p}$, $dN_p$ using Equation 4-23 and Equation 4-24
  - $d\bar{p} = -q_{osci}\frac{\bar{B}_o}{V_p c_t}dt$
  - $dN_p = q_{osci}dt$

**Is the average reservoir pressure below the bubble point?**
- Yes
  - Switch to secondary recovery (waterflooding)
- No
  - Does the open flow rate exceed the constant initial flow rate? (Equation 4-30)
    - $q_{osc,of} = \frac{N_w I_r (\bar{p} - p_{wf})}{B_n} > q_{osci}$
  - No
  - Yes
  - Yes
  - No
4.3.5 Well Pattern Assumptions

When calculating the productivity index in Equation 4-30, it is assumed the wells are spread evenly over the field in a square pattern, the field has a simple bounded square areal geometry, and $p_{wf,min}$ is the same for each well. Thus the total number of wells in the field is determined by the number of patterns arranged on each side of the square field. If there are $n_{pat.side}$ patterns per side, then there are $n_{pat.tot} = n_{pat.side}^2$ total patterns in the reservoir and the total number of wells in the reservoir is given by:

$$N_w = (n_{pat.side} + 1)^2$$

Equation 4-33

Figure 4-4 illustrates a field developed in this manner with five patterns per side, 25 total patterns in the field and 36 total wells. Each black dot represents a producer.
The drainage area for each well is taken to be the total areal extent of the field, shaded in blue, divided by the number of wells:

\[ A = \frac{\text{Area}}{N_w} \]

Equation 4-34

Each well has a square drainage area, so the shape factor \( C_A \) is 30.88 (Walsh and Lake p. 199).

4.4 Example Tank Model Calculation for a Hypothetical Reservoir

Using the reservoir fluid properties in Table 4-5, the physical parameters given below in Table 4-6, and the operational parameters given in Table 4-7, the performance of a hypothetical solution gas drive reservoir primary recovery is estimated. For these calculations, the time step \( dt \) is 0.1 months.
Table 4-6: Physical properties of a hypothetical reservoir

<table>
<thead>
<tr>
<th>Reservoir Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Reservoir Pressure</strong></td>
<td>$p_i$ 4000 psi</td>
</tr>
<tr>
<td><strong>Reservoir Temperature</strong></td>
<td>$T$ 150 °F</td>
</tr>
<tr>
<td><strong>Reservoir Depth</strong></td>
<td>$D$ 6000 ft</td>
</tr>
<tr>
<td><strong>Reservoir Type</strong></td>
<td>Sandstone</td>
</tr>
<tr>
<td><strong>Areal Extent of Reservoir</strong></td>
<td>$A_{res}$ 2000 acres</td>
</tr>
<tr>
<td><strong>Reservoir Thickness</strong></td>
<td>$H$ 60 ft</td>
</tr>
<tr>
<td><strong>Absolute Thickness-Weighted Permeability</strong></td>
<td>$\bar{k}$ 30 mD</td>
</tr>
<tr>
<td><strong>Oil Permeability</strong></td>
<td>$k_o$ 24 mD</td>
</tr>
<tr>
<td><strong>Initial Porosity</strong></td>
<td>$\varphi_i$ 0.26</td>
</tr>
<tr>
<td><strong>Porosity at Switch to Waterflooding</strong></td>
<td>$\varphi_{WF}$ 0.256</td>
</tr>
<tr>
<td><strong>Skin Factor</strong></td>
<td>$s$ 2</td>
</tr>
<tr>
<td><strong>Initial Oil Saturation</strong></td>
<td>$S_{oi,discovery}$ 0.8</td>
</tr>
<tr>
<td><strong>Oil Saturation at Switch to WF</strong></td>
<td>$S_{oiWF}$ 0.774</td>
</tr>
<tr>
<td><strong>Formation Rock Compressibility</strong></td>
<td>$c_f$ 5.00E-06 psi$^{-1}$</td>
</tr>
<tr>
<td><strong>Water Viscosity</strong></td>
<td>$\mu_w$ 0.6 cp</td>
</tr>
<tr>
<td><strong>Water Compressibility</strong></td>
<td>$c_w$ 3.00E-06 psi$^{-1}$</td>
</tr>
</tbody>
</table>

39 Assumes an endpoint relative permeability to oil of 0.8. See Table 7-5 below for relative permeability information.
Table 4-7: Operational parameters of a hypothetical reservoir

<table>
<thead>
<tr>
<th>Operational/Other Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well Radius</td>
<td>$r_w$</td>
</tr>
<tr>
<td>Injector Tubing Diameter</td>
<td>$D_{bg}$</td>
</tr>
<tr>
<td>Surface Temperature</td>
<td>$T_{surf}$</td>
</tr>
<tr>
<td>Shape Factor</td>
<td>$C_A$</td>
</tr>
<tr>
<td>Drainage Area Of A Well (Primary Production)</td>
<td>$A$</td>
</tr>
<tr>
<td>Min. Bottomhole Pressure</td>
<td>$p_{wf,min}$</td>
</tr>
<tr>
<td>Max. Rate Per Well for Primary Recovery</td>
<td>$q_{o,max}$</td>
</tr>
<tr>
<td>Number Of Wells (Primary Production)</td>
<td>$N_w$</td>
</tr>
<tr>
<td>Patterns Per Side Of Square (Square Root of Total Primary Recovery Patterns For Reservoir)</td>
<td>$n_{pat,side}$</td>
</tr>
<tr>
<td>Total Primary Recovery Patterns For Reservoir</td>
<td>$n_{pat,total}$</td>
</tr>
<tr>
<td>Pattern Area</td>
<td>$A_{pat}$</td>
</tr>
</tbody>
</table>

Figures 4-5 and 4-6 summarize the modified tank model’s predictions for oil production, recovery efficiency, and average reservoir pressure versus time. Table 4-8 provides calculations of initial pore and fluid volumes. The model estimates that oil production falls dramatically after the average reservoir pressure falls below the bubble point pressure; soon thereafter the reservoir enters the decline period. The average reservoir pressure reaches the bubble point 29 months into production. Recovery efficiency plateaus relatively soon after the bubble point has been reached, as can be seen in Figure 4-6. When one considers that waterflooding can help maintain a relatively high oil production rate, switching from primary to secondary recovery when the average
reservoir pressure reaches the bubble point intuitively makes sense as it prevents the steep decline in production.

As the modified tank model does not account for the development of a trapped gas saturation and gas flow below the bubble point, predictions beyond the time when reservoir pressure reaches the bubble point should be interpreted with caution. Although the numerical predictions of oil production are not reliable for this period, the general pattern of steep oil production decline once a solution gas drive reservoir has depleted below the bubble point is consistent with operator experience (Walsh and Lake p. 428). Rigorously accounting for both oil and gas-phase flow requires a more substantial modeling effort and is more easily accomplished through numerical simulation. Doing so would substantially increase the computational power and time necessary to evaluate the field for economic planning purposes. Since the operator is assumed to switch from primary to secondary recovery when the average reservoir pressure reaches the bubble point, a widespread gas saturation should not form.

Even though the average reservoir pressure may be above the bubble point, pressures in locations closer to the wellbore may be below it. Therefore some solution gas will evolve and reduce the oil relative permeability close to the wellbore before the switch to waterflooding. Although these effects are not explicitly captured, the skin factor can be modified to account for them.
Figure 4-5: Estimated field production and average reservoir pressure versus time

Figure 4-6: Estimated field recovery efficiency and average reservoir pressure versus time
Table 4-8: Pore and fluid volumes

<table>
<thead>
<tr>
<th>Reservoir Volumes</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Pore Volume</strong></td>
<td>$V_p$</td>
<td>242,043,110</td>
<td>RB</td>
</tr>
<tr>
<td><strong>Initial Water Volume</strong></td>
<td>$V_w$</td>
<td>48,408,622</td>
<td>RB</td>
</tr>
<tr>
<td><strong>Total Pore Volume at Switch to WF</strong></td>
<td>$V_{pwf}$</td>
<td>238,417,735</td>
<td>RB</td>
</tr>
<tr>
<td><strong>OOIP</strong></td>
<td></td>
<td>156,205,394</td>
<td>STB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>=</td>
<td>193,634,488</td>
</tr>
<tr>
<td><strong>Oil in Place at Start of WF</strong></td>
<td></td>
<td>147,608,065</td>
<td>STB</td>
</tr>
</tbody>
</table>

4.5 CONCLUSION

This modified tank model discussed here is used in conjunction with displacement models outlined in subsequent chapters to model oil production over the entire life cycle of a reservoir. Owing to the nature of production from solution gas drive reservoirs, primary recovery is relatively short in duration and yields relatively low recovery efficiency. For continuity, physical properties of the reservoir, such as average water and oil saturations and fluid characteristics, at the beginning of waterflooding are taken to be those predicted by the tank model at the switch time.
Chapter 5: Waterflooding and CO₂ Flooding

5.1 INTRODUCTION

Displacement methods offer the potential to deliver significant additional oil recovery from a reservoir beyond what is possible solely through primary recovery. This potential comes with higher costs and challenges with estimating the amount of and speed at which oil is recovered. Displacement methods include waterflooding, the injection of water into a reservoir to induce immiscible displacement of oil, and most forms of enhanced oil recovery (EOR), which involves the injection of polymers, surfactants, solvents, and other substances not normally present in a reservoir to recover its remaining oil (Lake 1989).

Depending on the characteristics of a particular reservoir, an operator may have multiple options for displacement methods that are technologically and economically feasible. Moreover, the screening process for determining feasibility can be quite complex. See Craig (1971) and Flaaten (2012) for discussions of the screening processes for waterflooding and chemical EOR methods. On the other hand, a reservoir’s physical properties oftentimes reduce the set of viable options for displacement methods, which may be evident by examining operators’ development choices for comparable nearby reservoirs.

This study examines one possible set of choices for displacement methods for a reservoir, namely a waterflood followed by a CO₂ flood. Waterflooding is a ubiquitous displacement technique, as it is relatively inexpensive and can deliver substantial incremental recovery. For reservoirs with light oils, CO₂ flooding is a commonly used
EOR method once the oil production from waterflooding has declined substantially. It involves the injection of carbon dioxide, usually alternated with slugs of water to lessen the mobility of the injected fluid, to miscibly or immiscibly displace remaining oil in the reservoir. Compared to waterflooding, CO₂ flooding requires some additional capital investment in processing facilities to separate CO₂ from the produced fluids and involves higher operational costs, such as those resulting from purchased CO₂. However, the process results in increased oil production more quickly than do most other EOR methods. Following primary recovery with a waterflood and a CO₂ flood is a common development strategy for suitable reservoirs in West Texas and other regions of the U.S. where CO₂ is relatively inexpensive and pipeline infrastructure is already present.

Given a fixed development strategy for the types and order of displacement methods to be used for a reservoir, an operator still faces the choice of when to switch from waterflooding to CO₂ flooding, as well as other design choices that impact the economics of the project. The predictive models employed in this study are designed to provide insight regarding the optimal values of this switching time and other important design parameters. While waterflooding and CO₂ flooding are discussed here, the modeling techniques can be generalized to other isothermal displacement methods.

Prediction techniques for oil recovery from displacement methods vary in complexity. The challenge for modeling is to limit the number of parameters and computational power necessary while still capturing the relevant physical phenomena and retaining the ability to explain the production history of a field. While there are no “silver bullet” prediction methods, are useful techniques developed by Andonyadis
(2010) and Mollaei (2011) that is applicable for waterflooding and isothermal EOR recovery, is capable of matching production histories very well, and is based on Koval (1963) and fractional flow theory. As these prediction methods assume relatively uniform water and oil before a CO₂ flood, this study extends the theory so they apply in reservoirs in which a waterflood is followed relatively closely by a CO₂ flood (i.e. the starting point is not spatially uniform).

5.2 FRACTIONAL FLOW THEORY

Fractional flow theory is crucial for characterizing displacements with the models discussed in this study. It is used to find the specific velocities, defined below, of the various fluid fronts that are assumed to move through a reservoir.

5.2.1 Fractional Flow in a Waterflood

Buckley and Leverett (1942) derive an equation for immiscible displacement that is central to fractional flow theory, which is the theoretical foundation of many models of displacements in permeable media. Equation 5-1 is a version of this equation, except expressed here in dimensionless time and distance terms, and describes the fractional flow of a one-dimensional displacement involving incompressible and immiscible fluids (oil and water) in a medium. This relationship, based on the conservation of mass, assumes that the medium does not adsorb any of the fluid:

\[
\frac{\partial s_w}{\partial t_D} + \frac{\partial f_w}{\partial x_D} = 0
\]

Equation 5-1
Both Andonyadis and Mollaei define the dimensionless time and distance $t_D$ and $x_D$ as follows:

$$t_D = \frac{\int_0^t qd\xi}{\int_0^t A\varphi d\xi}$$

Equation 5-2

$$x_D = \frac{\int_0^x A\varphi d\xi}{\int_0^L A\varphi d\xi}$$

Equation 5-3

In other words, $t_D$ and $x_D$ reparameterize time and distance in the medium as fractions of the medium’s pore volume and length. When $A$, the cross-sectional area of the medium, and $\varphi$ are constant across the medium, dimensionless distance is simply distance $x$ expressed as a fraction of the total length of the medium $L$: $x_D = \frac{x}{L}$. Dimensionless time characterizes time in terms of the quantity of fluid injected into the medium, expressed as a fraction of its pore volume. As discussed in the next chapter, estimating injection rates into a medium is crucial to converting dimensionless time, which is useful for predicting displacement behavior, to actual time, which is useful for calculating the economic viability of a process.

The fractional flow of water $f_w$ at a given point in the medium is defined as the flow rate of water at that point divided by the total fluid flow rate:

$$f_w = \frac{q_w}{q_{total}}$$

Equation 5-4
The fractional flows of oil or other fluids moving through the medium are similarly defined.

Walsh and Lake (1989) rewrite Equation 5-1 using the chain rule, which facilitates the use of dimensionless interstitial velocity (specific velocity) \( v_{s_w} \) of a particular water saturation contour:

\[
\frac{\partial s_w}{\partial t_D} + \frac{\partial f_w}{\partial s_w} \frac{\partial s_w}{\partial x_D} = 0
\]

Equation 5-5

\[
\frac{\partial f_w}{\partial s_w} = v_{s_w}
\]

Equation 5-6

\[
\frac{\partial s_w}{\partial t_D} + v_{s_w} \frac{\partial s_w}{\partial x_D} = 0
\]

Equation 5-7

Walsh and Lake (1989) also show, from the definition of a total derivative and the fact that the specific velocity of a particular water saturation contour is constant, the following relationship holds:

\[
\left( \frac{df_w}{ds_w} \right)_{s_w} = v_{s_w} = \frac{x_D}{t_D}
\]

Equation 5-8

Figure 5-1 illustrates the specific velocity, which is the slope of a fractional flow curve for saturations greater than \( S_{wf} \). It also illustrates the concept of a shock, which forms because the specific velocity at saturation \( S_{wf} \) is greater than those of saturations less than \( S_{wf} \). \( S_{wf} \) therefore overtakes these slower saturations, resulting in a front moving
through the medium at which the water saturation jumps from its initial saturation before injecting water in the medium, $S_{wi}$ (equal to $1 - S_{oi}$), to $S_{wf}$. The specific shock velocity $v_{\Delta S}$ of the waterflood front is given by the slope of a line drawn on the fractional flow curve from the initial water saturation to the tangency at $(S_{wf}, f_{wf})$:

$$v_{\Delta S} = \frac{f_{wf} - f_{wi}}{S_{wf} - S_{wi}}$$

Equation 5-9

Figure 5-1: Fractional flow diagram for oil and water
The tangent chord drawn from \((1 - S_{ol}, 0)\) to \((S_{wf}, f_{wf})\) for the shock velocity provides the average saturation behind the shock front, \(S_{oWF}\), where it intersects \(f_w = 1\). From the fractional flow construction in Figure 5-1, the following expression is true:

\[
\left( (1 - S_{oWF}) - S_{wl} \right) v_{\Delta S} = 1
\]

Equation 5-10

Solving for \(S_{oWF}\) yields:

\[
S_{oWF} = 1 - \frac{1}{v_{\Delta S}} - S_{wl}
\]

Equation 5-11

While traditional Buckley-Leverett theory holds that the saturation behind the shock front in a water-oil displacement varies according to the velocities determined by the fractional flow curve until the residual oil saturation \(S_{or}\) is reached (Willhite 1986), Andonyadis (2010) and Mollaei (2011) make the simplifying assumption that the displacement in a medium is piston-like. In the context of the fractional flow curve, this type of displacement means that a region of saturation \(S_{wl}\) is swept by a shock front, behind which there is a region of constant saturation \(S_{oWF}\). Figure 5-2 illustrates this displacement in a one-dimensional medium. Even with this assumption in their models, Mollaei and Andonyadis were able to obtain reasonable history matches because they also account for the heterogeneity of permeable media. The modeling of heterogeneity is discussed in later sections.
5.2.2 Relative Permeability

To determine the specific shock velocity of the waterflood front, it is necessary to characterize the fractional flow curve for a medium saturated with oil and water. Buckley and Leverett (1942) relied on the following relationship, which neglects the effects of gravity for simplicity:

\[ f_w = \frac{1}{1 + \frac{1}{M_{w,o}}} \]

Equation 5-12

The water-oil mobility ratio function \( M_{w,o} \) is defined as follows:

\[ M_{w,o} = \frac{kk_{rw}}{\mu_w \frac{r_{rw}}{r_{ro}}} \]

Equation 5-13

Figure 5-2: Piston-like displacement for a waterflood
where \( k \) is the absolute permeability of the medium, \( k_{rw} \) and \( k_{ro} \) are the relative permeabilities of oil and water as functions of water saturation \( S_w \), and \( \mu_w \) and \( \mu_o \) are the viscosities of water and oil.

Figure 5-3 plots relative permeability curves based on the form Corey (1954) proposes and given the parameters in Table 5-1. The water and oil relative permeabilities are described by the following equations:

\[
k_{rw}(S_w^*) = (k_{rw}^*)(S_w^*)^n
\]

Equation 5-14

\[
k_{ro}(S_w^*) = (k_{ro}^*)(1 - S_w^*)^m
\]

Equation 5-15

\[
S_w^* = \frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}}
\]

Equation 5-16

The endpoint relative permeabilities are given by \( k_{ro}^* \) and \( k_{rw}^* \); these along with the exponents \( m \) and \( n \) are determined for a particular medium through core floods. Equation 5-16 re-scales water saturations between residual saturations for the medium to a dimensionless saturation \( S_w^* \) that varies between 0 and 1. Intuitively, the slopes of the curves in Figure 5-3 mean that water flows through the permeable medium more easily as the water saturation increases; the same relationship holds for the oil relative permeability and oil saturation.
Figure 5-3: Corey relative permeability curves

Table 5-1: Parameters for Corey relative permeability curves

<table>
<thead>
<tr>
<th>Relative Permeability and Mobility</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual Oil Saturation to Waterflooding</td>
<td>$S_{or}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Oil Saturation at Switch to WF</td>
<td>$S_{oiWF}$</td>
<td>0.774</td>
</tr>
<tr>
<td>Water Saturation at Switch to WF</td>
<td>$S_{wiWF}$</td>
<td>0.226</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Oil</td>
<td>$k_{ro^o}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Water</td>
<td>$k_{rw^o}$</td>
<td>0.2</td>
</tr>
<tr>
<td>Oil Corey Exponent</td>
<td>$m$</td>
<td>2</td>
</tr>
<tr>
<td>Water Corey Exponent</td>
<td>$n$</td>
<td>2</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>$\mu_w$</td>
<td>0.6 cp</td>
</tr>
<tr>
<td>Oil Viscosity at Bubble Point</td>
<td>$\mu_{oB}$</td>
<td>0.626 cp</td>
</tr>
<tr>
<td>Endpoint Mobility Ratio</td>
<td>$M^o$</td>
<td>0.261</td>
</tr>
</tbody>
</table>
5.2.3 Solving for the Shock Velocity with Newton’s Method

Substituting the definitions for relative permeability into the water fractional flow expression in Equation 5-12 and rearranging, we obtain the following equation relating fractional flow to dimensionless water saturation (Willhite 1986):

\[
f_W(S_w^*) = \frac{(S_w^*)^n}{(S_w^*)^n + \frac{\mu_0 k_{rw}}{\mu_{wq}}(1-S_w^*)^m} = \frac{(S_w^*)^n}{(S_w^*)^n + A(1-S_w^*)^m}
\]

Equation 5-17

where \( A = \frac{\mu_0 k_{rw}}{\mu_{wq}} = \frac{1}{M_{wp}} \). This is the inverse of the endpoint mobility for oil and water.

Differentiating Equation 5-17 with respect to \( S_w \) yields:

\[
\frac{df_W}{dS_W} = \frac{ABn(S_w^*)^{n-1}(1-S_w^*)^m + ABm(S_w^*)^n(1-S_w^*)^{m-1}}{[(S_w^*)^n + A(1-S_w^*)^m]^2}
\]

Equation 5-18

where \( B = \frac{dS_w}{ds_w} = \frac{1}{1-S_{or}-S_{wi}} \).

To determine the tangent chord from the initial oil saturation before waterflooding to the tangent point at \((S_{wf}, f_{wf})\), we equate Equation 5-9 with Equation 5-18. This relationship is true because the slope at the tangent point must be equal to the slope of the chord drawn from the initial saturation in the medium to the tangent point:

\[
\frac{f_{wf}-f_{wt}}{S_{wf}-S_{wi}} = \frac{f_w(S_{wf}^*)}{\frac{1}{B_s}S_{wf}^*} = \frac{df_w}{dS_w} \left( S_{wf}^* \right)
\]

Equation 5-19
Substituting and rearranging to derive an expression in terms of the dimensionless saturation at the tangent point and other constants results in the following equation:

$$g(S_{wf}^*) = (S_{wf}^*)^n + A(1 - n)(1 - S_{wf}^*)^m - AmS_{wf}^*(1 - S_{wf}^*)^{m-1} = 0$$

Equation 5-20

As the exponents in Equation 5-20 may be noninteger,\textsuperscript{40} in many cases it will be difficult to solve for $S_{wf}^*$ directly using algebra.

Newton’s Method is a numerical technique that is applicable for approximating the roots of equations like those in the form of Equation 5-20 (Newton 1712). The procedure for implementing the method is relatively straightforward. One begins with a first guess for the saturation at the tangent point, $S_{wf}^*_0$. A better approximation $S_{wf}^*_1$ is then given by:

$$S_{wf}^*_1 = S_{wf}^*_0 - \frac{g(S_{wf}^*_0)}{g'(S_{wf}^*_0)}$$

Equation 5-21

Successive approximations for $S_{wf}^*$ are then given by:

$$S_{wf}^*_{i+1} = S_{wf}^*_i - \frac{g(S_{wf}^*_i)}{g'(S_{wf}^*_i)}$$

Equation 5-22

with the derivative of the function $g$ calculated as follows:

\textsuperscript{40} They still need to be greater than one, though.
\[ g'(S_{wf}^*) = n(S_{wf}^*)^{n-1} + Am(1 - n)(1 - S_{wf}^*)^{m-1} - Am \left[ (1 - S_{wf}^*)^{m-1} - (m - 1)S_{wf}^*(1 - S_{wf}^*)^{m-2} \right] \]

Equation 5-23

One computes additional approximations until the absolute difference between successive approximations \( |S_{wf}^*_{i+1} - S_{wf}^*_i| \) is smaller than a nonnegative desired tolerance, and the final approximation for \( S_{wf}^* \) is the solution. This dimensionless water saturation is readily converted to an actual saturation using Equation 5-16, and the shock velocity \( v_{\Delta S} \) is then calculated using Equation 5-9. This calculation process is relatively straightforward using a custom function defined in Excel. See Appendix C for Visual Basic code that implements the algorithm.

### 5.2.4 Fractional Flow in a CO₂ Flood

Walsh and Lake (1989) extended the application of fractional flow theory to CO₂ flooding for a water alternating gas (WAG) process. For this EOR process it is assumed that the fluids in the reservoir are incompressible and the injected fluid is miscible with the oil. The latter characteristic allows the water-CO₂ mixture to dissolve remaining oil in the reservoir, as CO₂ is a solvent, thereby reducing the residual oil saturation. While the use of WAG injection for CO₂ in field applications typically involves injecting discrete slugs of water and gas, Walsh and Lake assume that both water and CO₂ are injected simultaneously. Injection occurs in a fixed volumetric proportion \( W_R \), measured
in reservoir volumes of water divided by reservoir volumes of CO₂, called the WAG ratio. The WAG ratio is related to the fractional flow of water at the injection side of the medium by the following expression:

\[ f_{wJ} = \frac{W_R}{1 + W_R} \]

Equation 5-24

Figure 5-4 illustrates the fractional flow construction of a WAG CO₂ flood. There is now a second fractional flow curve for the water and solvent in addition to the water-oil fractional flow curve. This new fractional flow curve is assumed to be defined using the water-solvent mobility ratio, \( M_{w,s} \):

\[ M_{w,s} = \frac{kk_{rw}}{kk_{rs}} \frac{\mu_w}{\mu_s} \]

Equation 5-25

The solvent relative permeability is defined in a similar manner as is the oil relative permeability:

\[ k_{rs}(S_w^{*}) = (k_{rs}^{o})(1 - S_w^{*})^m \]

Equation 5-26

For now it is assumed that \( k_{rs}^{o} = k_{ro}^{o} \) and the Corey exponent \( m \) is the same for oil and solvent alike.

---

41 In the case where slugs are injected, the ratio of water-solvent slug size is still called the WAG ratio.
Figure 5-4: Fractional flow diagram for WAG CO$_2$ injection

Point F illustrates the final oil saturation $S_{oF}$ once the water-solvent mixture has swept the medium. Mollaei (2011) predicts $S_{oF}$ using the results of a multivariate regression on numerical simulation results, as discussed in Appendix D.

Point J indicates the location on the water-solvent fractional flow curve corresponding to the fractional flow of injected fluid $f_{wJ}$. The slope of the line connecting points F and J is the specific velocity of the solvent front, $v_C$. Extending the line connecting F and J to the intersection with the oil-water fractional flow curve...
determines the oil bank saturation at point B. The specific velocity of the oil bank front \( v_{oB} \) is then given by the slope of the line between point I, denoting the point on the water-oil fractional flow curve associated with the oil saturation after the completion of a waterflood, and point B.

The physical interpretation of the fractional flow diagram is perhaps more easily conceptualized in terms of one-dimensional piston-like displacement. Figure 5-5 illustrates this concept for the fractional flow diagram in Figure 5-4. Points I, B and F in Figure 5-4 correspond to the constant saturation regions of \( S_{oWF} \), \( S_{oB} \), and \( S_{oF} \) in Figure 5-5. The specific velocities of the oil bank and solvent fronts, along with dimensionless time, determine their locations in the medium.

![Piston-like displacement for a CO₂ flood](image)

Figure 5-5: Piston-like displacement for a CO₂ flood

### 5.2.5 Solving for Specific Velocities of CO₂ Flood Fronts

Calculating the specific velocities \( v_{oB} \) and \( v_c \) from the fractional flow construction illustrated in Figure 5-4 is necessary to predict the speed of displacements.
Since the fractional flow curves may have non-integer exponents, it is necessary to use numerical solution methods to compute these velocities.

To determine \( v_c \), the water saturation on the water-solvent fractional flow curve associated with the injected fluid fractional flow \( f_{wj} \) is needed. The dimensionless version of this water saturation, \( S_{wj}^* \), is found simply by equating water-solvent fractional flow, expressed as a function of dimensionless water saturation, with \( f_{wj} \):

\[
\frac{(S_{wj}^*)^n}{(S_{wj}^*)^n + \frac{\mu_s k_{rw}}{\mu_w k_{rs}} (1 - S_{wj}^*)^m} = \frac{(S_{wj}^*)^n}{(S_{wj}^*)^n + A_s(1 - S_{wj}^*)^m} = f_{wj}
\]

Equation 5-27

where \( A_s = \frac{\mu_s k_{rw}^o}{\mu_w k_{rs}^o} = \frac{1}{M_{ws}} \). The exponents \( m \) and \( n \) may be non-integer, so it is appropriate to solve for \( S_{wj}^* \) using Newton’s Method as was done in section 5.2.3. The procedure here is the same, except the goal now is to solve for \( S_{wj}^* \) instead of \( S_{wf}^* \) using the following expressions in place of \( g \) and \( g' \):

\[
\theta(S_{wj}^*) = (f_{wj} - 1)(S_{wj}^*)^n + A_s f_{wj}(1 - S_{wj}^*)^m = 0
\]

Equation 5-28

\[
\theta'(S_{wj}^*) = B \left[ n(f_{wj} - 1)(S_{wj}^*)^{n-1} - A_s m f_{wj}(1 - S_{wj}^*)^{m-1} \right] = 0
\]

Equation 5-29

where \( B = \frac{dS_w^*}{dS_w} = \frac{1}{1 - S_{or} - S_{wi}} \)
Once Newton’s Method has converged on a solution for $S_{wj}^*$, Equation 5-16 is used to convert the dimensionless saturation to a physical saturation $S_{wj}$. The specific velocity $v_c$ is then the slope of the line between the points $(S_{wj}, f_{wj})$ and $(1 - S_{oF}, 1)$.

$$v_c = \frac{1 - f_{wj}}{1 - S_{oF} - S_{wj}}$$

Equation 5-30

Now, given $v_c$, the point $(S_{wj}, f_{wj})$, and the equation for the water-oil fractional flow curve as a function of water saturation, it is possible to solve for the oil bank saturation. The specific velocity $v_c$ is also the slope of the line between points J and B on Figure 5-4:

$$v_c = \frac{f_{wj} - f_{WB}}{S_{wj} - S_{WB}}$$

Equation 5-31

This equation may be rewritten to convert water saturations to dimensionless values and express $f_{WB}$ as a function of the dimensionless water saturation of the oil bank $S_{WB}^*$:

$$v_c = \frac{f_{wj} - f_{WB}}{(S_{wj} - S_{wl}) - (S_{wb} - S_{wl})} = \frac{f_{wj} - \frac{(S_{wb}^*)^n}{(S_{wb})^n + A(1 - S_{wb}^*)^m}}{S_{wj}^* - S_{wb}^*}$$

Equation 5-32

With $A$ defined as it is for Equation 5-17, since the expression for $f_{wb}$ corresponds to the water-oil fractional flow curve. Equation 5-32 may also be rearranged to obtain the following equations to solve for $S_{wb}^*$ using Newton’s Method:
\[ \beta(S_{WB}^*) = f_{wJ}[(S_{WB}^*)^n + A(1 - S_{WB}^*)^m] - \\
(S_{WB}^*)^n - \frac{v_c}{B}(S_{WB}^* - S_{WB}^*)[n(S_{WB}^*)^n + A(1 - S_{WB}^*)^m] = 0 \]

Equation 5-33

\[ \beta'(S_{WB}^*) = B\left[n\left(f_{wJ} - 1\right)(S_{WB}^*)^{n-1} + Amf_{wJ}(1 - S_{WB}^*)^{m-1}\right] - v_c\left[(S_{WB}^*)^n + A(1 - S_{WB}^*)^m\right] - v_c\left(S_{wJ}^* - S_{WB}^*\right)\left[n(S_{WB}^*)^{n-1} + Am(1 - S_{WB}^*)^{m-1}\right] = 0 \]

Equation 5-34

Once the algorithm has converged on a solution for \( S_{WB}^* \), Equation 5-16 and Equation 5-17 are used to convert \( S_{WB}^* \) to \( S_{WB} \) and find \( f_{WB} \).

With the oil bank saturation and fractional flow, it is straightforward to calculate the specific velocity of the oil bank using points I and B as shown in Figure 5-4:

\[ v_{oB} = \frac{f_{wWF} - f_{WB}}{S_{wWF} - S_{WB}} \]

Equation 5-35

Where \( f_{wWF} \) is calculated using the fractional flow curve in Equation 5-17 using the water saturation after waterflood sweep \( S_{wWF} = 1 - S_{oWF} \).

Appendix C also contains the VBA code for Newton’s Method algorithms used to calculate water saturation values at intersections at points B and J on the CO\(_2\) flood fractional flow diagram as described above.
Using Newton’s Method to solve for specific velocities helps build the framework for the oil recovery prediction model described here to apply to a variety of relative permeability curves.

5.2.6 Special Case of Merged Fronts

If an operator chooses to develop a reservoir so that a waterflood is followed shortly by a CO₂ flood, it is possible that the oil bank from the CO₂ flood could “catch up” to the waterflood shock front. This situation could occur when \( v_{oB} \) in Figure 5-4 is greater than \( v_{\Delta s} \) in Figure 5-1 and the waterflood shock front depicted in Figure 5-2 has incompletely swept the reservoir upon initiation of the CO₂ flood. It is necessary to hypothesize what might occur in this situation to have a comprehensive method for estimating future oil recovery. The analysis in this section extends in part the models developed by Andonyadis (2010) and Mollaei (2011), which rely on the assumption that a reservoir is uniformly saturated immediately before initiating an EOR process. The possibility of the oil bank front reaching the waterflood front before it has swept the reservoir also requires further modification of Mollaei’s model to account for the locations of the fronts in a multilayer heterogeneous medium.

Fractional flow theory can provide some guidance for the instance in which the reservoir is developed with a short waterflood followed by a CO₂ flood. Consider the fractional flow diagram in Figure 5-6, which combines the constructions of Figure 5-1 and Figure 5-4. It is evident in this case that the oil bank specific velocity is greater than that of the waterflood front. The existence of the overlap of fronts depends on both the
velocities and the time at which the CO\textsubscript{2} flood is started. Note the specific velocities \( v_h \) and \( v_c \) are not necessarily equal, even though they may appear to be so in this diagram.

Now consider a hypothetical situation in a one-dimensional reservoir where a region of the oil bank saturation given at point B contacts and displaces a region with the oil saturation present in the reservoir before the waterflood was initiated, denoted by point U. The interface between these two regions would be a shock with a specific velocity given by \( v_h \), the slope of the orange line connecting points B and U. If the specific velocity of each saturation between points B and U were taken instead to be the derivative of the oil-water fractional flow curve, then the saturations closer to the displacing saturation \( S_{oB} \) would overtake those closer to the unswept region oil saturation \( S_{oil} \). Thus a shock forms in this hypothetical displacement involving points B and U. This is the same reasoning used to justify shock formation for the waterflood case above.

The specific velocity \( v_h \) is less than that of the waterflood front on the same fractional flow diagram. This result is also more generally true. If the WAG ratio were higher, which would increase \( f_{wJ} \) and move point B closer to point I on the water-oil fractional flow curve, the condition \( v_h \leq v_{DS} \) should always hold. If B were above the point at which the waterflood shock line is tangent to the oil-water fractional flow curve, then the hypothetical displacement involving B and U would form a shock similar to that of the waterflood depicted on the fractional flow diagram with a specific shock velocity of \( v_{DS} \). Moreover, \( v_{oB} \) would also be less than \( v_{DS} \) in this scenario with a high WAG ratio. The WAG ratio is constrained here to a maximum of 5, which generally precludes this situation from occurring.

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The intuition from the hypothetical displacement described above applies to the case of a one-dimensional medium in which a relatively faster oil bank catches a relatively slower waterflood front. If the oil bank front were to overtake the waterflood front completely, the shock from point B to point U would occur, but since $v_h \leq v_{\Delta S}$, this would entail the B-U shock moving slower than the existing waterflood front. Thus for the purposes of further analysis, it is assumed that in the limiting case of the faster oil bank front coming very close to the waterflood front, the fronts “merge” and the oil bank
front then moves at the same velocity as the waterflood front. This assumption regarding merging means that the fronts have not physically merged, as this would mean that the displacement would now move at the slower specific velocity $v_h$. In other words, there is still an infinitesimally small distance between the oil bank front and the waterflood front, and the oil bank front does not completely overtake the waterflood front.

This possibility for merging is illustrated in a time-distance diagram in Figure 5-7. Each line depicts the dimensionless position of a front in the reservoir as a function of time. The oil bank forms when the CO$_2$ flood is initiated at $t_{D,ST}$ and eventually catches the waterflood front. For the case of merging described above, both the oil bank front and waterflood front move along the line describing the waterflood front after the merging point.

![Time-distance diagram for front merging possibilities](image)

Figure 5-7: Time-distance diagram for front merging possibilities

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$^42$ The solvent front is omitted here to reduce clutter.
The assumption that the oil bank front and waterflood fronts merge in the manner described above in this special case is potentially a simplification of actual displacement behavior. Another possible outcome could be that the oil bank front fully overtakes the waterflood front and the resulting interface moves at the slower velocity $v_h$, also shown in Figure 5-7 for dimensionless times after the fronts collide. If this alternative outcome were true, then the model with the assumption of merging would overpredict the speed at which oil is recovered and suggest an earlier switch to CO$_2$ flooding than would actually be economically optimal. The alternative outcome suggests that, since $v_h \leq v_{\Delta S}$, a very early switch to CO$_2$ flooding might slow the waterflood front in this special case to the point where oil would have been recovered more quickly, at least in the near term, by continuing with the waterflood instead. This result seems a bit counterintuitive, so more investigation is needed to verify behavior in the special case where it is assumed the fronts merge. Future experimental results and field data from reservoirs developed with early switching times to CO$_2$ flooding could be used to confirm the assumption regarding front merging.

5.3 ACCOUNTING FOR HETEROGENEITY; BACKGROUND ON MOLLAEI’S METHOD

5.3.1 Introduction

As reservoirs are almost never homogeneous, it is important to account for the effects of large-scale heterogeneity for displacements in permeable media. Heterogeneity is central to Mollaei’s model for forecasting oil recovery (2011). He combines insights from fractional flow theory, along with contributions from Koval (1963) and Lake (1989)
to produce a tool that he successfully history matches to the production of oil from several reservoirs. He conducts a sensitivity analysis on several reservoir parameters to determine those that have the greatest impact on the model output and runs regressions on samples of numerical simulation results to predict Koval factors used in his model. These factors forecast future recovery from a reservoir based on a few physical parameters, which is useful for economic evaluation because it allows for relatively rapid assessment of a reservoir’s potential oil recovery \textit{a priori}. An update to his model described in section 5.4 ensures it is theoretically applicable to cases where switching recovery methods occurs relatively quickly, as is suggested for reservoir life cycle optimization by Parra Sanchez (2010). This section reviews Mollaei’s method, focusing particularly on waterfloods and defining consistent nomenclature for the model as adapted for the purposes of this study. Those who are familiar with the subject matter may wish to skip to section 5.4.

5.3.2 Flow Capacity and Storage Capacity

Mollaei’s forecasting model is based on the assumption of a uniformly layered model of a reservoir and the concept of parameterizing the heterogeneity of a reservoir in terms of flow capacity $F_n$ and storage capacity $C_n$. Stiles (1949) defines these variables as follows:

$$F_n = \frac{\sum_{l=1}^{\eta} (kh)_l}{\sum_{l=1}^{\eta} (kh)_l}$$

Equation 5.36
Equation 5-37

\[ C_n = \frac{\sum_{i=1}^{n} (\varphi h)_i}{\sum_{i=1}^{N_L} (\varphi h)_i} \]

Where \( k_i \), \( \varphi_i \) and \( h_i \) are the permeability, porosity and thickness of an individual layer \( l \) and there are \( N_L \) total layers in the reservoir. Before computing the sums, the layers are listed in decreasing order of permeability. It is useful to think of a value for \( F \) for a particular layer as the amount of flow capacity for all layers with permeability greater than or equal to that of a given layer divided by the total flow capacity of the reservoir. \( F \) and \( C \) are therefore restricted to values between 0 and 1. This method transforms physical reservoir dimensions into flow capacity and storage capacity.

The concept of flow capacity and storage capacity can be related to the distribution of flow in a reservoir. Based on reasoning discussed in Lake (1989), Mollaei notes that the slope of the curve relating \( F \) and \( C \) (the F-C curve, found by calculating \( F \) and \( C \) for all \( N_L \) layers in the reservoir) for the continuous case where \( N_L \) approaches infinity is equal to the ratio of the interstitial velocity at a given capacity \( C \) to the average interstitial velocity in the reservoir. In this model, \( n \) can be taken to represent the layer at which the displacing agent is just breaking through an arbitrary cross-section of a reservoir. When assuming piston-like displacement, the proportion of oil flowing across a cross-section of a reservoir can be characterized using the F-C curve. Thus the concept of flow capacity and storage capacity is very useful as it directly relates the heterogeneity of the reservoir to oil production and saturations when using a displacement recovery method.
As previously described in the discussion regarding fractional flow, Mollaei (2011) makes the key assumption for his model that displacement is locally segregated, or piston-like. The assumption of piston-like displacement is justified by the assertion that it is the most common for light oils and that any deviations are captured through the use of a Koval factor (Mollaei 2011). Figure 5-2 and Figure 5-5 illustrate piston-like displacement for waterflooding and CO$_2$ flooding in one dimension, assuming that the reservoir initially is uniformly saturated. Figure 5-8 illustrates piston-like displacement in a two-dimensional model with reservoir height on the vertical axis and dimensionless distance from injector to producer $x_D$ on the horizontal. The upper plot (a) shows a case with one front, which is applicable to a waterflood, while (b) shows the generic EOR case with two fronts. For CO$_2$ flooding, the displacing fluid in (b) is the injected solvent-water mixture. The final oil saturation $S_{oF}$ corresponds to a different value in each plot, as they depict different displacement processes.
The transformation from the physical dimension of height to the F-C parameterization that is central to Mollaei’s model is shown in Figure 5-9; the upper and lower plots correspond to (a) and (b) in Figure 5-8. Storage capacity $C$ is now on the vertical axis while $x_D$ is still on the horizontal. Each value of $C$ is related to a value of $F$ not displayed on the plot. The circled letters I, B, and F represent regions of constant oil saturation associated with the initial, oil bank, and final conditions. The distance of each front from the injector monotonically decreases as $C$ increases because of the

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43 He assumes the “initial” oil saturation condition before the CO$_2$ flood begins is the post-waterflood oil saturation.
manner in which $F$ and $C$ are defined. As small values of $C$ are associated with large permeabilities, breakthrough of the displacing agent occurs most rapidly for these values.

Figure 5-9: Piston-like displacement using F-C parameterization (from Mollaei 2011)

### 5.3.3 Heterogeneity and Dimensionless Time

Koval (1963) modified the Buckley-Leverett model to predict the behavior of miscible displacements, such as CO$_2$ floods. He expresses the fractional flow of solvent, defined analogously to the water cut given by Equation 5-4, as a function of the solvent
saturation and a Koval factor. This factor combines the effects of viscosity, fingering of the displacing fluid, and heterogeneity:

\[ K = H_k E; \quad E = \left(0.78 + 0.22\nu^2\right)^4 \]

Equation 5-38

Where \( H_k \) is a heterogeneity factor and the expression for \( E \), the effective viscosity ratio, is derived from experimental results. The variable \( \nu \) is the ratio of oil viscosity to solvent viscosity. Lake (1989) notes that \( H_k \) can be defined as follows:

\[ H_k = \frac{1-C}{C} \times \frac{F}{1-F} \]

Equation 5-39

Rearranging in terms of \( F \), this expression becomes

\[ F = \frac{1}{\frac{1}{1+\frac{1}{H_k\left(\frac{1-C}{C}\right)}}} \]

Equation 5-40

Mollaei’s (2011) model predicts the dimensionless location of each front in the reservoir as a function of the storage capacity \( C \), as seen in Figure 5-9. He observes that for generic displacement, given a dimensionless time \( t_D \) representing the total cumulative amount of fluid injected into a reservoir, each flow capacity/layer will have a different dimensionless time representing the amount of fluid that has flowed into that capacity/layer. Mollaei (2011) derives an expression for this dimensionless time
corresponding to a layer \( l \) when there is a finite number of layers in the reservoir, given by:

\[
t_{DL} = t_D \left( \frac{\bar{\omega} k_i}{k \varphi_i} \right)
\]

Equation 5-41

As the number of layers approaches infinity, he notes that Equation 5-41, which now expresses the amount of displacing fluid that has entered a particular flow capacity \( C \), becomes:

\[
t_{D|C} = t_D \left( \frac{dF}{dC} \right)
\]

Equation 5-42

Mollaei (2011) notes the need for an expression for \( F \) in terms of \( C \) to compute the derivative in Equation 5-42, so he

“use[s] Koval fractional flux model because of it’s [sic] abilities, simplicity and wide application even in very heterogeneous permeable media” (p. 52):

\[
F = \frac{1}{1 + \frac{1}{R(1-c)}}
\]

Equation 5-43

Contrast Equation 5-43, where the Koval factor \( K \) incorporates the effects of both heterogeneity and mobility of fluids, with Equation 5-40, which relates \( F \) and \( C \) with \( H_k \) and therefore appears more consistent with the heterogeneity-based derivation used to arrive at Equation 5-42. The derivative of Equation 5-43 yields

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\[
\frac{dF}{dc} = \frac{K}{(1+(K-1)c)^2}
\]

Equation 5-44

5.3.4 Front Locations

Based on the relationship given in Equation 5-8, Mollaei predicts the position of a front with specific velocity \( v \), obtained from fractional flow analysis, in a given storage capacity \( C \) with the following piecewise equation:

\[
X_{Dfront}\bigg|_C = \begin{cases} 
1 & 0 < C < C^* \\
\left\{ \frac{t_D v}{dc} \right\} & C^* < C < 1
\end{cases}
\]

Equation 5-45

Where \( C^* \) is the storage capacity at which the front has just reached the producer at \( t_D \), as can be seen in Figure 5-9.

Mollaei uses the subscripts \( B \) and \( C \) to signify variables associated with the oil bank and displacing EOR fluid bank fronts, respectively, and no subscripts for the waterflood front. For clarity here, the subscripts \( B \) and \( C \) will still correspond to the oil bank and solvent fronts, while the subscript \( W \) will signify variables associated with the waterflood front. Therefore Equation 5-45 becomes:

\[
X_{Dwaterflood\ front}\bigg|_C = \begin{cases} 
1 & 0 < C < C^*_W \\
\left\{ t_D v_{AS} \left( \frac{dF}{dc} \right)_W \right\} & C^*_W < C < 1
\end{cases}
\]

Equation 5-46
Since Mollaei assumes piston-like displacement, he takes the overall oil saturation of a given \( C \) is a simple weighted average of the oil saturations in the unswept and swept areas of a storage capacity.\(^{44}\) Since he expresses the location of the fronts in terms of dimensionless distance, this calculation is relatively straightforward. The overall oil saturations for each storage capacity in a waterflood, which only has one front and is depicted in the upper plot of Figure 5-9, are given as follows:

\[
S_{o|c} = \begin{cases} 
 t_D v_{AS} \left( \frac{dF}{dc} \right)_W S_{oWF} + \left[ 1 - t_D v_{AS} \left( \frac{dF}{dc} \right)_W \right] S_{ol} & \text{if } C_W < C < 1 \\
S_{oWF} & \text{if } 0 < C < C_W
\end{cases}
\]

Equation 5-47

Here the saturations \( S_{ol} \) and \( S_{oWF} \) have been used in place of \( S_{oR} \) and \( S_{oF} \) in Mollaei’s diagram, for clarity in later cases where multiple fronts are present. These oil saturations correspond to those depicted in the fractional flow construction of Figure 5-1. I and II correspond to two regions of storage capacity; for capacities in I, the front has already reached the producer. The specific velocity of the waterflood front \( v_{AS} \) is determined from fractional flow as shown in Figure 5-1. Also note that \( \left( \frac{dF}{dc} \right)_W \) is used here instead of \( \left( \frac{dF}{dc} \right) \); it and its associated flow capacity function \( F_W \) are defined as follows:

\[
F_W = F(K = K_W) = \frac{1}{1 + \frac{1}{K_W} \left( \frac{1-c}{c} \right)}, \quad \left( \frac{dF}{dc} \right)_W = \frac{K_W}{(1+(K_W-1)c)^2}
\]

Equation 5-48

\(^{44}\) As a particular storage capacity \( C \) is considered here for the limiting case of the number of layers approaching infinity, the “area” corresponding to each saturation in \( C \) is best conceptualized as the dimensionless length of a line. The dimensionless location of the front is the “area” corresponding to the swept saturation and the difference between one and the front location is the “area” corresponding to the unswept saturation.
5.3.5 Predicting Average Reservoir Oil Saturation

To find the average oil saturation in the reservoir, which is key to measuring oil recovery from a displacement process using Mollaei’s model, Mollaei integrates the expression for oil saturation as a function of \( C \) given in Equation 5-47 over the entire reservoir. Remember that \( F \) and \( C \) are restricted to values between 0 and 1 and \( F(C = 0) = 0 \) and \( F(C = 1) = 1 \).\(^{45}\) The integral is divided into the subregions I and II of \( C \). Here, \( C^*_W \) is the same as \( C^* \) in Figure 5-9; again, the subscript is used to signify that the value corresponds to the waterflood front:

\[
\bar{S}_o = \int_{C=0}^{C=1} S_o|_C \, dC = \int_{C=0}^{C=C^*_W} S_o|_C \, dC + \int_{C=C^*_W}^{C=1} S_o|_C \, dC
\]

\[
= S_{oWF} C^*_W + \int_{C=C^*_W}^{C=1} \left[ (S_{oWF} - S_{ol}) v_{\Delta s} t_D \left( \frac{dF}{dC} \right)_W + S_{oWF} \right] dC
\]

\[
= S_{oWF} - (S_{oWF} - S_{ol}) [C^*_W + v_{\Delta s} t_D (1 - F^*_{WW})]
\]

Equation 5-49

where

\[ F^*_{WW} = F_W(C = C^*_W) \]

Equation 5-50

The need for the complexity of notation in Equation 5-50 becomes apparent later when considering the multi-front cases for the model. When \( C^*_W = 0 \), the font has not yet

\(^{45}\) These conditions hold for any F-C curve. For example, \( F_W(C = 0) = 0 \) and \( F_W(C = 1) = 1 \). The same will apply later for fronts associated with a CO\(_2\) flood.
broken through or is just breaking through at the producer \( (x_D = 1) \) whereas when \( C^*_W = 1 \), the waterflood front has completely swept the reservoir and \( \bar{S}_o = S_{oWF} \).

The expression for average oil saturation in the reservoir requires a definition of \( C^*_W \) as a function of \( t_D \). Mollaei derives this expression from the definition of \( C^*_W \) as the storage capacity at which the waterflood front has just reached \( x_D = 1 \):\(^{46}\)

\[
t_D v_{\Delta S} \left( \frac{dF}{dC} \right)_{W} \left. \right|_{C^*_W} = t_D v_{\Delta S} \frac{K_W}{(1 + (K_W - 1)C^*_W)^2} = 1
\]

Equation 5-51

Solving for \( C^*_W \), restricting its potential values between 0 and 1, and then substituting into Equation 5-50 yields:

\[
C^*_W = \begin{cases} 
0 & t_{DBT}^W \leq t_D \\
\frac{\sqrt{t_D v_{\Delta S} K_W - 1}}{K_W - 1} & t_{DBT}^W < t_D < t_{DSW}^W \\
1 & t_{DSW}^W \leq t_D
\end{cases}
\]

Equation 5-52

\[
F^*_{WW} = \begin{cases} 
0 & t_{DBT}^W \leq t_D \\
\frac{K_W [\sqrt{t_D v_{\Delta S} K_W - 1}]}{(K_W - 1)\sqrt{t_D v_{\Delta S} K_W}} & t_{DBT}^W < t_D < t_{DSW}^W \\
1 & t_{DSW}^W \leq t_D
\end{cases}
\]

Equation 5-53

Dimensionless times \( t_{DBT}^W \) and \( t_{DSW}^W \) represent the breakthrough at the producer and sweepout of the waterflood front in the reservoir. Mollaei derives these quantities

\(^{46}\) This description for \( C^*_W \) is the intuitive definition, but is not strictly true before breakthrough or after sweepout, when it is assumed that \( C^*_W \) is 0 or 1.
simply by solving for \( t_D \) when the expression \( \frac{\sqrt{t_D v_{AS} K_W - 1}}{K_W - 1} \) is exactly equal to 0 and 1, yielding:

Waterflood Breakthrough Time: \( t_D^{W BT} = \frac{1}{K_W v_{AS}} \)

Waterflood Sweepout Time: \( t_D^{SW} = \frac{K_W}{v_{AS}} \)

Equation 5-54

The relatively basic expressions described above are used to calculate the average oil saturation in a reservoir during a waterflood as a function of dimensionless time. Estimation of the Koval factor \( K_W \) is discussed later. For \( \mathrm{CO}_2 \) flooding, reasoning similar to that which is discussed in this section applies, but the resulting analytical expressions become more complicated due to the presence of multiple fronts.

5.4 Modeling a \( \mathrm{CO}_2 \) Flood

5.4.1 Base Case: Three Fronts Present and No Front Merging

Mollaei model also can be used predict displacement behavior for a generic isothermal EOR process. The main difference between his modeling of waterflooding versus a generic EOR process is the assumption that two fronts, one corresponding to an oil bank and the other the injected fluid, are present in the reservoir in the latter case. As he also models a reservoir as uniformly saturated before an EOR process, his model will be extended here to account for incomplete waterflood sweep at the initiation of a \( \mathrm{CO}_2 \) flood. Incomplete waterflood sweep means that there will be three banks present in the
reservoir in the early stages of the CO$_2$ flood. It is assumed that the oil bank front has not merged with the waterflood front here; scenarios involving merging will be considered later. Figure 5-10 is similar to Mollaei’s plots in Figure 5-9 but illustrates a scenario where three fronts are present.$^{47}$

![Diagram of a reservoir with three fronts present](image)

**Figure 5-10:** F-C parameterization of a reservoir with three fronts present

Mollaei’s expressions for locations of the oil bank and solvent fronts for a given storage capacity $C$, as shown in the diagram above, follow the same form as was used in the previous section for the waterflood front:

---

$^{47}$ This is a schematic and does not represent the actual shapes of the fronts. For actual diagrams of the fronts, see example calculations in chapter seven.
\[
x_{D_{\text{oil bank front}}} = \begin{cases} 
1 & 0 < C < C_B^* \\
t_{DCO_2}v_B \left(\frac{dF}{dC}\right)_B & C_B^* < C < 1
\end{cases}
\]

\[
x_{D_{\text{solvent front}}} = \begin{cases} 
1 & 0 < C < C_C^* \\
t_{DCO_2}v_C \left(\frac{dF}{dC}\right)_C & C_C^* < C < 1
\end{cases}
\]

Equation 5-55

Note the subscript \( C \) denotes “solvent front” in the second expression of Equation 5-55 for the specific velocity, the derivative of the F-C function, and the storage capacity at which the front is reaching the producer. This notation is consistent with Mollaei’s. The specific velocities \( v_B = v_{oB} \) and \( v_C \) correspond to the oil bank front and the solvent front; these are found using fractional flow-based calculations as discussed earlier and are depicted graphically in Figure 5-4. Oil saturations depicted in Figure 5-10 also correspond to those labeled on in the fractional flow construction of Figure 5-4. The dimensionless time \( t_{DCO_2} \) is the amount of solvent-water mixture injected since the switch from waterflooding to CO\(_2\) flooding, given at reservoir conditions and measured as a proportion of reservoir pore volumes.

As Mollaei considers EOR and waterflooding as completely separate scenarios, he implicitly assumes each has its own definition of dimensionless time. This time in each scenario is measured as the amount of injected fluid in reservoir pore volumes since the initiation of the process in question. With waterflood and CO\(_2\) flood fronts alike present in the reservoir for the three-front case considered here, it is necessary to relate
$t_{D\text{CO}_2}$ to the dimensionless time $t_D$ since the start of the waterflood. The latter measures
the total dimensionless time (reservoir barrels of fluid injected) since the switch from
primary recovery to displacement processes:

$$t_{D\text{CO}_2} = t_D - t_{D,ST}$$

Equation 5-56

Where $t_{D,ST}$ is the dimensionless time at which an operator switches from a waterflood to
a CO$_2$ flood. $t_{D\text{CO}_2}$ will also be used in definitions below, but note that it is a function of
$t_D$ as defined above.

The derivatives in Equation 5-55 correspond to F-C functions particular to the oil
bank and solvent fronts:

$$F_B = F(K = K_B) = \frac{1}{1 + \frac{1-C}{K_B C}}; \quad F_C = F(K = K_C) = \frac{1}{1 + \frac{1-C}{K_C C}}$$

Equation 5-57

$$\left( \frac{dF}{dC} \right)_B = \frac{K_B}{(1+(K_B-1)C)^2}; \quad \left( \frac{dF}{dC} \right)_C = \frac{K_C}{(1+(K_C-1)C)^2}$$

Equation 5-58

Methods for estimation of the oil bank front and solvent front Koval factors $K_B$ and $K_C$
are discussed in Appendix D.

The storage capacities $C_B^*$ and $C_C^*$, corresponding to the oil bank and solvent fronts
and appearing in Equation 5-55, are defined in a very similar manner as is $C_W^*$ in
Equation 5-52:
The flow capacities \( F_{B}^* = F_B(C = C_B^*) \) and \( F_{C}^* = F_C(C = C_C^*) \) are also similarly defined:

\[
C_B^* = \begin{cases} 
0 & t_{DBT}^B \leq t_D \\
\frac{\sqrt{D CO_2 v_B k_B^{-1}}}{K_B^{-1}} & t_{DBT}^B < t_D < t_{DSW}^B \\
1 & t_{DSW}^B \leq t_D
\end{cases}
\]

\[
C_C^* = \begin{cases} 
0 & t_{DBT}^C \leq t_D \\
\frac{\sqrt{D CO_2 v_C k_C^{-1}}}{K_C^{-1}} & t_{DBT}^C < t_D < t_{DSW}^C \\
1 & t_{DSW}^C \leq t_D
\end{cases}
\]

Equation 5-59

Mollaei derives the breakthrough and sweepout times for the oil bank front and
the solvent front using the same logic as was employed to derive Equation 5-54 for the
waterflood front. Shifting these times so that they are measured in the dimensionless
time \( t_D \) that is common to both displacement processes by using the definition in
Equation 5-56 yields:

Oil Bank Front Breakthrough Time: \( t_{DBT}^B = \frac{1}{k_B v_B} + t_{DST} \)
Oil Bank Front Sweepout Time: \[ t_{D,SW}^{B} = \frac{K_B}{v_B} + t_{D,ST} \]

Solvent Front Breakthrough Time: \[ t_{D,BT}^{C} = \frac{1}{k_C v_C} + t_{D,ST} \]

Solvent Front Sweepout Time: \[ t_{D,SW}^{C} = \frac{K_C}{v_C} + t_{D,ST} \]

There are four regions of \( C \) in Figure 5-10 labeled I through IV, each with a different expression for oil saturation at a given \( C \). Again, since piston-like displacement is assumed, overall oil saturation at a particular \( C \) in each region is simply a weighted average of the four saturations \( S_{oI} \), \( S_{oWF} \), \( S_{oB} \), and \( S_{oF} \) using the dimensionless positions of the three fronts present in the reservoir. The equations for \( S_{o|c} \) are given below in Table 5-2.

<table>
<thead>
<tr>
<th>Region</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV: ( C_{W} &lt; C &lt; 1 )</td>
<td>( S_{o</td>
</tr>
<tr>
<td>III: ( C_{B} &lt; C &lt; C_{W} )</td>
<td>( S_{o</td>
</tr>
<tr>
<td>II: ( C_{C} &lt; C &lt; C_{B} )</td>
<td>( S_{o</td>
</tr>
<tr>
<td>I: ( 0 &lt; C &lt; C_{C} )</td>
<td>( S_{o</td>
</tr>
</tbody>
</table>

---

48 These numerals are distinct from those used in the previous section for the waterfording case.
49 The inequalities describing the boundaries of each storage capacity region apply for Figure 5-10, but are not generally applicable for other \( t_D \).
The average oil saturation in the reservoir can be found by integrating the piecewise function for oil saturation at a given $C$ in Table 5-2 over all storage capacities in the reservoir, as was done in the waterflooding case in Equation 5-49. Before doing so, it is worth noting that the appearance of the schematic in Figure 5-10 is continuously changing with time. Depending on how many of the three fronts have achieved breakthrough at a particular $t_D$, some of the storage capacity regions listed in Table 5-2 may not be present. Figure 5-11 illustrates how the schematic varies with time, assuming all three fronts are present at some $C$ in the reservoir. Case (4) corresponds to the schematic in Figure 5-10.

Figure 5-11: Three-front CO2 flood cases (no merging)
The integration of the piecewise function in Table 5-2 to find average oil saturation in a reservoir therefore requires different limits of integration for each of the cases depicted in Figure 5-11. These expressions for average oil saturation for cases (1) through (4) are derived below in Table 5-3. In computing similar types of integrals, Mollaei notes that \( \int_{c=0}^{c=1} \left( \frac{df}{dc} \right) dc = 1 \) for any F-C function. He also defines a few variables to aid in the evaluation of the integrals; these generally cancel using algebraic manipulations to achieve the final expression for each case as listed in the table. These variables are similar to \( F_{Ww}^* \) defined in Equation 5-50 as well as \( F_{Bb}^* \) and \( F_{Cc}^* \):

\[
F_{Cc}^* = F_C(C = C_{Ww}) \quad F_{Bw}^* = F_B(C = C_{Ww}) \quad F_{Cc}^* = F_C(C = C_B^*)
\]

Equation 5-61

Table 5-3: Average oil saturation in a reservoir with three fronts and no merging

\[
(1) \quad \bar{S}_o = \int_{c=0}^{c=1} S_o dc \quad (\text{Region IV})
\]

\[
\bar{S}_o = t_{DCO_2} v_C S_{OF} \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_c dc + t_{DCO_2} S_{OB} \left[ v_B \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_B dc - v_C \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_C dc \right]
\]

\[
+ S_{OWF} \left[ t_D v_{\Delta S} \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_W dc - t_{DCO_2} v_B \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_B dc \right]
\]

\[
+ S_{ot} \left[ \int_{c=0}^{c=1} dc - t_D v_{\Delta S} \int_{c=0}^{c=1} \left( \frac{df}{dc} \right)_W dc \right]
\]

\[
= t_{DCO_2} v_C S_{OF} + t_{DCO_2} S_{OB} (v_B - v_C) + S_{OWF} (t_D v_{\Delta S} - t_{DCO_2} v_B) + S_{ot} (1 - t_D v_{\Delta S})
\]
Table 5-3: Average oil saturation in a reservoir with three fronts and no merging (continued)

\[
\bar{S}_o = \int_{C=0}^{C=C_W} S_o dC + \int_{C=C_W}^{C=1} S_o dC \quad \text{(Regions III and IV)}
\]

\[
\bar{S}_o = t_{DCO_2} v_C S_{oF} \left[ \int_{C=0}^{C=C_W} \left( \frac{dF}{dC} \right) \frac{dC}{dC} + t_{DCO_2} S_{oB} \left[ \int_{C=0}^{C=C_W} \left( \frac{dF}{dC} \right) \frac{dC}{dC} \right] \right]
\]

\[
+ S_{oWF} \left[ \int_{C=0}^{C=C_W} dC - t_{DCO_2} v_B \left[ \int_{C=0}^{C=C_W} \left( \frac{dF}{dC} \right) \frac{dC}{dC} \right] \right]
\]

\[
+ t_{DCO_2} v_C S_{oF} \left[ \int_{C=C_W}^{C=1} \left( \frac{dF}{dC} \right) \frac{dC}{dC} + t_{DCO_2} S_{oB} \left[ \int_{C=C_W}^{C=1} \left( \frac{dF}{dC} \right) \frac{dC}{dC} \right] \right]
\]

\[
+ S_{oWF} \left[ t_D v_{\Delta S} \left[ \int_{C=C_W}^{C=1} \left( \frac{dF}{dC} \right) \frac{dC}{dC} - t_{DCO_2} v_B \left[ \int_{C=C_W}^{C=1} \left( \frac{dF}{dC} \right) \frac{dC}{dC} \right] \right] \right]
\]

\[
+ S_{ol} \left[ \int_{C=C_W}^{C=1} dC - t_D v_{\Delta S} \left[ \int_{C=C_W}^{C=1} \left( \frac{dF}{dC} \right) \frac{dC}{dC} \right] \right]
\]

\[
= t_{DCO_2} v_C S_{oF} + t_{DCO_2} S_{oB} (v_B - v_C)
\]

\[
+ S_{oWF} \left[ C_W^* + t_D v_{\Delta S} (1 - F_{W_W}) - t_{DCO_2} v_B \right] + S_{ol} \left[ 1 - C_W^* - t_D v_{\Delta S} (1 - F_{W_W}) \right]
\]
Table 5-3: Average oil saturation in a reservoir with three fronts and no merging
(continued)

\[ S_o = \int_{C=0}^{C=C_B} S_o dC + \int_{C=C_B}^{C=C_W} S_o dC + \int_{C=C_W}^{C=1} S_o dC \quad \text{(Regions II, III and IV)} \]

\[ \tilde{S}_o = t_{DCO_2} v_C S_{oF} \int_{C=0}^{C=C_B^*} \left( \frac{dF}{dC} \right)_C dC + S_{oB} \int_{C=C_B^*}^{C=C_W^*} \left( \frac{dF}{dC} \right)_C dC \]

\[ \quad + t_{DCO_2} v_C S_{oF} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_C dC \]

\[ \quad + t_{DCO_2} S_{oB} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_B dC - t_{DCO_2} v_C \int_{C=C_B^*}^{C=C_W^*} \left( \frac{dF}{dC} \right)_C dC \]

\[ \quad + S_{oWF} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_B dC \]

\[ \quad + t_{DCO_2} v_C S_{oF} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_B dC \]

\[ \quad + t_{DCO_2} S_{oB} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_B dC - t_{DCO_2} v_C \int_{C=C_B^*}^{C=C_W^*} \left( \frac{dF}{dC} \right)_C dC \]

\[ \quad + S_{oWF} \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_B dC \]

\[ \quad + S_{ol} \left( \int_{C=C_W^*}^{C=C_W} \left( \frac{dF}{dC} \right)_W dC \right) \]

\[ = t_{DCO_2} v_C S_{oF} + S_{oB} C_B^* + t_{DCO_2} S_{oB} \left[ v_B (1 - F_{BB}^*) - v_C \right] \]

\[ + S_{oWF} \left[ (C_W^* - C_B^*) + t_{DCO_2} v_B \left( F_{BB}^* - 1 \right) + t_D v_{\Delta S} (1 - F_{WW}^*) \right] + S_{ol} \left[ 1 - C_W^* \right] - t_D v_{\Delta S} (1 - F_{WW}^*) \]
Table 5-3: Average oil saturation in a reservoir with three fronts and no merging (continued)

\[
\tilde{S}_o = \int_{C=0}^{C=C^*_B} S_o \, dC + \int_{C=C^*_B}^{C=C^*_W} S_o \, dC + \int_{C=C^*_W}^{C=1} S_o \, dC \quad \text{(Regions I, II, III and IV)}
\]

\[
\begin{align*}
\tilde{S}_o &= S_{oF} \int_{C=0}^{C=C^*_B} \, dC + t_{DCO_2} v_C S_{oF} \int_{C=C^*_B}^{C=C^*_C} \left( \frac{dF}{dC} \right)_C \, dC \\
&+ S_{oB} \left[ \int_{C=C^*_C}^{C=C^*_B} \, dC - t_{DCO_2} v_C \int_{C=C^*_C}^{C=C^*_B} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
&+ t_{DCO_2} v_C S_{oF} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_C \, dC \\
&+ t_{DCO_2} S_{oB} \left[ \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=C^*_C}^{C=C^*_B} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
&+ t_{DCO_2} v_C S_{oF} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_C \, dC + t_{DCO_2} S_{oB} \left[ \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=C^*_C}^{C=C^*_B} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
&+ t_{DCO_2} v_C S_{oF} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_C \, dC + t_{DCO_2} S_{oB} \left[ \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=C^*_C}^{C=C^*_B} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
&+ S_{oWF} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_W \, dC - t_{DCO_2} v_B \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_B \, dC \\
&+ S_{oWF} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_W \, dC - t_{DCO_2} v_B \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_B \, dC \\
&+ S_{oI} \int_{C=C^*_C}^{C=C^*_W} \left( \frac{dF}{dC} \right)_W \, dC \\
&= S_{oF} \left[ C^*_C + t_{DCO_2} v_C \left( 1 - F_{CC}^* \right) \right] + S_{oB} \left( C^*_B - C^*_C \right) \\
&+ t_{DCO_2} S_{oB} \left[ v_B \left( 1 - F_{BB}^* \right) + v_C \left( F_{CC}^* - 1 \right) \right] \\
&+ S_{oWF} \left[ \left( C^*_W - C^*_B \right) + t_D v_{\Delta S} \left( 1 - F_{WW}^* \right) + t_{DCO_2} v_B \left( F_{BB}^* - 1 \right) \right] + S_{oI} \left[ 1 - C^*_W - t_D v_{\Delta S} \left( 1 - F_{WW}^* \right) \right]
\end{align*}
\]

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Thus Table 5-3 provides analytical expressions for estimating the reservoir’s average oil saturation during a CO\textsubscript{2} flood when the waterflood front is still present, but has not merged with the oil bank front. While these some of these equations are a bit long, they are simple enough to implement in an Excel model.

5.4.2 After Waterflood Front Sweepout

Mollaei derives analytical expressions for average reservoir oil saturation that apply for a CO\textsubscript{2} flood when the initial oil saturation in the reservoir is uniform. These equations will apply to instances where the waterflood front has attained sweepout. His integrations are repeated here using notation described in previous sections, for clarity. Figure 5-12 outlines the possible cases that could occur with only the two fronts corresponding to the CO\textsubscript{2} flood present in the reservoir. The numerals still relate to storage capacity regions whose oil saturations are characterized by the equations in Table 5-2, except here the boundaries of each region differ.
Figure 5-12: Two-front CO₂ flood cases

Table 5-4 lists the expressions for average reservoir oil saturation for cases (1s) through (5s) as depicted in Figure 5-12.
Table 5-4: Average oil saturation in a reservoir with two CO₂ flood fronts

\( \bar{S}_o = \int_{C=0}^{C=1} S_o \, dC \)  \hspace{1cm} (Region III)

\[
\bar{S}_o = t_{DCO_2} v_C S_{OF} \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right) C \, dC + t_{DCO_2} S_{OB} \left[ v_B \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC \right] \\
+ S_{OWF} \left[ \int_{C=0}^{C=1} dC - t_{DCO_2} v_B \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC \right]
\]

= \sum \left[ t_{DCO_2} v_C S_{OF} + t_{DCO_2} S_{OB} (v_B - v_C) + S_{OWF} (1 - t_{DCO_2} v_B) \right]

\[
\bar{S}_o = \int_{C=C_B}^{C=C_B} S_o \, dC + \int_{C=C_B}^{C=1} S_o \, dC \)  \hspace{1cm} (Regions II and III)

\[
\bar{S}_o = t_{DCO_2} v_C S_{OF} \int_{C=0}^{C=C_B} \left( \frac{dF}{dC} \right)_C \, dC + S_{OB} \left[ v_B \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC \right] \\
+ t_{DCO_2} v_C S_{OF} \int_{C=C_B}^{C=C_B} \left( \frac{dF}{dC} \right)_C \, dC \\
+ t_{DCO_2} S_{OB} \left[ v_B \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC - v_C \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC \right] \\
+ S_{OWF} \left[ \int_{C=C_B}^{C=C_B} dC - t_{DCO_2} v_B \int_{C=C_B}^{C=C_B} \left( \frac{dF}{dC} \right)_B \, dC \right]
\]

= \sum \left[ t_{DCO_2} v_C S_{OF} + S_{OB} C_B + t_{DCO_2} S_{OB} [v_B (1 - F_B) - v_C] + S_{OWF} [(1 - C_B) - t_{DCO_2} v_B (1 - F_B)] \right]
Table 5-4: Average oil saturation in a reservoir with two CO₂ flood fronts (continued)

\( \bar{S}_o = S_{oF} \int_{C=0}^{C=C_B^o} S_o dC + \int_{C=C_B^o}^{C=C_C^o} S_o dC + \int_{C=C_C^o}^{C=1} S_o dC \quad \text{(Regions I, II and III)} \)

\[
\bar{S}_o = S_{oF} \int_{C=0}^{C=C_B} dC + t_{DCO2} v_C S_{oF} \int_{C=C_B}^{C=C_C} \left( \frac{dF}{dC} \right)_C dC \\
+ S_{oB} \int_{C=C_C}^{C=1} dC - t_{DCO2} v_C \int_{C=C_C}^{C=1} \left( \frac{dF}{dC} \right)_C dC \\
+ t_{DCO2} v_C S_{oF} \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B dC \\
+ t_{DCO2} S_{oB} \int_{C=C_B}^{C=1} v_B \left( 1 - F_{BB}^* \right) dC - v_C \int_{C=C_B}^{C=1} \left( \frac{dF}{dC} \right)_B dC \\
+ S_{oWF} \int_{C=C_B}^{C=1} dC - t_{DCO2} v_B \left( 1 - F_{BB}^* \right) dC \]

\( S_{oF} \left[ C_C^* + t_{DCO2} v_C \left( 1 - F_{CC}^* \right) \right] + S_{oB} \left( C_B^* - C_C^* \right) \\
+ t_{DCO2} S_{oB} \left[ v_B \left( 1 - F_{BB}^* \right) + v_C \left( F_{CC}^* - 1 \right) \right] \\
+ S_{oWF} \left[ \left( 1 - C_B^* \right) - t_{DCO2} v_B \left( 1 - F_{BB}^* \right) \right] \)

(4s) \( \bar{S}_o = \int_{C=0}^{C=C_C^*} S_o dC + \int_{C=C_C^*}^{C=1} S_o dC \quad \text{(Regions I and II)} \)

\[
\bar{S}_o = S_{oF} \int_{C=0}^{C=C_C^*} dC + t_{DCO2} v_C S_{oF} \int_{C=C_C^*}^{C=1} \left( \frac{dF}{dC} \right)_C dC \\
+ S_{oB} \int_{C=C_C^*}^{C=1} dC - t_{DCO2} v_C \int_{C=C_C^*}^{C=1} \left( \frac{dF}{dC} \right)_C dC \]

\[
S_{oF} C_C^* + t_{DCO2} v_C S_{oF} \left( 1 - F_{CC}^* \right) + S_{oB} \left[ \left( 1 - F_{CC}^* \right) - t_{DCO2} v_C \left( 1 - F_{CC}^* \right) \right] \]

(5s) \( \bar{S}_o = \int_{C=0}^{C=1} S_o dC \quad \text{(Region I)} \)

\[
\bar{S}_o = S_{oF} \int_{C=0}^{C=1} dC \]

\[
= S_{oF} \]
5.4.3 Conditions for Front Merging in a Layer of the Reservoir

If an operator switches a reservoir from a waterflood to a CO$_2$ flood relatively early in the waterflood and/or the oil bank front moves much faster than the waterflood front, it is hypothesized that the fronts can merge. The reasoning behind this hypothesis is discussed earlier in the context of fractional flow theory for a one-dimensional medium. The analytical condition for the merging of banks in the F-C parameterization of a reservoir is derived here.

At any dimensionless time $t_D$, the following relationship can be used to determine the location at which the oil bank front is just merging with the waterflood front:

\[
t_D v_{\Delta S} \left( \frac{dF}{dC} \right)_W = t_{DCO_2} v_B \left( \frac{dF}{dC} \right)_B
\]

Equation 5-62

The particular value of $\zeta$ at which the fronts merge will be called $C_x$. If $C_x$ has a nonphysical value, which could be greater than 1 or less than 0, the fronts could be fully merged or will not have merged, respectively.\(^{50}\) To find $C_x$, the definitions of the derivatives $\left( \frac{dF}{dC} \right)_W$ and $\left( \frac{dF}{dC} \right)_B$ and $t_{DCO_2}$ are substituted into Equation 5-62:

\[
t_D v_{\Delta S} \frac{K_W}{(1+K_W-1/C_x)^2} = (t_D + t_{DST}) v_B \frac{K_B}{(1+(K_B-1/C_x)^2}
\]

Equation 5-63

Rearranging this equation yields:

\(^{50}\) This is not to say that the fronts have partially merged if $0 < C_x < 1$. See below.
\[
\frac{1+(K_W-1)C_X}{1+(K_B-1)C_X} = \frac{t_D v_{AS}}{\sqrt{(t_D + t_{D,ST})v_B K_B}} = \sqrt{A}
\]

Equation 5-64

Where \(\sqrt{A}\) is used as shorthand for the right hand side of the equation in order to make
the final expression for \(C_X\) more readable. Solving for \(C_X\) yields the following
relationship as a function of dimensionless time:

\[
C_X(t_D) = \frac{1 - \sqrt{A}}{\sqrt{A}(K_B-1)-(K_W-1)}
\]

Equation 5-65

The oil bank front from the CO\(_2\) flood has partially merged with the waterflood
front at some storage capacity \(C\) in the medium if \(C_W \leq C_X < 1\). With partial merging,
there are some values of \(C\) below \(C_X\) where the fronts have merged whereas there are
other values above \(C_X\) where the fronts have not merged.

If \(C_X \geq 1\) and \(C_W^* < 1\), the waterflood front has not fully swept the reservoir and
oil bank front has fully merged with it. This situation is depicted in Figure 5-13.
Figure 5-13: Fully merged fronts and incomplete waterflood sweep

For both partial and full merging, there is a new region of storage capacity that replaces region III listed in Table 5-2:

$$III_m: \left. S_o \right|_c = t_{DCO_2} v_c \left( \frac{dF}{dc} \right)_c S_{oF} + \left[ t_D v_{WS} \left( \frac{dF}{dc} \right)_W - t_{DCO_2} v_c \left( \frac{dF}{dc} \right)_c \right] S_{oB} +$$

$$\left[ 1 - t_D v_{WS} \left( \frac{dF}{dc} \right)_W \right] S_{oi}$$

Equation 5-66

Also note that once merging occurs, $C_B^* = C_W^*$. Region $III_m$ spans from $C_W^* < C < C_X$.

5.4.4 Partial Front Merging

There are three types of possible scenarios that can occur with partial merging of the oil bank and waterflood fronts. These outcomes are illustrated below in Figure 5-14,
which classifies outcomes based on how many fronts have achieved breakthrough at the producer.

Figure 5-14: Three-front CO$_2$ flood cases (partial merging)

Once the waterflood front has swept out, the schematic of the reservoir appears as it does in (4s) of Figure 5-12. The progression of diagrams in Figure 5-14 does not necessarily depict how a flood will progress in a particular reservoir; the process may move from (1s) to (2s) in Figure 5-12 to (2p) in Figure 5-14 and finally to (4s) followed by (5s) in Figure 5-12.

The average oil saturation in the reservoir is obtained again by integration over the entire range of storage capacities in the reservoir. These integrations for cases (1p) through (3p) are outlined in Table 5-5, and rely on the following definitions:
\[ F_{C_x} = F_C(C = C_x) \quad F_{B_x} = F_B(C = C_x) \quad F_{W_x} = F_W(C = C_x) \]

Equation 5-67

Table 5-5: Average oil saturation in a reservoir with three fronts and partial merging

\[ 
\bar{S}_o = \int_{C=0}^{C=C_x} S_o dC + \int_{C=C_x}^{C=1} S_o dC \quad \text{(Regions IIIm and IV)}
\]

\[ 
\bar{S}_o = t_{DCO_2} v_C S_o F \int_{C=0}^{C=C_x} \left( \frac{dF}{dC} \right) C dC
\]

\[ + S_{oB} \left[ t_{Dv_\Delta S} \int_{C=0}^{C=C_x} \left( \frac{dF}{dC} \right)_W dC - t_{DCO_2} v_C \int_{C=0}^{C=C_x} \left( \frac{dF}{dC} \right) C dC \right]
\]

\[ + S_{oi} \left[ \int_{C=0}^{C=1} dC - t_{Dv_\Delta S} \int_{C=0}^{C=C_x} \left( \frac{dF}{dC} \right)_W dC \right] + t_{DCO_2} v_C S_o F \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right) C dC
\]

\[ + t_{DCO_2} S_{oB} \left[ \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_B dC - t_{DCO_2} v_B \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right) B dC \right]
\]

\[ + S_{oWF} \left[ t_{Dv_\Delta S} \int_{C=0}^{C=C_x} \left( \frac{dF}{dC} \right)_W dC - t_{DCO_2} v_B \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_B dC \right]
\]

\[ + S_{oi} \left[ \int_{C=C_x}^{C=1} dC - t_{Dv_\Delta S} \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_W dC \right]
\]

\[ = S_{oB} \left( t_{Dv_\Delta S} F_{W_x} \right) + t_{DCO_2} v_C S_o F + t_{DCO_2} S_{oB} \left[ v_B (1 - F_{B_x}) - v_C \right]
\]

\[ + S_{oWF} \left[ t_{Dv_\Delta S} (1 - F_{W_x}) - t_{DCO_2} v_B (1 - F_{B_x}) \right] + S_{oi} (1 - t_{Dv_\Delta S}) \]
Table 5-5: Average oil saturation in a reservoir with three fronts and partial merging (continued)

(2p) \[
\bar{S}_o = \int_{c=0}^{c=c_W} S_o dC + \int_{c=c_W}^{c=c_x} S_o dC + \int_{c=c_x}^{c=1} S_o dC \quad \text{(Regions II, III, and IV)}
\]

\[
\bar{S}_o = t_{DCO_2} v_c S_{sof} \int_{c=0}^{c=c_W} \left( \frac{dF}{dC} \right)_c dC + S_{ob} \int_{c=c_W}^{c=c_x} \left( \frac{dF}{dC} \right)_w dC + t_{DCO_2} v_c \int_{c=c_x}^{c=1} \left( \frac{dF}{dC} \right)_w dC \\
+ S_{oi} \left[ \int_{c=c_W}^{c=c_x} dC - t_{Dv_{AS}} \int_{c=c_W}^{c=c_x} \left( \frac{dF}{dC} \right)_w dC \right] + t_{DCO_2} v_c S_{sof} \int_{c=c_x}^{c=1} \left( \frac{dF}{dC} \right)_c dC \\
+ t_{DCO_2} v_B \int_{c=c_x}^{c=1} \left( \frac{dF}{dC} \right)_B dC - t_{DCO_2} v_B \int_{c=c_x}^{c=1} \left( \frac{dF}{dC} \right)_w dC \\
+ S_{oWF} \left[ t_{Dv_{AS}} \int_{c=c_W}^{c=c_x} \left( \frac{dF}{dC} \right)_w dC - t_{DCO_2} v_B \int_{c=c_W}^{c=c_x} \left( \frac{dF}{dC} \right)_w dC \right] \\
+ S_{oi} \left[ \int_{c=c_x}^{c=1} dC - t_{Dv_{AS}} \int_{c=c_W}^{c=c_x} \left( \frac{dF}{dC} \right)_w dC \right] \\
= t_{DCO_2} v_c S_{sof} + S_{ob} \left[ c_W + t_{Dv_{AS}} (F_{w_x} - F_{w_W}) \right] + t_{DCO_2} S_{ob} \left[ v_B (1 - F_{B_x}) - v_c \right] \\
+ S_{oWF} \left[ t_{Dv_{AS}} (1 - F_{w_x}) - t_{DCO_2} v_B (1 - F_{B_x}) \right] \\
+ S_{oi} (1 - t_{Dv_{AS}} + t_{Dv_{AS}} (F_{w_W} - c_W))
\]
Table 5-5: Average oil saturation in a reservoir with three fronts and partial merging (continued)

\[
\bar{S}_o = \int_{C=0}^{C=C_c^*} S_o \, dC + \int_{C=C_c^*}^{C=C_W^*} S_o \, dC + \int_{C=C_W^*}^{C=C_x} S_o \, dC + \int_{C=C_x}^{C=1} S_o \, dC \quad \text{(Regions I, II, III, and IV)}
\]

\[
\bar{S}_o = S_{oF} \int_{C=0}^{C=C_c^*} dC + t_{DCO_2} v_c S_{oF} \int_{C=C_c^*}^{C=C_W^*} \left( \frac{dF}{dC} \right)_c \, dC \\
+ S_{oB} \left[ \int_{C=C_W^*}^{C=C_c^*} dC - t_{DCO_2} v_c \int_{C=C_c^*}^{C=C_W^*} \left( \frac{dF}{dC} \right)_c \, dC \right] \\
+ t_{DCO_2} v_c S_{oF} \int_{C=C_W^*}^{C=C_x} \left( \frac{dF}{dC} \right)_c \, dC
\]

\[
+ S_{oB} \left[ v_B \int_{C=C_W^*}^{C=C_x} \left( \frac{dF}{dC} \right)_B \, dC - v_c \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
+ t_{DCO_2} S_{oB} \left[ v_B \int_{C=C_W^*}^{C=C_x} \left( \frac{dF}{dC} \right)_B \, dC - v_c \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_C \, dC \right] \\
+ S_{oWF} \left[ t_D v_{\Delta S} \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_W \, dC - t_{DCO_2} v_B \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_B \, dC \right] \\
+ S_{oi} \left[ t_D v_{\Delta S} \int_{C=C_x}^{C=1} \left( \frac{dF}{dC} \right)_W \, dC \right]
\]

\[
= S_{oF} \left[ C_c^* + t_{DCO_2} v_c (1 - F_{C_c}^*) \right] + S_{ob} \left[ (C_W^* - C_c^*) + t_D v_{\Delta S} (F_{W_x} - F_{W_W}^*) \right] \\
+ t_{DCO_2} S_{oB} \left[ v_B (1 - F_{B_y}) - v_c (1 - F_{C_c}^*) \right] \\
+ S_{oWF} \left[ t_D v_{\Delta S} (1 - F_{W_x}) - t_{DCO_2} v_B (1 - F_{B_x}) \right] \\
+ S_{oi} \left( 1 - t_D v_{\Delta S} + t_D v_{\Delta S} F_{W_W}^* - C_W^* \right)
\]
5.4.5 Full Front Merging

There are three potential outcomes associated with the oil bank front and waterflood front merging for all relevant $C$ in the reservoir ($C_w^* \leq C \leq 1$). Figure 5-15 illustrates these possibilities. Note that region IV is no longer present, as the storage capacity $C_X$ at which the oil bank and waterflood fronts intersect is no longer less than 1.

Figure 5-15: Three-front CO$_2$ flood cases (full merging)

Again, a reservoir characterized by the schematic in (3f) at a given $t_D$ will look more like (4s) in Figure 5-12 once the waterflood front has attained sweepout ($t_D > t_{D_{SW}}^w$).

Table 5-6 outlines the integrations and expressions for determining the average reservoir oil saturation when full merging has occurred.
Table 5-6: Average oil saturation in a reservoir with three fronts and full merging

(1f) \[ \bar{S}_o = \int_{C=0}^{C=1} S_o dC \] (Region III_m)

\[ \bar{S}_o = t_{DCO_2} v_C S_{OF} \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_C dC \]

\[ + S_{ob} \left[ t_D v_{\Delta S} \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_W dC - t_{DCO_2} v_C \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_C dC \right] \]

\[ + S_{ol} \left[ \int_{C=0}^{C=1} dC - t_D v_{\Delta S} \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_W dC \right] \]

\[ = t_{DCO_2} v_C S_{OF} + S_{ob} \left( t_D v_{\Delta S} - t_{DCO_2} v_C \right) + S_{ol} \left( 1 - t_D v_{\Delta S} \right) \]

(2f) \[ \bar{S}_o = \int_{C=0}^{C=C^*_W} S_o dC + \int_{C=C^*_W}^{C=1} S_o dC \] (Regions II and III_m)

\[ \bar{S}_o = t_{DCO_2} v_C S_{OF} \int_{C=0}^{C=C^*_W} \left( \frac{dF}{dC} \right)_C dC + S_{ob} \left[ \int_{C=C^*_W}^{C=1} dC - t_{DCO_2} v_C \int_{C=0}^{C=1} \left( \frac{dF}{dC} \right)_C dC \right] \]

\[ + t_{DCO_2} v_C S_{OF} \int_{C=C^*_W}^{C=1} \left( \frac{dF}{dC} \right)_C dC \]

\[ + S_{ob} \left[ t_D v_{\Delta S} \int_{C=C^*_W}^{C=1} \left( \frac{dF}{dC} \right)_W dC - t_{DCO_2} v_C \int_{C=C^*_W}^{C=1} \left( \frac{dF}{dC} \right)_C dC \right] \]

\[ + S_{ol} \left[ \int_{C=C^*_W}^{C=1} dC - t_D v_{\Delta S} \int_{C=C^*_W}^{C=1} \left( \frac{dF}{dC} \right)_W dC \right] \]

\[ = t_{DCO_2} v_C S_{OF} + S_{ob} \left[ C^*_W + t_D v_{\Delta S} \left( 1 - F^*_W \right) - t_{DCO_2} v_C \right] \]

\[ + S_{ol} \left[ \left( 1 - C^*_W \right) - t_D v_{\Delta S} \left( 1 - F^*_W \right) \right] \]
Table 5-6: Average oil saturation in a reservoir with three fronts and full merging (continued)

\[
\bar{S}_o = \int_{c=0}^{c=C_c^*} S_o \, dc + \int_{c=C_c^*}^{c=C_w^*} S_o \, dc + \int_{c=C_w^*}^{c=1} S_o \, dc \quad \text{(Regions I, II and III)}
\]

\[
\bar{S}_o = S_{oF} \int_{c=0}^{c=C_c^*} dC + t_{DCO_2} v_c S_{oF} \int_{c=C_c^*}^{c=C_w^*} \left( \frac{dF}{dc} \right)_c \, dc + S_{oB} \int_{c=C_c^*}^{c=C_w^*} dC - t_{DCO_2} v_c \int_{c=C_c^*}^{c=C_w^*} \left( \frac{dF}{dc} \right)_c \, dc + S_{oB} t_D v_{\Delta S} \int_{c=C_w^*}^{c=1} \left( \frac{dF}{dc} \right)_w \, dc - t_{DCO_2} v_c \int_{c=C_w^*}^{c=1} \left( \frac{dF}{dc} \right)_c \, dc + S_{oI} \int_{c=C_w^*}^{c=1} dC - t_D v_{\Delta S} \int_{c=C_w^*}^{c=1} \left( \frac{dF}{dc} \right)_w \, dc \right]
\]

\[
= S_{oF} \left[ C_c^* + t_{DCO_2} v_c (1 - F_c^{*c}) \right] + S_{oB} \left[ (C_w^* - C_c^*) + t_D v_{\Delta S} (1 - F_w^{*w}) + t_{DCO_2} v_c (F_c^{*c} - 1) \right] + S_{oI} \left[ (1 - C_w^*) - t_D v_{\Delta S} (1 - F_w^{*w}) \right]
\]

5.5 PREDICTING RECOVERY; ASSUMPTIONS ABOUT FORMATION VOLUME FACTOR

5.5.1 Introduction

The expressions defined in previous sections for average oil saturation are crucial to estimating recovery from waterflooding and CO₂ flooding, but a few logistical hurdles remain before it is possible to use the model to estimate the economic viability of a reservoir. This section discusses how to calculate oil recovery and includes assumptions regarding the formation volume factor. It also summarizes a process for determining when to use each portion of the reservoir model described above. Estimating fluid
injection rates is a more involved subject matter, and is therefore discussed in the next chapter. Methods for determining Koval factors, $S_{oWF}$, and $S_{oF}$ are discussed in Appendix D.

In chapter four, it is assumed for primary recovery that the areal geometry of the reservoir is a square and that wells are drilled in an evenly-spaced square pattern, as depicted in Figure 4-4. It will also be assumed that when the operator switches to waterflooding for a reservoir, he undertakes a campaign of infill drilling that adds additional wells to the center of each pattern and around the outer perimeter of the reservoir. Thus the number of wells $N_w$ drilled in the reservoir increases by $(n_{pat,side} + 2)^2$. This drilling campaign creates five-spot well patterns. Figure 5-16 illustrates how this infill drilling would occur with 25 existing primary recovery patterns. Black dots indicate existing wells from primary recovery, which are converted to injectors, while red ones represent the infill wells, which serve as producers. Infill drilling leads to 36 five-spot patterns in the reservoir.
5.5.2 Recovery Efficiency, Oil Production, and the Formation Volume Factor

Recovery efficiency, or the proportion of the original oil in place (OOIP) extracted from a reservoir at a given time, is used to calculate oil production directly from estimated reservoir oil saturation. Mollaei (2011) defines recovery efficiency in his model as the difference between the uniform oil saturation before a displacement process and the current average reservoir oil saturation, all normalized by the initial oil saturation in the reservoir.

A more refined version of this recovery efficiency definition is required to use the model described above, as the oil formation volume factor and saturation in the reservoir change during primary recovery. It is imperative to account for oil recovered in STB to...
measure consistently across recovery methods. Here, recovery efficiency since initiation of a waterflood\(^{51}\) is defined as follows:

\[
E_{R_{displ}}(t_D) = \frac{S_{oWF} - \bar{S}_o(t_D)}{S_{oWF}}
\]

Equation 5-68

where \(S_{oWF}\) is the oil saturation at the start of waterflooding, and \(\bar{S}_o(t_D)\) is estimated using the expressions obtained through integration as discussed in previous sections. For fractional flow calculations as described in previous sections, \(S_{oi}\) is assumed to be equal to \(S_{oWF}\), the oil saturation at the start of waterflooding.

The amount of oil recovered from displacement methods, measured in reservoir barrels, is simply this “displacement method recovery efficiency” multiplied by the amount of oil in place at the start of a waterflood, also measured in reservoir barrels:

\[
N_{P_{displ,RB}}(t_D) = \left[E_{R_{displ}}(t_D)\right] \times (OIP_{WF,RB})
\]

Equation 5-69

The amount of oil recovered since waterflood initiation, measured in reservoir barrels, is simply \(N_{P_{displ,RB}}\) divided by the oil formation volume factor at the switch from primary recovery to waterflooding. Since it is assumed that the operator switches to waterflooding at the bubble point, the latter value is taken simply to be \(B_{oB}\):

\[
N_{P_{displ,STB}}(t_D) = \frac{N_{P_{displ,RB}}(t_D)}{B_{oB}}
\]

Equation 5-70

\(^{51}\) “displ” denotes that this recovery efficiency is for the displacement processes.
At any dimensionless time, the total oil recovered since the beginning of primary recovery, expressed in STB, is simply the following sum:

\[ N_{P_{total,STB}}(t_D) = N_{P_{primary,STB}} + N_{P_{displ,STB}}(t_D) \]

Equation 5-71

Where \( N_{P_{primary,STB}} \) is the total amount of oil produced from primary recovery, expressed in STB. From Equation 5-71, it is straightforward to calculate the total recovery efficiency for a reservoir:

\[ E_R(t_D) = \frac{N_{P_{total,STB}}(t_D)}{OOIP_{total,STB}} \]

Equation 5-72

Where \( OOIP_{total,STB} \) is the original oil in place before primary recovery begins, measured in STB.

A key assumption implicit in Equation 5-70 is that the oil formation volume factor remains constant at its bubble point value \( B_oB \) after the operator switches from primary recovery to a waterflood. This assumption is necessary from an accounting standpoint, as the model used to estimate \( S_o(t_D) \) as described above assumes that fluids in the reservoir are incompressible. In reality, a depleted reservoir usually experiences an increase in pressure at the beginning of a waterflood that in turn changes the compressibility of the resident fluid, which is reflected by a shift in the formation volume factor. This fill-up effect is especially relevant in solution gas drive reservoirs with pressures far below the bubble point at waterflood initiation, as they contain an initial gas
saturation that is highly compressible (Willhite 1986). Even if there is no trapped gas saturation at waterflood initiation, raising the reservoir pressure will change the oil formation volume factor, although the effect will be less pronounced. Since it is assumed that the operator switches to a waterflood when the average reservoir pressure is at the bubble point, there should not be a substantial trapped gas saturation present in the reservoir.

5.5.3 Piston-Like Displacement

The models discussed in this chapter assume piston-like displacement, which is an approximation of actual displacement behavior in permeable media. Relaxing this assumption would add substantial and unnecessary complexity to the model, which is not desirable as it would essentially require numerical simulation to predict oil production for tertiary recovery. Although piston-like displacement approximates actual displacement behavior, Mollaei’s model is still capable of predicting observed oil recoveries from displacement methods using few parameters compared to those needed for simulation. This simplicity makes it a great candidate for economic evaluations.

5.5.4 A Process Diagram for Using the Modified Displacement Model

The modeling of a CO₂ flood as described in section 5.4 involves many possible cases, each with its own formula for determining the average oil saturation in the reservoir. Each case is associated with a range of dimensionless times. Many of these cases have possible dimensionless times that overlap with those of other cases since it is
assumed that fronts can merge. To avoid confusion, Figure 5-17 below helps clarify the logic used to determine which case applies for a given dimensionless time.

Figure 5-17: Determining which schematic applies to a reservoir at a given $t_D$
Chapter 6: Estimating Injection Rates

6.1 INTRODUCTION

The model discussed in the previous chapter provides a framework for forecasting the performance of waterfloods and CO₂ floods. One noteworthy feature of this model is that it expresses recovery efficiency and oil production as a function of dimensionless time. This feature precludes one from using it in economic evaluations without information on fluid injection rates, which are used to convert dimensionless time to actual time. Therefore it is also necessary to estimate injection rates into a reservoir for these displacement methods in the context of the theory of the previous chapter.

This chapter focuses on deriving analytical expressions for estimating injection rates into a formation. One could alternatively use field data from similar projects to estimate injection rates, although the idiosyncrasies of a prospective reservoir could mean that this method would not be particularly useful. At the very least, injection rates from comparable reservoirs could be used to verify that estimations from analytical expressions are reasonable to the operator.

Methods for estimating the constraints on operating pressures of injection and production wells are also discussed here, as the pressure difference between wells is necessary to compute injection rates.
6.2 WATERFLOOD INJECTION RATES

The literature for estimating the performance of waterfloods is relatively mature; the goal of this section is to enhance existing methods to apply them to a model based on flow capacity and storage capacity.

Caudle and Witte (1959) conducted a series of laboratory experiments for a five-spot pattern and found a correlation between a conductance ratio \( \gamma \), the areal sweep efficiency \( E_A \) and the mobility ratio \( M \). This correlation is shown in Figure 6-1; note that it applies for mobility ratios from 0.1 to 10. The results of these experiments are used in analytical methods developed by Craig (1971) for predicting waterflood performance, as discussed by Willhite (1986). The conductance ratio captures changes in injectivity because of shifts in mobility of fluids in the reservoir as a waterflood progresses; it is defined as follows:

\[
\gamma = \frac{i}{i_b}
\]

Equation 6-1

where \( i \) is the injection rate at any time after the flood has begun for a medium of constant permeability and \( i_b \) is the base (initial) injection rate, which is a form of Darcy’s Law in radial coordinates given by Willhite (1986):

\[
i_b = \frac{3.541 k_o h (p_{wi} - p_{wp})}{\mu \left( \ln \left( \frac{d}{r_w} \right) - 0.619 \right)}
\]

Equation 6-2
where $d$ is the distance in feet between an injector and producer in a pattern, assumed to be a five-spot here (Willhite 1986):\footnote{Based on the infill drilling plan discussed in the previous chapter, the pattern area for a five-spot $A_{pat}$ in this model, expressed in acres, is the same as the drainage area of an individual well in primary recovery.}

\[
d = \frac{\sqrt{A_{pat} \left(\frac{43,560 \text{ ft}^2}{\text{acre}}\right)}}{\sqrt{2}}
\]

Equation 6-3

$\tau_w'$ is the effective wellbore radius, also in feet, and $p_{wi}$ and $p_{wp}$ are the bottomhole pressures in psi at an injector and producer, respectively. Permeability is in Darcies and $\mu$ is in centipoise. The effective wellbore radius captures the effects of the skin factor $s$, or near-wellbore damage, and is also related to the actual wellbore radius (Economides et al. 2012):

\[
\tau_w' = \tau_w e^{-s}
\]

Equation 6-4

Finally, Equation 6-2 assumes a single homogeneous layer of constant permeability.

If the operator desires to inject water into a homogeneous medium at a constant rate, Equation 6-2 can be rearranged to obtain an equation for determining the injector-producer pressure difference needed to attain that rate. Here it will be assumed that the operator wishes to maintain a constant injector-producer pressure drop; this parameter and others are used to calculate the time-varying injection rate into the formation.
The base injection rate is evaluated assuming the oil saturation at the start of the waterflood is used to calculate relative permeability. This oil saturation $S_{oilWF}$ of the unswept region of the medium is assumed to correspond to the endpoint of the oil relative permeability curve, so the oil permeability $k_o$ in Equation 6-2 is evaluated accordingly:\textsuperscript{53}

$$k_o = k k_{ro}^o; \quad k_{ro}^o = k_{ro}|_{S_{oilWF}}$$

Equation 6-5

\textsuperscript{53} This initial oil saturation at the start of waterflooding differs from the oil saturation at the beginning of primary recovery, since the tank model assumes that oil saturation decreases, albeit slightly, due to primary production. For the example calculation in the next chapter, the oil saturation decreases by 0.026 over the course of this recovery phase, decreasing from 0.8 to 0.774.
With $\Delta p = p_{wi} - p_{wp}$, the injection rate into a layer $l$ can be expressed as
follows:

$$i_l = (\gamma_i b)_l = \gamma_l \frac{3.541 k_l \alpha_{u_l} \Delta p}{\ln\left(\frac{d}{r_w}\right)^{0.619}}$$

Equation 6-6

The mobility and relative mobility of the unswept region of layer $l$ are given by:

$$\lambda_{u_l} = \left(\frac{k_l k_{ro}}{\mu_o}\right)_{S_{oWF}} = \frac{k_l k_{ro}}{\mu_o}$$

$$\lambda_{ru} = \left(\frac{k_{ro}}{\mu_o}\right)_{S_{oWF}} = \frac{k_{ro}}{\mu_o}$$

Equation 6-7

Since an actual reservoir has many layers of varying permeability, it is necessary to derive an expression that accounts for heterogeneity. The definition of flow capacity as discussed in the previous chapter can be rewritten in terms of the average permeability and total thickness $H$ of a heterogeneous medium with $N_L$ layers:

$$F_l = \frac{\sum_{i=1}^{l} (kh)_l}{\sum_{i=1}^{N_L} (kh)_l} = \frac{\sum_{i=1}^{l} (kh)_l}{kH}, \quad \sum_{i=1}^{N_L} (kh)_l = \bar{k}H$$

Equation 6-8

For the case of a discrete number of layers in the reservoir, the change in flow capacity from layer $l - 1$ to layer $l$ is given by:

$$\Delta F_l = \left[\frac{\sum_{i=1}^{l} (kh)_l - \sum_{i=1}^{l-1} (kh)_l}{kH}\right] = \frac{(kh)_l}{kH}$$

Equation 6-9
The intuition of Equation 6-9 is that the change in the flow capacity at a given value of $F$ is equal to the product $kh$ for the incremental layer divided by the representative product of permeability and height $kH$, which characterizes all layers in the reservoir. It is therefore possible to express the injection rate into layer $l$ as:

$$i_l = \gamma_l \frac{3.541 kh \lambda r u \Delta p}{\ln \left( \frac{d}{r_w} \right) - 0.619} = \gamma_l \frac{3.541 \lambda r u \Delta p}{\ln \left( \frac{d}{r_w} \right) - 0.619} kH \Delta F_l$$

Equation 6-10

The total injection rate $I$ into a single five-spot pattern at a given time is the sum of the injection rates into the individual layers. This means that the incremental injection rate into the reservoir $\Delta I$ from a layer $l$ can be represented as:

$$\Delta I = i_l$$

Equation 6-11

Given the injection rate expression for an individual layer in the reservoir parameterized in terms of flow capacity, the total injection rate into the reservoir can be expressed as:

$$I = \sum_{l=1}^{N_L} \Delta I = \sum_{l=1}^{N_L} i_l = \sum_{l=1}^{N_L} kH \gamma_F \frac{3.541 \lambda r u \Delta p}{\ln \left( \frac{d}{r_w} \right) - 0.619} \Delta F_l$$

Equation 6-12

As the number of layers $N_L$ approaches infinity, $\Delta F_l \to dF$, $\Delta I \to dI$, and the sums in Equation 6-12 can be expressed as integrals:

$$I = \int_0^1 dI = \int_0^1 kH \gamma_F \frac{3.541 \lambda r u \Delta p}{\ln \left( \frac{d}{r_w} \right) - 0.619} dF$$
Equation 6-13

The conductance ratio $\gamma_F$ for a given flow capacity (associated with an individual layer) depends on the areal sweep efficiency $E_A$ of the waterflood, which varies over the life of the flood. The procedure for estimating $\gamma_F$ is defined below.

As discussed in the previous chapter, for a heterogeneous medium with continuous parallel layers, Lake (1989) expresses the flow capacity $F$ as a function of the heterogeneity factor $H_k$, a measure of the heterogeneity in a reservoir, and storage capacity $C$:

$$F = \frac{1}{1 + \frac{1}{H_k C}}$$

Equation 6-14

$H_k$ is empirically related to the Dykstra Parsons coefficient, another measure of heterogeneity (Paul et al. 1982):

$$log(H_k) = \frac{V_{DP}}{(1-V_{DP})^{0.2}}$$

Equation 6-15

Differentiating Equation 6-14 results in the following expression for $dF$ as a function of $H_k$ and the storage capacity $C$:

$$\frac{dF}{dC} = \left(\frac{H_k}{(1+(H_k-1)C)^2}\right)$$

$$dF = \left(\frac{H_k}{(1+(H_k-1)C)^2}\right) dC$$

Equation 6-16
The areal sweep efficiency $E_a$ in a waterflood is the amount of area in a well pattern invaded by injected water (Willhite 1986). This definition is illustrated by Figure 6-2. For analytical methods of predicting the performance of a waterflood, such as the Craig-Geffen-Morse method, it is assumed water breakthrough at the producer well for a single homogeneous layer occurs before $E_a = 100$ percent. The areal geometry of the swept area can be rather complicated. By contrast, the model discussed in the previous chapter approximates displacement behavior using a F-C parameterization with a dimensionless distance measured from injector to producer and piston-like fronts. The latter assumption means that for any given layer (or $C$) of constant permeability, breakthrough occurs when the layer is completely swept by the waterflood front. Thus applying existing methods for estimating waterflood injection rates to the F-C-based model discussed in the previous chapter requires some geometric approximation.
To evaluate the integral in Equation 6-13, it is necessary to calculate the conductance ratio for individual flow/storage capacities. This calculation in turn requires an approximation for $E_A$ as a function of storage capacity.\textsuperscript{54} For a given $C$ and $t_D$, Mollaei’s model predicts the dimensionless location of the waterflood front. Intuitively, this front location is analogous to $E_A$ from the waterflooding literature, as both measure the extent to which injected water has contacted a layer of the reservoir. This approximation for $E_A$ in a F-C-based model is therefore assumed to be:

$$E_A(C) = v_{\Delta} s t_D \left( \frac{dF}{dc} \right)_W = v_{\Delta} s t_D \left( \frac{K_w}{1 + (K_w - 1)C} \right)^2$$

Equation 6-17

\textsuperscript{54} Each curve in depicted in the correlation in Figure 6-1 applies to a different $E_A$, hence the need to determine $E_A$ in each layer/capacity.
with variables defined as they were in the previous chapter. This equation is specified in terms of \( C \) and the waterflood Koval factor \( K_W \), in contrast to Equation 6-16, which is defined in terms of \( C \) and \( H_K \).

Calculating the conductance ratio also requires a definition of the mobility ratio. Craig (1971) provides two, depending on whether the injected water has achieved breakthrough at the producer well. Before breakthrough, he calculates the mobility ratio using the average water saturation behind the front whereas after breakthrough he uses the average water saturation in the swept region. These saturations vary since he assumes that behind the shock front in each layer, the oil saturation continuously decreases (and water saturation increases) until it reaches the residual saturation. Since piston-like displacement is assumed in the model discussed in the previous chapter and implies a constant oil saturation behind the waterflood front, the average water saturation behind the front is equal to the average water saturation in the swept region. The mobility ratio for estimating the conductance ratio is therefore evaluated as follows, with \( S_{oWF} = 1 - S_{wWF} \) as the oil saturation behind the waterflood front:

\[
M_S = \frac{(k_{rw})}{(k_{ro})} \frac{S_{oWF}}{S_{ol}}
\]

Equation 6-18

Wasson and Schrider (1968) estimate a regression to summarize the correlation for conductance developed by Caudle and Witte shown in Figure 6-1; this is very useful
for incorporating the information into an analytical solution for the injection rate. They claim the “goodness of fit” of the regression to the diagram is greater than 99 percent:

\[
y = 1.9669 - 0.5262M_S - 3.3378E_A + 1.5599(M_S)(E_A) - 0.00437M_S^2 + 1.3965E_A^2
\]

Equation 6-19

Substituting into Equation 6-12 to find an expression for total injection rate for one well into a single five-spot pattern yields:

\[
I = 3.541\lambda_{ru}(\Delta p)\bar{k}H \int_0^1 1.9669 - 0.5262M_S - 3.3378v_{\Delta S}t_D \left( \frac{K_w}{(1+(K_w-1)C)^2} \right) + 1.5599(M_S) \left( v_{\Delta S}t_D \left( \frac{K_w}{(1+(K_w-1)C)^2} \right) \right) - 0.00437M_S^2 + 1.3965 \left( v_{\Delta S}t_D \left( \frac{K_w}{(1+(K_w-1)C)^2} \right) \right)^2 \left[ \frac{H_k}{(1+(H_k-1)C)^2} \right] \ln \left( \frac{d}{r_w} \right) + 0.619 \right) dC
\]

Equation 6-20

While this expression is a bit complicated, it can be integrated numerically using the trapezoidal rule. The integral is evaluated at each \( t_D \) for calculations discussed below using a step size of 0.05 for \( C \). Smaller steps were not found to change injection rate estimations substantially but added significant computation time. \( I \) is measured in reservoir barrels per day, and it is assumed that the formation volume factor for water is
1.0. Multiplying $I$ by the number injector wells in the reservoir\textsuperscript{55} yields the total injection rate into the reservoir as a function of dimensionless time:

$$I_{\text{tot}}(t_D) = (I)(N_{w,\text{primary}})$$

Equation 6-21

where $N_{w,\text{primary}}$ indicates the number of wells drilled for primary recovery.

One final step is necessary to map dimensionless time to actual time for the waterflood forecast. Craig (1971) provides a method using the trapezoidal rule to approximate this conversion, and Table 6-1 illustrates this method for four time steps. The dimensionless times in the first column are arbitrarily chosen; for the purposes of this study they begin at 0 and increase with at step size of 0.005. For each value of dimensionless time, oil recovery is calculated using the methods discussed in the previous chapter and the instantaneous fluid injection rate into the reservoir $I_{\text{tot}}$ is calculated using the method described above. The average injection rate into the reservoir $I_{\text{avg}}$ is then calculated between adjacent dimensionless time steps; each of these average injection rates are then used to calculate the actual time increment between each dimensionless time step. The time increment is calculated as the change in dimensionless time (measured in pore volumes), multiplied by the pore volume of the reservoir at the switch to waterflooding $V_{p,WF}$\textsuperscript{56} divided by the average injection rate into the reservoir

\textsuperscript{55} Remember from the previous chapter that the infill drilling campaign is designed such that the wells from primary recovery are converted to injectors.

\textsuperscript{56} With the tank model for primary recovery, it is assumed that the reservoir exhibits a degree of compaction because of drainage, which causes the initial pore volume to shrink. It is assumed that this pore volume at the switch to waterflooding remains constant for the rest of the displacement processes, just as fluids are assumed to be incompressible in situ.
for the increment. The final column measures the actual time since the initiation of the waterflood and corresponds to the dimensionless time in the first column. The initial time is also assumed to be zero, and subsequent times are found simply by adding the time increment found in the previous column to the time in the previous row.

Table 6-1: Converting dimensionless time to actual time

<table>
<thead>
<tr>
<th>$t_D$ ($RB$)</th>
<th>$I_{tot}$ ($RB/D$)</th>
<th>$I_{avg}$ ($RB/D$)</th>
<th>$\Delta t$</th>
<th>$t$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{D1} = 0$</td>
<td>$I_{tot 1}$</td>
<td>$I_{avg 1} = \frac{I_{tot 1} + I_{tot 2}}{2}$</td>
<td>$\Delta t_1 = \frac{(t_{D2} - t_{D1})(V_{p,WF})}{(365.25 \text{ d}) I_{avg 1}}$</td>
<td>$t_1 = 0$</td>
</tr>
<tr>
<td>$t_{D2}$</td>
<td>$I_{tot 2}$</td>
<td>$I_{avg 2} = \frac{I_{tot 2} + I_{tot 3}}{2}$</td>
<td>$\Delta t_2 = \frac{(t_{D3} - t_{D2})(V_{p,WF})}{(365.25 \text{ d}) I_{avg 2}}$</td>
<td>$t_2 = t_1 + \Delta t_1$</td>
</tr>
<tr>
<td>$t_{D3}$</td>
<td>$I_{tot 3}$</td>
<td>$I_{avg 3} = \frac{I_{tot 3} + I_{tot 4}}{2}$</td>
<td>$\Delta t_3 = \frac{(t_{D4} - t_{D3})(V_{p,WF})}{(365.25 \text{ d}) I_{avg 3}}$</td>
<td>$t_3 = t_2 + \Delta t_2$</td>
</tr>
<tr>
<td>$t_{D4}$</td>
<td>$I_{tot 4}$</td>
<td></td>
<td></td>
<td>$t_4 = t_3 + \Delta t_3$</td>
</tr>
</tbody>
</table>

6.3 **CO$_2$ Flood Injection Rates**

Estimating fluid injection rates for a CO$_2$ flood is more complicated, as there are multiple fronts present in the reservoir. This issue is especially true in WAG flooding where discrete slugs of water and solvent are injected, which creates multiple banks in the reservoir of fluids with different mobilities.
Christman and Gorell (1990) derive analytical expressions for the injection rate and apply them to a few WAG CO₂ flood field cases. Their model is essentially a radial version of Darcy’s law that accounts for the resistance in series of multiple fluid banks in a reservoir. This model makes the simplifying assumptions that fluids in situ are incompressible, miscible, and dispersion-free, with stable fronts that have no viscous fingering or vertical communication between layers. These assumptions are essentially validated by data from observation wells in the fields they consider. Although not all of these apply for the model discussed in the previous chapter, it will be assumed that Christman and Gorell’s analytical expression can be adapted to provide a first approximation of injection rates before acquiring any substantial production data for a field.

Equation 6-22 is Christman and Gorell’s general expression for estimating the fluid flow rate at the base of an injection well \( q \), assuming radial flow in a homogeneous layer, in consistent units (1990).

\[
q = 2\pi k h (p_{wl} - p_{wp}) \left[ \frac{\ln \left( \frac{1}{r_{DM}} \right)}{\lambda_{M-1}} + \sum_{j=M}^{P-1} \frac{\ln \left( \frac{r_{Dj}}{r_{Dj+1}} \right)}{\lambda_j} + \frac{\ln \left( \frac{r_{DP}}{r_{WDT}} \right)}{\lambda_p} \right]^{-1}
\]

Equation 6-22 essentially assumes that the producer well is a circular outer boundary a specified distance from the injector well. \( q \) is analogous to \( i \) in the expression derived in the last section based on the Caudle-Witte correlation. \( r_{wD'} \) is the dimensionless effective wellbore radius, which is the effective wellbore radius in Equation 6-4 divided by the
injector-producer distance. The other radii $r_{DP}$, $r_{Dj}$ and $r_{DM}$ are also dimensionless quantities and represent the distances of each of the fluid fronts in the reservoir from the injector. $P$ and $M$ are parameters that track the first and last banks of fluid in the reservoir at a given time. The mobility\textsuperscript{57} of the jth slug $\lambda_j$ is calculated based on the fluids in each bank; Christman and Gorell define it as:

$$\lambda_j = \sum_i k_{ri} / \mu_i$$

Equation 6-23

where each subscript $i$ denotes a component in slug j, corresponding to either oil, water, or solvent.

From Equation 6-22, Christman and Gorell derive expressions for injectivity normalized to initial injectivity of the CO\textsubscript{2} flood. Figure 6-3 compares the normalized injectivities predicted by their analytical expressions to the actual data for two field cases; the theory from the model fits these field data very well.

\textsuperscript{57} Christman and Gorell call $\lambda_j$ a “mobility”, but “relative mobility” is a more appropriate term for it here since its expression does not include multiplication by an absolute permeability.
The expression for fluid injection in Equation 6-22 can be adapted for use with CO₂ floods as modeled with the theory in the previous chapter. While simultaneous injection of solvent and water is assumed here, the same basic idea of modeling injection rates assuming a series of fluid banks and piston-like displacement still applies. This process maps the F-C-based model with dimensionless injector-producer distance to an actual geometry, which in this case is radial.

Assuming radial flow and multiple fluid fronts because of incomplete sweepout of a waterflood before initiating a WAG CO₂ flood, the following equations express the dimensionless distances of the three fronts in a heterogeneous reservoir at any time and storage capacity. These equations account for the possible cases of front merging as
discussed in the previous chapter. Dimensionless distances are expressed as dimensionless radii here, since they will be used in an expression derived from Equation 6-22:

\[
\begin{align*}
r_{WFD} &= \min \left(t_D v_{AS} \left(\frac{dF}{dc}\right)_W, 1\right) \\
r_{BD} &= \min \left(t_{DCO_2} v_B \left(\frac{dF}{dc}\right)_B, r_{WFD}, 1\right) \\
r_{CD} &= \min \left(t_{DCO_2} v_C \left(\frac{dF}{dc}\right)_C, 1\right)
\end{align*}
\]

Equation 6-24

As discussed in the previous chapter, the dimensionless times \(t_D\) and \(t_{DCO_2}\) correspond to the times since the initiation of the waterflood and CO_2 flood, respectively. The “1” in each expression is the dimensionless distance of the producer, the furthest a front can travel. Figure 6-4 illustrates three front locations, each for a different arbitrary storage capacity \(C\). As before, the model assumes there are at most four zones of constant saturation present in the reservoir.
Figure 6-4: Illustration of dimensionless radii in Equation 6-24

The derivatives in Equation 6-24 are defined in the previous chapter.

Applying Equation 6-22 with the dimensionless radii defined in Equation 6-24 and assuming a single layer with homogeneous permeability yields, in consistent units:

\[ q = 2\pi kh(p_{wi} - p_{wp}) \left[ \frac{\ln\left(\frac{1}{r_{WFD}}\right)}{\lambda_{roi}} + \frac{\ln\left(\frac{r_{WFD}}{r_{BD}}\right)}{\lambda_{rWF}} + \frac{\ln\left(\frac{r_{BD}}{r_{CD}}\right)}{\lambda_{roB}} + \frac{\ln\left(\frac{r_{CD}}{r_{wD}'}\right)}{\lambda_{rS}} \right]^{-1} \]

Equation 6-25

\( r_{wD}' \) is the dimensionless effective wellbore radius. Here, the relative mobilities are given by Equation 6-23 and depend on the two fluids in each of the regions of constant saturation:
\[ \lambda_{roi} = \left( \frac{k_{ro}}{\mu_o} \right) S_{oilWF} + \left( \frac{k_{rw}}{\mu_w} \right) S_{oilWF} \]

\[ \lambda_{WF} = \left( \frac{k_{ro}}{\mu_o} \right) S_{oilWF} + \left( \frac{k_{rw}}{\mu_w} \right) S_{oilWF} \]

\[ \lambda_{roB} = \left( \frac{k_{ro}}{\mu_o} \right) S_{oB} + \left( \frac{k_{rw}}{\mu_w} \right) S_{oB} \]

\[ \lambda_{rS} = \left( \frac{k_{rs}}{\mu_s} \right) S_{wJ} + \left( \frac{k_{rw}}{\mu_w} \right) S_{wJ} \]

Equation 6-26

Where \( S_{oilWF}, S_{oilWF}, S_{oB}, \) and \( S_{wJ} \) are the oil saturation before waterflood, oil saturation after the waterflood bank has swept, oil bank saturation, and water saturation of the injected water-CO₂ mixture. \( S_{wJ} \) corresponds to the area with oil saturation \( S_{oF} \) in the diagram. Once a front has reached the producer, its resistance term in Equation 6-25 disappears. For example, when the waterflood front location \( r_{WFD} \) reaches \( x_D = 1 \), the term \( \frac{\ln\left(\frac{1}{r_{WFD}}\right)}{\lambda_{roi}} \) becomes 0. Based on how the dimensionless radii have been defined in Equation 6-24, the term \( \frac{\ln\left(\frac{r_{WFD}}{r_{BD}}\right)}{\lambda_{WF}} \), which is the resistance associated with the post-waterflood saturation region, also becomes zero if the oil bank front merges with the waterflood front.

For a discrete layer \( l \) in a heterogeneous medium with parallel layers, Equation 6-25 becomes:
Equation 6-27

\[ q_l = \Delta p 2\pi(kh)_l \left[ \frac{\ln \left( \frac{1}{r_{WFD_l}} \right)}{\lambda_{roi}} + \frac{\ln \left( \frac{r_{WFD_l}}{r_{BD_l}} \right)}{\lambda_{rWF}} + \frac{\ln \left( \frac{r_{BD_l}}{r_{CD_l}} \right)}{\lambda_{roB}} + \frac{\ln \left( \frac{r_{CD_l}}{r_{WD_l}} \right)}{\lambda_{rS}} \right]^{-1} \]

Since the dimensionless radii in this expression are defined using the F-C parameterization, it is more natural to define the entire equation in this manner. Using the same argument as is employed above for waterflood injection rates, as the number of layers in a heterogeneous medium approaches infinity, the \((kh)_l\) term can be replaced by \(\bar{k}HdF\):

\[ dQ = \Delta p 2\pi \bar{k}HdF \left[ \frac{\ln \left( \frac{1}{r_{WFD}} \right)}{\lambda_{roi}} + \frac{\ln \left( \frac{r_{WFD}}{r_{BD}} \right)}{\lambda_{rWF}} + \frac{\ln \left( \frac{r_{BD}}{r_{CD}} \right)}{\lambda_{roB}} + \frac{\ln \left( \frac{r_{CD}}{r_{WD}} \right)}{\lambda_{rS}} \right]^{-1} \]

Equation 6-28

where \(Q\) is the total injection rate into the reservoir for a single injector and \(dQ\) is the incremental injection rate from the incremental flow capacity \(dF\).

Integrating over the range of storage capacities for the reservoir yields the following expression for the total injection rate of the water-solvent mixture into the reservoir for a single pattern’s injection well:

\[ Q = \left( \frac{1}{14.12} \right) \Delta p \bar{k}H \int_0^1 \frac{H_k}{(1+(H_k-1)C)^2} \left[ \frac{\ln \left( \frac{1}{r_{WFD}} \right)}{\lambda_{roi}} + \frac{\ln \left( \frac{r_{WFD}}{r_{BD}} \right)}{\lambda_{rWF}} + \frac{\ln \left( \frac{r_{BD}}{r_{CD}} \right)}{\lambda_{roB}} + \frac{\ln \left( \frac{r_{CD}}{r_{WD}} \right)}{\lambda_{rS}} \right]^{-1} dC \]

Equation 6-29
where the dimensionless radii are functions of \( C \) as defined above. This equation assumes oilfield units, and \( Q \) is measured in reservoir barrels per day. Note again that here \( dF \) is defined as a function of \( C \) and the heterogeneity factor \( H_k \), as it strictly replaces the product of permeability and layer thickness and does not account for fluid mobilities. This is in contrast to the derivatives used to calculate the dimensionless radii for fluid fronts, which employ Koval factors that do incorporate fluid mobilities.

Equation 6-29 also can be integrated numerically using the trapezoidal rule. The integral is evaluated for subsequent calculations at each \( t_D \) using a step size of 0.05 for \( C \). Multiplying by the number of wells from primary recovery yields the total injection rate of water-solvent mixture into the reservoir as a function of dimensionless time:

\[
Q_{tot}(t_D) = (Q)(N_{w,primary})
\]

Equation 6-30

Dimensionless time for the \( CO_2 \) flood can be mapped to actual time using the same method outlined in Table 6-1, except with \( Q_{tot} \) replacing \( I_{tot} \).

**6.4 ALTERNATIVE WATERFLOOD METHOD**

Indeed, the derivation in the previous section for \( CO_2 \) flood injection rates could also be applied to waterflooding, which results in a simpler version of Equation 6-29:

\[
I_{CG} = \left( \frac{1}{141.2} \right) \Delta p \bar{k}H \int_0^1 \left( \frac{H_k}{(1+(H_k-1)C)^2} \right) \left[ \ln \left( \frac{1}{r_{WFD}} \right) \lambda_{roi} + \ln \left( \frac{r_{WFD}}{r_{WDF}} \right) \lambda_{WDF} \right]^{-1} dC
\]

Equation 6-31
Since the oil bank and solvent fronts do not apply for waterflooding, the resistance terms for the regions behind these fronts simply disappear from Equation 6-29, and the term associated with the post-waterflood saturation now includes the dimensionless effective wellbore radius.

Equation 6-20 and Equation 6-31 (for \( I \) and \( l_{CG} \)) are both used to estimate waterflood injection rates but they differ and it is difficult to compare them analytically to determine how substantially their estimations diverge. Therefore it is worthwhile to consider actual calculations to examine their behavior. Table 6-2 provides reservoir properties and operational parameters for three waterflood cases that will be illustrated in subsequent figures. The first and second cases involve a light oil with viscosity determined by correlations discussed in chapter four, assuming an API gravity of 40. For these two cases, the endpoint relative permeabilities were chosen so that Case 1 represents a more water wet reservoir and Case 2 represents a more oil wet reservoir. Case 3 has relative permeability characteristics like those of Case 1, but the oil viscosity was selected to be an arbitrary higher value so as to examine differences in the prediction methods at a higher mobility ratio. This higher viscosity also increases the time to reach the ultimate recovery efficiency of the waterflood.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Thickness (ft)</td>
<td>$H$</td>
<td>220</td>
<td>220</td>
</tr>
<tr>
<td>Dykstra-Parsons Coefficient</td>
<td>$V_{DP}$</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Average Absolute Permeability ($md$)</td>
<td>$\bar{k}$</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Skin Factor</td>
<td>$s$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Wellbore Radius (ft)</td>
<td>$r_w$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Effective Wellbore Radius (ft)</td>
<td>$r_w'$</td>
<td>0.068</td>
<td>0.068</td>
</tr>
<tr>
<td>Injector-Producer Distance (ft)</td>
<td>$d$</td>
<td>733</td>
<td>733</td>
</tr>
<tr>
<td>Pressure Drop (psi)</td>
<td>$\Delta p$</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>Oil Saturation at Switch to Waterflood</td>
<td>$S_{oi}$</td>
<td>0.768</td>
<td>0.768</td>
</tr>
<tr>
<td>Residual Oil Saturation</td>
<td>$S_{or}$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Oil Gravity ($^\circ$API)</td>
<td>$\gamma_o$</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Oil Viscosity (cp)</td>
<td>$\mu_o$</td>
<td>0.404</td>
<td>0.404</td>
</tr>
<tr>
<td>Water Viscosity (cp)</td>
<td>$\mu_w$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Oil Endpoint Relative Permeability</td>
<td>$k_{ro}$</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Water Endpoint Relative Permeability</td>
<td>$k_{rw}$</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Oil Corey Exponent</td>
<td>$m$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Water Corey Exponent</td>
<td>$n$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Relative Mobility of Swept Region (for Christman-Gorell-Based Formula, cp$^{-1}$)</td>
<td>$\lambda_{AWF}$</td>
<td>0.368</td>
<td>1.169</td>
</tr>
<tr>
<td>Relative Mobility of Unswept Region (for Christman-Gorell-Based Formula, cp$^{-1}$)</td>
<td>$\lambda_{ROI}$</td>
<td>1.978</td>
<td>1.731</td>
</tr>
<tr>
<td>Mobility Ratio for Conductance Ratio Calculation (for Caudle-Witte-Based Formula)</td>
<td>$M_S$</td>
<td>0.184</td>
<td>0.649</td>
</tr>
</tbody>
</table>
Figures 6-5 and 6-6 plot the estimated injection rate into one five-spot pattern versus $t_D$ and incremental oil recovered from waterflooding versus time, respectively, for Case 1. The data labels correspond to the authors whose contributions were used to derive each injection rate prediction method, as outlined above. The subsequent figures mimic Figures 6-5 and 6-6 for Cases 2 and 3.

For Cases 1 and 2, oil is recovered quickly because of its small viscosity. The injection rates for the two estimation methods are initially substantially different for Case 1 (water wet) but nearly converge for later dimensionless times. This difference results in Equation 6-20 (Caudle-Witte-based) predicting faster recovery than Equation 6-31 (Christman-Gorell-based), although the difference is not large since the time scale of recovery is already relatively short for Case 1. For Case 2 (oil wet), the estimated injection rates for each method are quite similar initially but then diverge substantially as injection progresses. Predictably, the progression of recovery efficiency as estimated by the two methods is similar for early times but proceeds more slowly for later times when using Equation 6-20. Again, the since the time scale of recovery is relatively short, the differences in the two estimation methods are not substantial.
Figure 6-5: Case 1 (small oil viscosity, water wet) injection rates into a single five-spot versus dimensionless time

Figure 6-6: Case 1 recovery efficiencies versus time
Figure 6-7: Case 2 (small oil viscosity, oil wet) injection rates into a single five-spot versus dimensionless time

Figure 6-8: Case 2 recovery efficiencies versus time
The two methods for estimating injection rates diverge substantially for all dimensionless times for Case 3, which also involves oil recovery over a longer time scale because of the larger viscosity of the oil. This displacement involves a mobility ratio greater than one and the injection rates increase from their initial values, in contrast to the results in Cases 1 and 2. The divergence of injection rates estimated by each method results in a large difference in the speed of recovery, as depicted in Figure 6-10. The time scale of the difference here is on the order of multiple years for equivalent recovery efficiencies, which is economically significant.

Figure 6-9: Case 3 (large viscosity, water wet) injection rates into a single five-spot versus dimensionless time
Figure 6-10: Case 3 recovery efficiencies versus time

Clearly, the different assumptions used to derive each of the two injection rate estimation methods do lead to significant variation in their predictions, all else equal. Equation 6-19 relies on the simplifying assumption that areal sweep efficiency is approximated by the dimensionless distance of the waterflood front, but also is based on actual experiments involving five-spot patterns. Equation 6-31 assumes a radial geometry for front movement in the reservoir, which only approximates front behavior in a five-spot for part of a displacement (Deppe 1961).

While both of these methods must be validated with waterflood field data, subsequent calculations will use Equation 6-31 for consistency since it is based on the same assumptions used for predicting injection rates for a CO₂ flood.
6.5 Operating Pressures

Regardless of which analytical expression is used to estimate injection rates, it is also necessary to predict the bottomhole pressures at which the injector and producer wells can operate. In particular, it is useful to determine the maximum pressure for the injectors and minimum pressure for producers, as these values will yield the largest pressure drop and fastest recovery that the model predicts is theoretically possible.

For injectors, the maximum pressure is constrained by the formation parting pressure, regardless of displacement method employed. The concern with exceeding this pressure is that the resulting fractures could serve as conduits for injected fluid to bypass most of the formation, flowing quickly to the producer wells. Rose et al. (2001) suggest using a gradient of 1.0 psi per foot to estimate this pressure, but advise that the gradient could vary from 0.8 to 1.4 psi per foot. Saripalli et al. (1999) document how the lower temperature of injected fluids relative to that of the formation can lead to thermally-induced fracturing over the course of a displacement process. Therefore for calculations in the next chapter, the more conservative gradient of 0.8 psi per foot will be used.

The minimum bottomhole pressure for producers will differ for a waterflood versus a miscible CO₂ flood. For a waterflood, artificial lift could in theory be used to lower the bottomhole pressure well below the bubble point, particularly for shallow wells. On the other hand, doing this would mean that a trapped gas saturation would form close to the producer well for a solution gas drive reservoir, impeding oil recovery. Thus there is a bit of a tradeoff at lower pressures. Moreover, for deeper wells, such as those considered in the next chapter, the weight of the produced fluids require high
bottomhole pressures to bring them to the surface unaided. For subsequent calculations it will be assumed that the producers operate at essentially the same bottomhole pressure for the waterflood as they do for the CO$_2$ flood. With a miscible CO$_2$ flood, producer wells need to operate at or above the MMP to ensure the displacement remains miscible. Otherwise, the recovery efficiency of the flood will be lower. Therefore subsequent calculations will assume that the MMP is the minimum pressure at which producer wells can operate during a CO$_2$ flood.
Chapter 7: Economic Modeling and Optimization of the Geologic-Parameter-Based Model

7.1 INTRODUCTION

The models for primary recovery, waterflooding and CO$_2$ flooding in chapters four through six are combined here to yield a geologic-parameter-based model for the entire life cycle of a hypothetical reservoir. The term “geologic-parameter-based model” is used to distinguish the combination of the tank model and Koval-derived models from the decline curve modeling discussed in chapter three, as it is grounded in physical properties of the reservoir and can be used to estimate production \textit{a priori}. For the purposes of economic evaluation, this model only captures the physical behavior of the reservoir; the costs and revenues associated with production also need to be estimated. This chapter focuses on characterizing both the technical and economic aspects of production and addresses optimization of a reservoir’s NPV over its life cycle.

For simplicity and continuity, many of the cost, royalty and tax assumptions from chapter three are retained here, but more detail is possible with the geologic-parameter-based model. In contrast to the decline-curve modeling of the Lost Soldier Tensleep reservoir discussed, the geologic-parameter-based model includes more technical parameters such as CO$_2$ injection rates, which allows for economic characterization of prospective reservoirs more closely tied to physical behavior. Therefore this chapter will expand on existing assumptions to provide a richer understanding of the reservoir life cycle optimization problem by including additional decision variables such as well spacing, injection pressures and the WAG ratio for the CO$_2$ flood.

Following the discussion in chapter three, the metric used to evaluate a project here is NPV, which is simply the sum of the discounted cash flows that a project generates and described by Equation 7-7. Revenues are modeled simply as the oil price
at a given time, adjusted for geographic/quality differences from WTI, and multiplied by the barrels of oil produced. Cost modeling is discussed in subsequent sections.

### 7.2 Reservoir Parameters and Example Calculations Using Geologic-Parameter-Based Model

A base case of how a field might normally be operated is considered here to illustrate calculations using the geologic-parameter-based model. Although in this section the operator is not assumed to be explicitly optimizing the overall NPV of the field, the switch from primary recovery to waterflooding occurs when the average reservoir pressure reaches the bubble point. For this base case, production from each well occurs at relatively modest rates of about 250 to 500 barrels per day. These rates are in line with those of the fields undergoing primary and secondary recovery considered in the EIA cost study (2010). Higher fluid flow rates are required for CO₂ flooding to attain recovery on an economically feasible time scale. In subsequent sections covering optimization of a reservoir’s NPV over its life cycle, the possibility of accelerating production by boosting fluid production rates is considered.

#### 7.2.1 Primary Recovery

As discussed in chapter three, the forecasting of oil production from primary recovery entails a finite-differencing of time for a tank model. At each time step, several reservoir and fluid properties are recalculated. Table 7-1 lists properties of a hypothetical reservoir used for this tank model calculation.
Table 7-1: Physical properties of a hypothetical reservoir

<table>
<thead>
<tr>
<th>Reservoir Properties</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Reservoir Pressure</td>
<td>$p_i$</td>
<td>4000 psi</td>
</tr>
<tr>
<td>Reservoir Temperature</td>
<td>$T$</td>
<td>150 °F</td>
</tr>
<tr>
<td>Reservoir Depth</td>
<td>$D$</td>
<td>6000 ft</td>
</tr>
<tr>
<td>Reservoir Type</td>
<td></td>
<td>Sandstone</td>
</tr>
<tr>
<td>Areal Extent of Reservoir</td>
<td>$A_{res}$</td>
<td>2000 acres</td>
</tr>
<tr>
<td>Reservoir Thickness</td>
<td>$H$</td>
<td>60 ft</td>
</tr>
<tr>
<td>Absolute Thickness-Weighted Permeability</td>
<td>$\bar{k}$</td>
<td>30 mD</td>
</tr>
<tr>
<td>Oil Permeability$^{58}$</td>
<td>$k_o$</td>
<td>24 mD</td>
</tr>
<tr>
<td>Initial Porosity</td>
<td>$\varphi_i$</td>
<td>0.26</td>
</tr>
<tr>
<td>Porosity at Switch to Waterflooding</td>
<td>$\varphi_{WF}$</td>
<td>0.256</td>
</tr>
<tr>
<td>Skin Factor</td>
<td>$s$</td>
<td>2</td>
</tr>
<tr>
<td>Initial Oil Saturation</td>
<td>$S_{oi,discovery}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Oil Saturation at Switch to WF</td>
<td>$S_{oiWF}$</td>
<td>0.774</td>
</tr>
<tr>
<td>Dykstra-Parsons Coefficient</td>
<td>$V_{DP}$</td>
<td>0.6</td>
</tr>
<tr>
<td>Dimensionless Correlation Length</td>
<td>$\lambda_x$</td>
<td>2</td>
</tr>
<tr>
<td>Formation Rock Compressibility</td>
<td>$c_r$</td>
<td>5.00E-06 psi$^{-1}$</td>
</tr>
</tbody>
</table>

Table 7-2 provides fluid properties, some at both the initial reservoir conditions and the bubble point. Some of these parameters are inputs to the correlations from Vasquez et al. while others are results of these calculations. Other parameters, such as water viscosity and compressibility, are assumed to be constant even as the average reservoir pressure declines.

$^{58}$ Assumes an endpoint relative permeability to oil of 0.8. See Table 7-5 below for relative permeability information.
Table 7-2: Fluid properties in a hypothetical reservoir

<table>
<thead>
<tr>
<th>Fluid Properties</th>
<th>μ oD</th>
<th>2.108 cp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Oil Viscosity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil Viscosity at Bubble Point</td>
<td>μ oB</td>
<td>0.626 cp</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>μ w</td>
<td>0.6 cp</td>
</tr>
<tr>
<td>Oil Gravity</td>
<td>γ o</td>
<td>40 °API</td>
</tr>
<tr>
<td>Gas Gravity at Separator Conditions of 100 psig</td>
<td>γ gs</td>
<td>1.50</td>
</tr>
<tr>
<td>Initial Solution Gas-Oil Ratio</td>
<td>R p</td>
<td>500 scf/STB</td>
</tr>
<tr>
<td>Predicted Bubble Point Pressure</td>
<td>p b</td>
<td>1060 psia</td>
</tr>
<tr>
<td>Initial Formation Volume Factor</td>
<td>B oi</td>
<td>1.24 rb/STB</td>
</tr>
<tr>
<td>Predicted Oil FVF at Bubble Point</td>
<td>B oB</td>
<td>1.262 rb/STB</td>
</tr>
<tr>
<td>Oil Compressibility at Bubble Point Pressure</td>
<td>c o</td>
<td>2.25E-05 psi⁻¹</td>
</tr>
<tr>
<td>Water Compressibility</td>
<td>c w</td>
<td>3.00E-06 psi⁻¹</td>
</tr>
</tbody>
</table>

Table 7-3 primarily lists parameters that characterize the operator’s design choices for the reservoir, such as well spacing and the maximum flow rate for each well in the field. The latter choice impacts the length of the constant rate production period; a smaller maximum flow rate leads to a longer time for the constant rate period. Although the minimum bottomhole pressure for producers is well below the bubble point, in practice the operator switches to waterflooding well before most of the reservoir has reached this pressure.
Finally, Table 7-4 provides calculations for the quantities of water and oil in the reservoir as well as total pore volume, both initially and at the switch to waterflooding. Since it is assumed that the reservoir tank is compressible, the pore volume shrinks slightly during primary production. The pore volume at the switch to waterflooding is then assumed to remain constant for the remainder of the field’s life.
Table 7-4: Pore and fluid volumes

<table>
<thead>
<tr>
<th>Reservoir Volumes</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Pore Volume</strong></td>
<td>$V_p$</td>
<td>242,043,110</td>
</tr>
<tr>
<td><strong>Initial Water Volume</strong></td>
<td>$V_w$</td>
<td>48,408,622</td>
</tr>
<tr>
<td><strong>Total Pore Volume at Switch to WF</strong></td>
<td>$V_{pwf}$</td>
<td>238,417,735</td>
</tr>
<tr>
<td><strong>OOIP</strong></td>
<td></td>
<td>156,205,394</td>
</tr>
<tr>
<td><strong>Oil in Place at Start of WF</strong></td>
<td></td>
<td>193,634,488</td>
</tr>
<tr>
<td><strong>Oil in Place at Start of WF</strong></td>
<td></td>
<td>147,608,065</td>
</tr>
</tbody>
</table>

Figures 7-1 and 7-2 illustrate the results of calculations using correlations from Vasquez et al. The viscosity and compressibility of oil vary substantially above the bubble point, whereas the oil formation volume factor and solution gas-oil ratio do not.

Figure 7-1: Oil formation volume factor and compressibility
Figures 7-3 and 7-4 plot the estimated oil production, recovery efficiency, and average reservoir pressure versus time. The constant rate period of production lasts roughly three years before production is projected to exhibit a very sharp decline. The bubble point is reached after 29 months. Behavior when the reservoir is below the bubble point as depicted in these figures is not indicative of how an actual solution gas drive reservoir might behave, as the tank model does not account for the presence of solution gas that would begin to form at lower pressures. If one wanted to estimate primary recovery behavior below the bubble point, using a more complex model would be necessary.
Figure 7-3: Estimated field production and average reservoir pressure versus time

Figure 7-4: Estimated field recovery efficiency and average reservoir pressure versus time
7.2.2 Waterflooding

With water injection, some information regarding relative permeability curves is necessary to do a fractional flow analysis that underpins Mollaei’s method for characterizing oil production. Table 7-5 provides parameters for the hypothetical reservoir considered here. The reservoir is a sandstone that is water-wet, so the endpoint relative permeability for oil is comparatively large while the endpoint relative permeability for water is comparatively small. The residual oil saturation to waterflooding is relatively large, leaving a substantial amount of oil that could potentially be recovered through CO₂ flooding. Finally, as the oil is light and therefore its viscosity is quite low, the endpoint mobility ratio is also low, which can enhance the sweep efficiency of the waterflood.

Table 7-5: Parameters for calculating relative permeability and mobility

<table>
<thead>
<tr>
<th>Relative Permeability and Mobility</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual Oil Saturation to Waterflooding</td>
<td>S₀₀</td>
</tr>
<tr>
<td>Oil Saturation at Switch to WF</td>
<td>Sₒ𝑖WF</td>
</tr>
<tr>
<td>Water Saturation at Switch to WF</td>
<td>S𝑤𝑖WF</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Oil</td>
<td>kᵣο</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Water</td>
<td>kᵣ𝑤ο</td>
</tr>
<tr>
<td>Oil Corey Exponent</td>
<td>m</td>
</tr>
<tr>
<td>Water Corey Exponent</td>
<td>n</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>µ𝑤</td>
</tr>
<tr>
<td>Oil Viscosity at Bubble Point</td>
<td>µₒ𝐵</td>
</tr>
<tr>
<td>Endpoint Mobility Ratio</td>
<td>M°</td>
</tr>
</tbody>
</table>

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Figure 7-5 illustrates the Corey-type relative permeability curves calculated according to the parameters listed in Table 7-5.

![Relative permeability curves](image)

Figure 7-5: Relative permeability curves for a hypothetical reservoir

Figure 7-6 depicts the oil-water fractional flow curve calculated using the relative permeability curves in Figure 7-5. The Welge tangent yields the specific velocity $v_{\Delta S}$ of the waterflood front that is later used to calculate the waterflood front location in each storage capacity $C$ in the reservoir.
Figure 7-6: Fractional flow and Welge tangent for the waterflood

Based on the fractional flow diagram, a few key values for water and oil saturation and fractional flow are calculated. These values appear in Table 7-6, along with \( v_{\Delta S} \). As Mollaei suggests that the oil saturation behind the waterflood front \( S_{oWF} \)\(^{59} \) could be calculated through a correlation. In addition to using fractional flow, the value calculated from the correlation also appears in the table for reference. \( S_{oWF} \) as calculated from the correlation does not diverge substantially from that predicted by the fractional flow construction, although for a more oil wet reservoir, the difference between the values of \( S_{oWF} \) predicted by these two calculation methods can be on the order of 0.1.

---

\(^{59}\) He calls this \( S_{of} \), the final oil saturation after flooding; he considers the waterflood and enhanced oil recovery cases separately.
### Table 7-6: Calculated parameters from fractional flow analysis

<table>
<thead>
<tr>
<th>Fractional Flow Results</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Saturation at Tangent Chord Intersection to Fractional Flow Curve</td>
<td>$S_{wf}$</td>
<td>0.558</td>
</tr>
<tr>
<td>Dimensionless Water Saturation at Tangent Chord Intersection to Fractional Flow Curve</td>
<td>$S_{wf}^\ast$</td>
<td>0.891</td>
</tr>
<tr>
<td>Fractional Flow at Tangent Chord Intersection to Fractional Flow Curve</td>
<td>$f_{wf}$</td>
<td>0.945</td>
</tr>
<tr>
<td>Specific Velocity of the Waterflood Shock Front</td>
<td>$v_{\Delta S}$</td>
<td>2.754</td>
</tr>
<tr>
<td>Average Water Saturation Behind the Waterflood Front</td>
<td>$S_{wWF}$</td>
<td>0.578</td>
</tr>
<tr>
<td>Oil Saturation Behind the Waterflood Front Calculated from Fractional Flow</td>
<td>$S_{oWF}$</td>
<td>0.422</td>
</tr>
<tr>
<td>Oil Saturation Behind the Waterflood Front Estimated from Mollaei’s Correlation</td>
<td></td>
<td>0.446</td>
</tr>
</tbody>
</table>

The Koval factor for the waterflood front $K_W$ is calculated using another correlation provided by Mollaei in Equation D1. From $K_W$ and $v_{\Delta S}$, the waterflood front breakthrough and sweepout times are calculated. These parameters appear in Table 7-7.

### Table 7-7: Parameters for reservoir-wide oil recovery calculations

<table>
<thead>
<tr>
<th>Koval Factor and Key Dimensionless Times</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterflood Front Koval Factor</td>
<td>$K_W$</td>
<td>1.903</td>
</tr>
<tr>
<td>Waterflood Front Dimensionless Breakthrough Time</td>
<td>$t_{p_{BT}}^W$</td>
<td>0.191</td>
</tr>
<tr>
<td>Waterflood Front Dimensionless Sweepout Time</td>
<td>$t_{p_{SW}}^W$</td>
<td>0.691</td>
</tr>
</tbody>
</table>
The location of the waterflood front at any given storage capacity \( C \) can be calculated for a particular dimensionless time. Figure 7-7 illustrates a profile of the dimensionless waterflood front location at each storage capacity in the hypothetical reservoir at \( t_D = 0.25 \). Since this time is between the breakthrough and sweepout times but closer to breakthrough, \( C_W^* \) is between 0 and 1, but closer to 0.

![Figure 7-7: Waterflood front location for each storage capacity at \( t_D = 0.25 \)](image)

The average oil saturation in the reservoir at each dimensionless time is calculated using Mollaei’s expression in Equation 5-49. This average saturation is used to calculate oil production since the start of the waterflood \( N_{P_{displ.STB}} \) as a function of dimensionless time, which is then used to determine the total recovery efficiency \( E_R \). Subtracting the recovery efficiency at the beginning of the waterflood from \( E_R \) yields the incremental recovery efficiency of the waterflood. Figure 7-8 displays the results of these
calculations and indicates that almost all additional recovery is obtained after 0.6 pore volumes have been injected.

![Graph showing incremental ER and So_bar versus dimensionless time](image)

**Figure 7-8:** Additional waterflood recovery efficiency and average reservoir oil saturation versus dimensionless time

To convert dimensionless times to actual times, the overall injection rate into the field is calculated using the expression in Equation 6-31, which estimates water injection rates in layered media and is based on Christman and Gorell’s model. Table 7-8 summarizes parameters necessary to estimate injection rates as well as the average injection rate for a single well over the first pore volume of injection.

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Table 7-8: Additional parameters for injection rate calculations

<table>
<thead>
<tr>
<th>Waterflood Injection Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Injector-Producer Distance</strong></td>
</tr>
<tr>
<td>Injector BHP</td>
</tr>
<tr>
<td>Producer BHP</td>
</tr>
<tr>
<td>Pressure Drop</td>
</tr>
<tr>
<td>Dimensionless Effective Wellbore Radius</td>
</tr>
<tr>
<td>Heterogeneity Factor</td>
</tr>
<tr>
<td>Total Relative Mobility of Post-Waterflood Region</td>
</tr>
<tr>
<td>Total Relative Mobility of Region at Initial Oil Saturation</td>
</tr>
<tr>
<td>Water Relative Permeability at Post-Waterflood Oil Saturation</td>
</tr>
<tr>
<td>Oil Relative Permeability at Initial Water Saturation</td>
</tr>
<tr>
<td>Average Fluid Injection/Production Rate per Well for the First Pore Volume of Production</td>
</tr>
</tbody>
</table>

Figure 7-9 illustrates the results of the injection rate calculation using the Christman-Gorell-based method. As the total relative mobility of the region behind the waterflood front is lower than the pre-front region, the injection rate decreases as time progresses. The water formation volume factor is assumed to be 1. Also included in the diagram is water cut at the producer wells versus time. This parameter is calculated using Mollaei’s definition for oil cut:

\[
f_w = 1 - f_o = 1 - \frac{s_{o,n+1} - s_{o,n}}{t_{D,n+1} - t_{D,n}}
\]

Equation 7-1
where $n$ and $n + 1$ denote the value of the relevant variable at adjacent time steps. As expected, the water cut at producers increases as the waterflood front progressively breaks through in more layers. The calculated water cut is useful for estimating water treatment costs as the flood matures, as is discussed later.

![Graph showing water injection rate and water cut over time](image)

**Figure 7-9:** Total water injection into reservoir and water cut at producers versus actual time

Finally, Figure 7-10 illustrates the waterflood oil recovery information in Figure 7-8 mapped from dimensionless time to actual time using the estimated injection rates and the method outlined in Table 6-1. Essentially all the additional recovery from the waterflood is attained by 25 years into the flood.
Figure 7-10: Additional recovery from waterflooding and average reservoir oil saturation versus actual time

7.2.3 CO$_2$ Flooding

For the hypothetical reservoir’s CO$_2$ flood, the relative permeability parameters used to calculate the water-solvent fractional flow curve are similar to those for the water-oil curve. CO$_2$ parameters appear in Table 7-9. The endpoint saturations (initial and residual) are assumed to be the same as those for the water-oil case listed in Table 7-5. These parameters yield relative permeability curves that are essentially the same as those in Figure 7-5, but with solvent instead of oil.
Table 7-9: Parameters for calculating relative permeability and fractional flow (water-solvent)

<table>
<thead>
<tr>
<th>Relative Permeability and Fractional Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endpoint Relative Permeability</strong> for Solvent</td>
</tr>
<tr>
<td><strong>Endpoint Relative Permeability</strong> for Water</td>
</tr>
<tr>
<td><strong>Solvent Corey Exponent</strong></td>
</tr>
<tr>
<td><strong>Water Corey Exponent</strong></td>
</tr>
<tr>
<td><strong>Water Viscosity</strong></td>
</tr>
<tr>
<td><strong>CO$_2$ Viscosity</strong></td>
</tr>
</tbody>
</table>

Figure 7-11: Fractional flow construction for the CO$_2$ flood
Figure 7-11 illustrates the fractional flow construction for the CO₂ flood while Table 7-10 summarizes both input and output parameters for the construction. The MMP, assumed here to be 2110 psi, along with the bottomhole pressure at which producers operate affect the dimensionless pressure difference $\Delta P_D$, which is a parameter used in one of Mollaei’s correlations to estimate the final oil saturation after solvent bank sweep $S_{oF}$. This procedure is outlined in Appendix D.

Table 7-10: Parameters from CO₂ flood fractional flow analysis

<table>
<thead>
<tr>
<th>Fractional Flow-Related Parameters</th>
<th>$W_R$</th>
<th>$\Delta P_D$</th>
<th>$S_{oF}$</th>
<th>$f_{wWF}$</th>
<th>$f_{wJ}$</th>
<th>$S_{wJ}$</th>
<th>$v_C$</th>
<th>$S_{WB}$</th>
<th>$S_{oB}$</th>
<th>$f_{wB}$</th>
<th>$v_{oB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAG Ratio</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MMP</td>
<td></td>
<td>2110 psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Dimensionless Pressure Deviation</td>
<td></td>
<td></td>
<td>0.0047</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>from MMP at Producers</td>
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<td></td>
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<td></td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>Final Oil Saturation After</td>
<td></td>
<td></td>
<td>0.236</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Solvent Front Sweep</td>
<td></td>
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<tr>
<td>Water Fractional Flow at</td>
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</tr>
<tr>
<td>Post-Waterflood Water Saturation</td>
<td></td>
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<td></td>
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<tr>
<td>(Point I)</td>
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<tr>
<td>Fractional Flow of Water at</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Injectors</td>
<td></td>
<td></td>
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<td>Water Saturation Associated with</td>
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<td>$f_{wJ}$ on Water-Solvent Fraction</td>
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<td>Oil-Water Fractional Flow at</td>
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<tr>
<td>Oil Bank Water Saturation (Point B)</td>
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<tr>
<td>Specific Velocity of Oil Bank</td>
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</tbody>
</table>

244
The Koval factors for the oil bank and solvent fronts $K_B$ and $K_C$ are calculated using Mollaei’s correlations in Appendix D. These Koval factors and the specific velocities of their associated fronts listed in Table 7-10 are used to calculate front breakthrough and sweepout times. The waterflood is assumed to last 411 months (34.25 years), which corresponds to about 0.7 pore volumes of injected water. At this point, the waterflood does not deliver more recovery. In reality, the project may be converted to a CO$_2$ flood well before this time due to economic considerations. These parameters appear in Table 7-11.

Table 7-11: Parameters for reservoir-wide CO$_2$ flood oil recovery calculations

<table>
<thead>
<tr>
<th>Koval Factors and Key Dimensionless Times</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oil Bank Front Koval Factor</strong> $K_B$</td>
</tr>
<tr>
<td><strong>Solvent Front Koval Factor</strong> $K_C$</td>
</tr>
<tr>
<td><strong>Oil Bank Front Dimensionless Breakthrough Time</strong> $t_{DBT}^B$</td>
</tr>
<tr>
<td><strong>Solvent Front Dimensionless Breakthrough Time</strong> $t_{DBT}^C$</td>
</tr>
<tr>
<td><strong>Oil Bank Front Dimensionless Sweepout Time</strong> $t_{DBW}^B$</td>
</tr>
<tr>
<td><strong>Solvent Front Dimensionless Sweepout Time</strong> $t_{DBW}^C$</td>
</tr>
<tr>
<td><strong>Length of Waterflood</strong> $t_{life2}$</td>
</tr>
<tr>
<td><strong>Dimensionless Switching Time from Waterflood to CO$_2$ Flood</strong> $t_{DSST}$</td>
</tr>
</tbody>
</table>

Figure 7-12 illustrates a profile of dimensionless front locations in the reservoir, parameterized by storage capacity, at $t_D = 1.1$. At this time, both the oil bank and
solvent fronts have broken through at the producer. The waterflood front already swept out before CO₂ injection started.

![Figure 7-12: Front locations for each storage capacity at $t_{DCO_2} = 0.4$ ($t_D = 1.1$)](image)

For comparison, a similar profile (Figure 7-13) of the reservoir for an earlier dimensionless time is calculated assuming $t_{D,ST} = 0.2$. In Figure 7-13, all three fronts are still present in the reservoir.
Equations derived in chapter five are used to estimate the average oil saturation in the reservoir as a function of dimensionless time. Similar to the waterflood case, total recovery efficiency $E_R$ is then calculated. The additional recovery efficiency delivered by the CO$_2$ flood is found by subtracting the recovery efficiency at the beginning of the CO$_2$ flood from $E_R$. Figure 7-14 displays the results of these calculations. Additional recovery is possible up to the final oil saturation by continuing the flood for a long time, but it would likely become uneconomic well before this point.
Figure 7-14: Additional recovery efficiency of the CO₂ flood and average reservoir oil saturation versus dimensionless time

Table 7-12 summarizes parameters associated with injection rate calculations in a layered medium for the solvent flood. This injection rate is computed using Equation 6-29, which is adaptation of Christman and Gorell’s model. The injection pressure is higher here than it is for the waterflood to ensure oil is recovered on a realistic time scale.
Table 7-12: Additional parameters for CO\textsubscript{2} SWAG injection rate calculations

<table>
<thead>
<tr>
<th>CO\textsubscript{2} SWAG Injection Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Injector-Producer Distance</strong></td>
</tr>
<tr>
<td><strong>Injector BHP</strong></td>
</tr>
<tr>
<td><strong>Producer BHP</strong></td>
</tr>
<tr>
<td>Incremental Pressure Above MMP at Which Producers Operate</td>
</tr>
<tr>
<td><strong>Pressure Drop</strong></td>
</tr>
<tr>
<td>Dimensionless Effective Wellbore Radius</td>
</tr>
<tr>
<td><strong>Heterogeneity Factor</strong></td>
</tr>
<tr>
<td>Total Relative Mobility of Region at Initial Oil Saturation</td>
</tr>
<tr>
<td>Total Relative Mobility of Post-Waterflood Region</td>
</tr>
<tr>
<td>Total Relative Mobility of Oil Bank</td>
</tr>
<tr>
<td>Total Relative Mobility of Region Behind Solvent Front</td>
</tr>
<tr>
<td>Average Fluid Injection/Production Rate per Well for the First Pore Volume of Production</td>
</tr>
</tbody>
</table>

Figure 7-15 plots the total relative mobility of alternative water-CO\textsubscript{2} mixtures, parameterized by water saturation. Larger values of \( S_w \) are associated with larger WAG ratios, based on the intersection of \( f_{wj} \) with the water-solvent fractional flow curve. This association can be seen in Figure 7-11 by observing the positive slope of the water-solvent fractional flow curve; increasing \( f_{wj} \) moves its intersection saturation with the curve to the right towards larger \( S_w \). Increasing the WAG ratio improves sweep in the reservoir because it lowers the total relative mobility of the water-CO\textsubscript{2} mixture. The tradeoff of this improved sweep is a smaller injection rate of the fluid because of its lower
total mobility. The determination of the optimal WAG ratio is ultimately rooted in economics, so this question will be considered later in the context of optimizing the reservoir’s NPV.

Figure 7-15: Total relative mobility of water-CO$_2$ mixtures

The Christman-Gorell-based method of injection rate estimation is illustrated in Figure 7-16 for the CO$_2$ flood of the hypothetical reservoir. The total mobility of the region behind the solvent front is greater than those of the post-waterflood and oil bank regions, so as the flood progresses and the solvent front sweeps more of the reservoir, the fluid injection rate increases.
Figure 7-16: Injection rate versus time for the CO₂ flood (constant Δp for injection)

These injection rates are used to map recovery efficiency and average oil saturation specified in terms of dimensionless time, shown in Figure 7-14, to actual time. This mapping is accomplished using the trapezoidal rule method illustrated in Table 6-1 in the previous chapter. The results of this process are in Figure 7-17.
The economics of a CO₂ flood are influenced by the quantity of CO₂ required for injection, the size of processing facilities for recycling produced fluids, and the rate at which CO₂ is recycled. The mechanics of how these technical factors operate are approximated here; the results are used later to estimate the flood’s costs. Here we neglect CO₂ solubility in water.

The total rate of CO₂ injected into the field at reservoir conditions at a given time is found by rearranging the definition of the WAG ratio, which yields the solvent injection rate as a function of the WAG ratio and the total fluid injection rate:
\[ Q_{CO_2,IC} = \frac{Q_{tot,IC}}{1+W_R} \]

Equation 7-2

where the “IC” portion of the subscripts indicate bottomhole injection conditions. Appropriate conversion factors are used to convert reservoir barrels to thousands of cubic feet.

\[ Q_{CO_2,IC} \] is converted to a rate at surface conditions simply by dividing \[ Q_{CO_2,IC} \] by the CO\(_2\) formation volume factor \( B_{CO_2} \). This formation volume factor is given by the density of CO\(_2\) at standard conditions of 70\(^\circ\)F and 14.7 psia divided by the density of CO\(_2\) at injection conditions:

\[ B_{CO_2} = \frac{\rho_{CO_2,sc}}{\rho_{CO_2,IC}} \]

Equation 7-3

Appendix E includes VBA code for a custom Excel function that combines tables from Jarrell et al. (2002) and correlations from Ouyang (2011) to calculate CO\(_2\) density given a temperature and pressure. This function yields a \( B_{CO_2} \) of 0.0221 RB/STB for the hypothetical reservoir.

Ettehadtavakkol (2013) performs an analysis of projects that combine CO\(_2\) EOR and storage in reservoirs. A crucial time-varying technical parameter he considers that impacts the economics of a CO\(_2\) flood is the reservoir CO\(_2\) recycle ratio \( R_{res} \), defined as the ratio of CO\(_2\) production rate from the reservoir to the fresh CO\(_2\) injection rate. To account for the fact that some CO\(_2\) is lost between the bottom of the producers and the
reinjection stream, he multiplies this reservoir recycle ratio 0.85 to yield the net recycle ratio $R_{net}$. He performs numerical simulations for representative sandstone and carbonate reservoirs to compute the reservoir recycle ratio as a function of dimensionless time; the results of these simulations appear in Figures 7-18 and 7-19.

Figure 7-18: Reservoir CO$_2$ recycle ratio for sandstone (from Ettehadtavakkol 2013)

Figure 7-19: Reservoir CO$_2$ recycle ratio for carbonate (from Ettehadtavakkol 2013)
The recycle ratios in these figures, which are dimensionless, are approximated here as functions of dimensionless time by the following piecewise linear functions:

\[
R_{res} = \begin{cases} 
  \text{sandstone} & \left(\frac{2.65}{1.38-0.05}\right)(t_{DCO_2} - 0.05) \quad \text{if } t_{DCO_2} < 0.05 \\
  \text{carbonate} & \left(\frac{1.75}{0.64-0.025}\right)(t_{DCO_2} - 0.025) \quad \text{if } t_{DCO_2} \geq 0.05 
\end{cases}
\]

Equation 7-4

To find the fresh CO\textsubscript{2} injection rate at standard conditions, the definition of the recycle ratio can be rearranged and reparameterized in terms of the total CO\textsubscript{2} injection rate, which is simply the sum of the fresh CO\textsubscript{2} injection rate and the recycled CO\textsubscript{2} injection rate:

\[
Q_{CO_2 \ fresh \ SC} = \frac{Q_{CO_2 \ SC}}{1+R_{net}}
\]

Equation 7-5

\[
Q_{CO_2 \ recycled \ SC} = Q_{CO_2 \ SC} - Q_{CO_2 \ fresh \ SC}
\]

Equation 7-6

Figure 7-20 illustrates the breakdown of fresh versus recycled CO\textsubscript{2} usage as the flood progresses for the hypothetical reservoir. Also plotted is the net recycle ratio. The estimations for later dimensionless times in the diagram do not necessarily represent economic portions of the flood and are shown to illustrate the theoretical dynamics of CO\textsubscript{2} would behave in the field. Predictably, the amount of recycled CO\textsubscript{2} reinjected increases over time as more solvent reaches the producers. This pattern leads to reduced
purchases of fresh CO\textsubscript{2} over time but increased costs from operating the CO\textsubscript{2} plant and compressors.

Figure 7-20: Breakdown of field CO\textsubscript{2} usage versus dimensionless time

CO\textsubscript{2} from the recycle plant must be repressurized for injection. Ettehadtavakkol (2013) also performs calculations to determine the capacity and costs of a representative compressor for this application. This representative compressor has a capacity of 3.6 MMscf per day. It is assumed that an operator can only buy compression capacity in increments of 3.6 MMscf per day, so if, for example, 3.60001 MMscf per day or more of capacity is needed, two compressors are purchased. Figure 7-21 shows the quantity of compressors the operator must have in operation as a function of time under these
assumptions. The number of compressors needed is a step function since they can only be purchased in fixed increments.

![Graph showing CO₂ recycled for field and compressors needed over time.]

Figure 7-21: Field compression capacity needs

7.3 **ECONOMIC ASSUMPTIONS**

7.3.1 Introduction

The economic assumptions considered here are adapted from those discussed in chapter three. With the geologic-parameter-based model, it is possible to estimate costs such as those associated with water treatment and CO₂ compression for the waterflood and CO₂ flood. Therefore changes in technical design variables such as injection pressures are reflected in economic calculations for the project.
In this chapter, cash flows are calculated in monthly time intervals. NPV is determined using the following formula:

$$NPV = \sum_{t=0}^{t_{project}} \frac{CF(t)}{(1+R)^t}$$

Equation 7-7

Since production and its associated cash flows are calculated on a monthly basis, a monthly discount rate of 0.565 percent is used as this is equivalent to the annual rate of 7 percent used in chapter three. When optimizing the value of the field, $t_{project}$ is chosen such that NPV is maximized. Cumulative discounted cash flow (CDCF) is also calculated in a similar fashion for any time $t^*$, except with monthly time steps here.

$$CDCF(t^*) = \sum_{t=0}^{t^*} \frac{CF(t)}{(1+R)^t}$$

Equation 7-8

7.3.2 Capital Costs

Drilling and completion occurs for both primary and secondary recovery; for the hypothetical reservoir considered here, 36 wells are drilled for primary recovery and 49 are drilled for secondary recovery. Using the drilling and completion cost regression given by Equation 3-12, the cost for drilling at the depth of the hypothetical reservoir is $2,183,899 per well. These costs are incurred at the beginning of a phase of production.

Other basic capital costs associated with equipment for primary, secondary, and tertiary recovery are estimated based on the EIA cost study (2010) and scaled to current values using the CERA UCCI, as is shown in chapter three. These costs depend on depth and geographic location and are reproduced in Table 7-13. A linear interpolation is used
to find the cost for the hypothetical reservoir’s depth. Again, costs such as those for separators, pumps, storage tanks, manifolds and flowlines are included in these figures while tubing costs have been excluded from the EIA estimates for primary recovery since they are assumed to be accounted for with drilling and completion costs. The EIA study also includes infill drilling costs for secondary recovery, which have been removed here. The additional equipment CAPEX costs for secondary recovery figures are quoted by the EIA on a per 11-injector-well basis. Here it is assumed that the additional equipment costs incurred at the beginning of secondary recovery, which include tubing replacement and pumping equipment, are also incurred at the beginning of CO₂ flooding. Other CAPEX relevant to CO₂ flooding will also be considered below.

Table 7-13: Equipment CAPEX incurred at the beginning of recovery phases

<table>
<thead>
<tr>
<th></th>
<th>Producing Depth (ft.)</th>
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<tbody>
<tr>
<td></td>
<td>4000</td>
<td>8000</td>
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<tr>
<td><strong>Costs per 10 Producers (2009 Dollars)</strong></td>
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<tr>
<td>Primary Recovery</td>
<td>$1,507,000</td>
<td>$2,250,400</td>
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<tr>
<td>(Rocky Mountain Region figures)</td>
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<tr>
<td><strong>Costs per 11 Injectors (2009 Dollars)</strong></td>
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<tr>
<td>Secondary Recovery</td>
<td>$1,931,800</td>
<td>$3,462,100</td>
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<tr>
<td>(West Texas figures)</td>
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<tr>
<td><strong>Costs per 10 Producers (2013 Dollars)</strong></td>
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<td></td>
</tr>
<tr>
<td>Primary Recovery</td>
<td>$1,690,780</td>
<td>$2,524,839</td>
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<tr>
<td>(Rocky Mountain Region figures)</td>
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<tr>
<td><strong>Costs per 11 Injectors (2013 Dollars)</strong></td>
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<tr>
<td>Secondary Recovery</td>
<td>$2,167,385</td>
<td>$3,884,307</td>
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<td>(West Texas figures)</td>
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</table>
The EIA cost study figures are based on the assumption that production wells and their associated equipment are capable of handling 200 barrels of liquid per day for primary production and 350 barrels per day for secondary production. As the wells considered in the model here might handle more liquid, especially for later cases where production is accelerated, it is necessary to account for increased equipment costs. For simplicity, a 100 percent increase in liquid production over these base values of 200 and 350 barrels per day are assumed to require a 20 percent increase in equipment costs. For primary recovery, the liquid production rate used to scale equipment costs is taken to be the maximum flow rate per well from the constant rate period while for secondary and tertiary recovery, it is taken to be the average injection rate into a well for the first pore volume of fluid injected into the reservoir.\(^{60}\) The average liquid production rates and calculated cost adjustment factors for the base case of the hypothetical reservoir considered here are illustrated in Table 7-14.

Table 7-14: Adjusting costs for higher liquid production rates

<table>
<thead>
<tr>
<th>Production Method</th>
<th>Assumed Liquid Handling Capacity from EIA Study (B/D)</th>
<th>Average Production Rate for Production Method (base case scenario, B/D)</th>
<th>Adjustment in EIA Costs Due to Higher Production Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>200</td>
<td>250.0</td>
<td>1.05</td>
</tr>
<tr>
<td>Secondary</td>
<td>350</td>
<td>368.2</td>
<td>1.01</td>
</tr>
<tr>
<td>Tertiary</td>
<td>350</td>
<td>1457.4</td>
<td>1.63</td>
</tr>
</tbody>
</table>

\(^{60}\) Since fluids in the reservoir are assumed to be incompressible, the liquid injection rate is equal to the production rate.
Finally, based on the development pattern assumed for the reservoir, 36 producer wells are used for primary recovery; these are converted to injectors for secondary and tertiary recovery. The 49 wells drilled for secondary recovery become producers for secondary and tertiary recovery. Wells incur the capital equipment costs associated with primary when they are drilled, regardless of recovery period, and the additional capital equipment costs for secondary recovery are applied to the 36 injectors at the beginning of secondary and tertiary recovery. The repeating of additional equipment costs for tertiary accounts for wear of equipment during secondary production.

The CO₂ flood has CAPEX associated with the recycle plant, as discussed in chapter three, and the purchase of compressors. The latter cost is given by Ettehadavakkol (2013) as $1,880,000 per compressor. The former cost varies depending on the capacity needed. This plant cost is calculated from capacity cost estimates from the 2006 Advanced Resources International study, adjusted to current dollars using the UCCI. Here, this capacity cost is multiplied by the maximum recycled CO₂ injection rate expected within the first two pore volumes of production from CO₂ flooding. Table 7-15 illustrates this recycle plant cost estimate.

Table 7-15: CO₂ plant cost estimate

<table>
<thead>
<tr>
<th>Cost of Plant from ARI Study ($/MMscf/day, 2006 Dollars)</th>
<th>Needed Capacity (Max. Recycled CO₂ for First Two Pore Volumes of Flood, MMscf/D)</th>
<th>Adjustment for Cost Inflation Since 2006</th>
<th>Total CO₂ Plant Cost (2013 Dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$700,000</td>
<td>35.0</td>
<td>1.55</td>
<td>$38,105,518</td>
</tr>
</tbody>
</table>
Table 7-16 summarizes the capital costs associated with each phase of production. The costs in this table are incurred at the beginning of each production phase, while the CAPEX for compressors occurs throughout the CO₂ flood as additional compressors are needed. Purchases of CO₂ are considered in the next section.

Table 7-16: CAPEX for each recovery phase for base case of hypothetical reservoir (excluding compressor purchases)

<table>
<thead>
<tr>
<th>Production Method</th>
<th>Drilling, CO₂ Plant, and Well Equipment CAPEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>$86,587,891</td>
</tr>
<tr>
<td>Secondary</td>
<td>$127,861,279</td>
</tr>
<tr>
<td>Tertiary</td>
<td>$54,274,925</td>
</tr>
</tbody>
</table>

7.3.3 Operating Costs

Basic operating costs are estimated from the EIA cost study in a similar manner as are equipment capital costs. Operating costs for primary production tallied in the study include supervision, overhead, labor, auto usage, chemicals, fuel, power, water, operative supplies, and surface and subsurface maintenance. A similar set of costs are tallied for secondary recovery. These costs are reported for sets of 10 producer wells for primary, 11 injector wells for secondary, at various well depths and by region of the U.S. As is also the case for equipment costs, secondary recovery costs are only reported for West Texas, whereas primary recovery costs are available for the Rocky Mountain region. The basic operating costs for secondary recovery are also assumed to apply to tertiary recovery.
Table 7-17 lists the annual operating costs reported in the EIA study, both in 2009 dollars and converted to current dollars using the CERA UOCI. Linear interpolation is used to calculate the operating cost for the depth of the hypothetical reservoir for each phase; the resulting costs are converted to a monthly per well basis and multiplied by the last column of Table 7-14 to adjust for production above what is assumed for the EIA cost study. The results of these calculations are given in Table 7-18. For primary recovery, there are 36 producers, which also happens to be the number of injectors used throughout secondary and tertiary production.

Table 7-17: Basic OPEX for recovery phases from EIA study

<table>
<thead>
<tr>
<th>Producing Depth (ft.)</th>
<th>4000</th>
<th>8000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Recovery</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Rocky Mountain Region figures)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual Costs per 10 Wells (2009 Dollars)</td>
<td>$285,400</td>
<td>$394,200</td>
</tr>
<tr>
<td>Annual Costs per 11 Injector Wells (2009 Dollars)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Recovery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(West Texas figures)</td>
<td>$706,900</td>
<td>$1,012,900</td>
</tr>
<tr>
<td>Annual Costs per 10 Wells (2013 Dollars)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Recovery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Rocky Mountain Region figures)</td>
<td>$320,864</td>
<td>$443,183</td>
</tr>
<tr>
<td>Annual Costs per 11 Injector Wells (2013 Dollars)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Recovery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(West Texas figures)</td>
<td>$794,740</td>
<td>$1,138,763</td>
</tr>
</tbody>
</table>
Table 7-18: OPEX on per-well bases for hypothetical reservoir

<table>
<thead>
<tr>
<th>Production Method</th>
<th>OPEX (Current $/Producer Well-Month for Primary, Current $/Injector Well-Month for Secondary and Tertiary)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>$3,343</td>
</tr>
<tr>
<td>Secondary</td>
<td>$7,400</td>
</tr>
<tr>
<td>Tertiary</td>
<td>$11,959</td>
</tr>
</tbody>
</table>

Additional OPEX is calculated based on the amount of fluid production from secondary and tertiary recovery from the field.

For waterflooding, produced water must be separated and treated for reinjection or disposal. Based on findings from Welch and Rychel (2004), these costs are estimated to be roughly $1.00 per barrel of produced water for Wyoming. For any given time, the water cut found in Equation 7-1 is multiplied by the fluid injection rate to yield the water production rate. Multiplying this water production rate by the length of time considered and the water treatment cost per barrel in turn yields the water treatment cost for a given time period. The water treatment costs for tertiary recovery are assumed to be included in the cost of the OPEX for the CO$_2$ recycle plant.

The CO$_2$ purchase costs, recycle plant OPEX, and compressor OPEX are considered for tertiary recovery. Fresh CO$_2$ is assumed to be priced at 1 percent of the Wyoming oil price$^{61}$ per Mscf plus a $0.50 per Mscf delivery charge (Cook 2012). Ettehadtavakkol (2013) calculates the fixed operational cost of a CO$_2$ recycling plant to be $3,150,000 per year for a plant with a 6,000 metric tonne per day CO$_2$ processing capacity; converting this capacity to a Mscf per day basis yields $27.27 per year of fixed operating costs for each Mscf per day of plant processing capacity. Multiplying this by

---

$^{61}$ Taken to be the WTI price discounted by $14.54.
the 35,000 Mscf per day capacity of the plant yields an annual fixed OPEX of $955,327 for the hypothetical reservoir’s plant. Ettehadtavakkol (2013) also estimates a recycling plant’s variable costs to be $21.70 per tonne of CO\textsubscript{2} processed; this is equivalent to $1.13 per Mscf processed. Finally, he finds the OPEX for a compressor with 3.6 MMscf per day of capacity to be $388,000 per year; on a monthly basis this is $32,333.

### 7.3.4 Other Assumptions and Example CDCF Calculation

The same assumptions regarding taxes and royalties outlined in chapter three are used here when calculating the cash flow for \( t \) months since the beginning of primary production:

\[
CF(t) = Revenues(t) - Revenues(t)(Roy_{total})(T_{sev}) - Revenues(t - 1) * (T_{advat}) - Costs(t)
\]

Since ad valorem taxes in Wyoming are assessed on 100 percent of the value of a prior year’s production, it is assumed here that these taxes are paid in the first month of each year for an entire previous year.

Revenues for a given month are calculated based on oil production, the real WTI price, and the discount from WTI for Wyoming oil:

\[
Revenues(t) = (OOP_{STB})[E_R(t) - E_R(t - 1)](P_{t,WTI} - D)
\]

Finally, the total costs of production in month \( t \) are the sum of the CAPEX and OPEX, which in turn depend on the production method used in that month and other variables discussed above:

\[
Costs(t) = CAPEX(t) + OPEX(t)
\]
Figure 7-22 illustrates the results of a calculation of cumulative discounted cash flow for the example discussed thus far in this chapter. The WTI oil price is taken to be constant at $79.15 for the life of the field. Primary recovery lasts 30 months while waterflooding lasts 411 months (34.25 years). As the CDCF declines towards the end of waterflooding and never recovers afterwards, an operator of this hypothetical reservoir with the cost assumptions and design choices discussed above would have realized more value from the field by not pursuing the CO₂ flood, even though it improves the recovery efficiency. This is by no means a general conclusion, as the operator has not been given the option to adjust development parameters yet; therefore the development choices for the field are far from optimal. Moreover, the project costs may differ even for a similar project in another geographic location.

Figure 7-22: CDCF and recovery efficiency for base case of a hypothetical reservoir
7.4 DEFINING THE OPTIMIZATION PROBLEM

7.4.1 Decision Variables

For essentially all private operators, the goal in developing a reservoir is to maximize its value. This optimization problem can have many decision variables; fortunately, with the way the geologic-parameter-based model has been defined, a select few can be isolated. Table 7-19 outlines the key variables used here.

Table 7-19: Decision variables for reservoir life cycle optimization

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>(t_{\text{life}2})</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>(n_{\text{pat,side}})</td>
</tr>
<tr>
<td>Production Rate for Wells in Constant Rate Period of Primary Production</td>
<td>(q_{\text{o,max}})</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>(p_{w_{i}}(WF))</td>
</tr>
<tr>
<td>CO₂ Flood Injection Pressure</td>
<td>(p_{w_{i}}(CO₂))</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>(W_R)</td>
</tr>
</tbody>
</table>

The length of primary recovery is already taken to be constrained by when the average reservoir pressure reaches the bubble point, but the length of the waterflood is crucial parameter the cooperator can directly control that impacts the economics of the reservoir’s development. Once the length of the waterflood is chosen, the length of the
CO₂ flood is defined based on the time when the CDCF is maximized. This time is the economic limit for the field, as increased production does not generate any more value. In extreme cases, the waterflood may be very short or the CO₂ flood may be entirely uneconomic. For the latter, the optimal solution would suggest terminating field development before the theoretical maximum recovery from waterflooding is attained, as the costs of production would be greater than the revenues.

Well spacing is a crucial parameter that impacts the speed at which oil is produced, as it determines drainage areas for primary recovery and injector-producer distances for secondary and tertiary recovery. It also determines the number of wells drilled; drilling and completion costs are a large portion of CAPEX. For simplicity, the square geometry of well patterns for primary recovery, which are converted to five spots through infill drilling for secondary and tertiary, as discussed in previous chapters, is assumed to be fixed here. Since the areal geometry of the reservoir is taken to be a square and the well patterns are evenly spaced, the well spacing is determined by the number of patterns fit into the reservoir. The decision variable for well spacing is taken to be the square root of the number of square primary recovery patterns in the reservoir. A primary recovery pattern is defined as a square cluster of four wells, as shown in Figure 4-4. Infill drilling occurs at the beginning of secondary recovery such that each production well for primary recovery is converted to an injector well placed at the center of a five spot pattern. This injector is used for secondary and tertiary recovery. Thus the choice for \( n_{\text{pat,side}} \) fixes other well spacing parameters.
The oil flow rate during the constant rate period of primary recovery impacts the length of time the operator spends on primary recovery, since a higher flow rate depletes the reservoir at a faster rate and hastens the time it takes for the average reservoir pressure at reach the bubble point (Walsh and Lake 2003). This flow rate also affects the costs the operator faces for primary recovery, since a higher production rate leads to higher equipment and operating costs.

Injection pressures for waterflooding and CO\textsubscript{2} flooding certainly have substantial impacts on the economics of the field, as they determine the fluid production rates of these phases of recovery. These production rates impact both the revenues from saleable oil and the costs of handling produced fluids. The bottomhole pressures of the producers are assumed to be 2100 psi for waterflooding and 2120 for CO\textsubscript{2} flooding. The latter is necessary to remain above the minimum miscibility pressure for the CO\textsubscript{2} flood, while the former is chosen since the produced fluids from the waterflood are relatively heavy and therefore need substantial bottomhole pressures to reach the surface, likely even with the aid of artificial lift equipment. An extension of the analysis that considers multiple artificial lift options is possible, although it is not pursued here since it requires knowledge of the cost-benefit tradeoffs of each option for a particular location. As granular cost information is not readily available, such an analysis is beyond the scope of this study.

Finally, the WAG ratio has a substantial impact on the economics of the CO\textsubscript{2} flood, since it alters the fractional flow calculations, Koval factors determined by
Mollaei’s regressions, and fluid injection rate. As discussed earlier in this chapter, the optimal WAG ratio entails a tradeoff between sweep efficiency and fluid injection rate.

Aside from the decision variables discussed here and their associated dependent variables as defined by the geologic-parameter-based model, other physical reservoir parameters discussed in this chapter are assumed to be the same as previously specified for the base case hypothetical reservoir for the optimization problem. Some of these reservoir characteristics and costs are modified later to determine the sensitivity of the optimal solution to various changes in parameters.

7.4.2 Objective Function

The objective function used in this chapter is the maximum of the CDCF function; choosing this is equivalent to maximizing the NPV. Since ongoing overhead operating costs can eventually overwhelm revenues and deteriorate the economics of production, there is a point beyond which NPV of the field would decline with additional production. Therefore the maximum of the CDCF function is appropriate since it gives the highest NPV that the field is projected to attain.

7.4.3 Constraints

This optimization problem also requires constraints to ensure the solutions that the algorithm produces are feasible:

\[
t_{project} = t_{Life1} + t_{Life2} + t_{Life3} \leq 120 \times 12 \text{ months}
\]

\[
t_{Life2} \geq 1 \text{ months}
\]

\[
t_{Life3} \geq 0 \text{ months}
\]
\[ n_{\text{pat,side}} \geq 1 \]
\[ q_{o,\text{max}} \leq 1500 \text{ STB/D} \]
\[ 2200 \text{ psi} \leq p_{wi,WF} \leq 4800 \text{ psi} \]
\[ 2200 \text{ psi} \leq p_{wi,CO_2} \leq 4800 \text{ psi} \]
\[ 1 \leq W_R \leq 5 \]
\[ S_{WB} < S_{WWF} \]
\[ v_C \leq f_w' \big|_{swb} \]

The final two constraints pertain to the CO\textsubscript{2} fractional flow construction; the water saturation corresponding to point B of Figure 5-4 cannot exceed that of point I. The line between points F and J, which determines the specific velocity of the solvent front, must intersect the water-oil fractional flow curve at a point B when extended. Generally, the tangent to the water-oil fractional flow curve will have a slope greater than the specific velocity of the solvent front. When the line that determines the solvent front specific velocity is exactly tangent to the water-oil fractional flow curve, \( f_w' \big|_{swb} \) is exactly equal to \( v_C \). Otherwise, the construction is not feasible as the line that determines \( v_C \) no longer intersects the water-oil fractional flow curve.

The injection pressures are assumed to be limited by the conservative estimate of the formation parting pressure and approximately the bottomhole pressures of the producers. Durations of each production phase and \( n_{\text{pat,side}} \) are assumed to take integer

---

\(^{62}\) \( f_w \) here corresponds to the water-oil fractional flow curve.
values. The WAG ratio is constrained by the range of values Mollaei (2011) considers for his analysis; he notes these values are typical for actual floods.

7.5 Deterministic Optimization of the Model

7.5.1 Water-Wet Sandstone

Palisade Evolver, an Excel add-in that includes genetic algorithms and the OptQuest optimization engine, is used to maximize the water-wet sandstone (base case) reservoir’s NPV, assuming a constant WTI price of $79.15 per barrel. Evolver adjusts the decision variables and searches for better solutions, constantly updating the best solution until it is unlikely to find a solution that is any better. New solutions are attempted by using “mutations” of existing solutions so that the algorithm avoids getting trapped in local maxima that may not represent the global maximum. While it is not possible to know whether the algorithm has found the global maximum for a problem, as it could search indefinitely to refine the best solution, for practical purposes a cutoff time of 10 minutes is chosen as it appears that no further progress is made minutes before this time; the resulting solution is considered optimal.

Table 7-20 summarizes the optimal values of the decision variables, the length of each recovery phase, and the estimated NPV of the project, as determined by Evolver.
Table 7-20: Optimal variables determined by Evolver, water-wet sandstone

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>$t_{\text{life2}}$</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{\text{pat,side}}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{\text{o,max}}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{\text{wi}}$ (WF)</td>
</tr>
<tr>
<td>CO$_2$ Flood Injection Pressure</td>
<td>$p_{\text{wi}}$ (CO$_2$)</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Key Parameters</th>
<th>Value Associated with Evolver’s Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
<td>$t_{\text{life1}}$</td>
</tr>
<tr>
<td>Length of CO$_2$ Flood</td>
<td>$t_{\text{life3}}$</td>
</tr>
<tr>
<td>Length of Project</td>
<td>$t_{\text{project}}$</td>
</tr>
<tr>
<td>NPV of Project</td>
<td></td>
</tr>
</tbody>
</table>

The results from Evolver in Table 7-20 indicate the optimal NPV is attained through an accelerated production strategy, but not necessarily via a heavy drilling campaign. 41 wells are drilled over the life of the field, which results in an injector-producer distance of 1650 feet for secondary and tertiary recovery. Particularly noteworthy is how the optimal solution essentially bypasses the waterflood by minimizing both its length and opting for a CO$_2$ flood with a high WAG ratio instead. Acceleration is achieved by using a relatively high flow rate per well for primary recovery, which means that primary lasts just over a year before the bubble point is reached. The CO$_2$ flood entails increasing the injection pressure to the
upper constraint of the formation parting pressure to ensure the production rate is maximized. The WAG ratio is also relatively high. Indeed, the CO₂ flood essentially serves as a carbonated waterflood, especially considering the incomplete sweep of the reservoir because of the immediate switch from waterflooding to CO₂ flooding.

Figure 7-23 illustrates the evolution of recovery efficiency and the project’s value over time. Most of the project’s NPV is realized within 40 years, with the majority of the project’s NPV delivered by the CO₂ flood. The latter result is in contrast to the conclusions the Lost Soldier Tensleep analysis by Parra Sanchez (2010) and in chapter three, which find that primary recovery delivers the majority of the field’s NPV. The difference here is likely because of the differences in reservoir drive mechanisms and modeling techniques used for the reservoirs. The Lost Soldier Tensleep reservoir exhibits water drive, which leads to high recovery efficiencies for primary recovery at a relatively low cost to the operator (Brokmeyer et al. 1996). Solution gas drive, which is assumed here, is a much weaker drive mechanism that leads to relatively low ultimate recovery efficiencies for primary recovery and leaves a substantial quantity of target oil in the reservoir for recovery via displacement processes. Moreover, the decline curve model used in chapter three does not provide the flexibility to alter design parameters that could be used to improve the value of displacement processes. By contrast, the model discussed here provides the ability to model acceleration of displacement production by techniques such as increasing fluid injection rates.
Figure 7-23: CDCF and recovery efficiency for optimal solution, water wet sandstone

If the oil price is assumed to fluctuate, the ultimate NPV outcome that occurs for the field will vary depending on the actual realization of the oil price over time. This price behavior is simulated here using a mean-reverting process with a constant mean and monthly parameters given in cells shaded in gray in Table 2-2. Half the fitted volatility is assumed here as this intuitively seems more plausible. The mean-reverting process is used to conduct a Monte Carlo simulation with 10,000 realizations of the water-wet sandstone reservoir’s NPV. This simulation yields a distribution of possible optimal NPV outcomes if the operator were to use the optimal decision variables listed in Table 7-20 (obtained by assuming the price is constant). Here, the operator does not alter the decision variables in response to fluctuations in price, but does cease the project when the maximum CDCF is attained for each price realization.
The histogram in Figure 7-24 summarizes this distribution of NPV outcomes assuming the mean-reverting price process governs future oil price behavior, costs are uncorrelated with the oil price with the exception of the CO₂ purchase cost, which is directly linked to it, and the operator has perfect information regarding the technical aspects of the reservoir but imperfect information regarding the realization of the oil price. This distribution of possible NPV outcomes on inspection looks like it is lognormal in nature, which is consistent with the way the mean-reverting process models oil prices and the results of similar Monte Carlo simulations for the decline-curve-based model discussed in chapter three.

Figure 7-24: NPV outcome distribution for water-wet sandstone – mean-reverting price and operator has imperfect information on price realizations
Figure 7-25 illustrates the front positions at a dimensionless time very close to when the oil bank front merges with the waterflood front. This condition occurs for the optimal solution given by parameters in Table 7-20. The occurrence of merging for the optimal solution underscores the importance of characterizing the behavior of fronts under these conditions.

![Front Locations Diagram](image)

Figure 7-25: Front locations for each storage capacity at $t_{DCO_2} = 0.37$, optimal solution for water wet sandstone

### 7.5.2 Oil-Wet Carbonate

An alternative case of an oil-wet carbonate reservoir that otherwise has the same characteristics as the reservoir described previously in this chapter is considered here.
The difference in the lithology of the reservoir affects the evolution of the CO$_2$ recycle ratio over time, as outlined in Equation 7-4. Table 7-21 itemizes the parameters used here to characterize the relative permeability of the oil-wet reservoir. The oil and water saturations at the switch to waterflooding will vary depending on the nature of primary recovery, which is in turn determined by some of the decision variables, but since the oil saturation does not change much during primary recovery, the values listed here are representative. The largest differences here versus the water wet case are the endpoint relative permeability for water and the oil Corey exponent.

Table 7-21: Parameters for calculating relative permeability and mobility, oil-wet carbonate

<table>
<thead>
<tr>
<th>Relative Permeability and Mobility</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual Oil Saturation to Waterflooding</td>
<td>$S_{or}$</td>
</tr>
<tr>
<td>Oil Saturation at Switch to WF</td>
<td>$S_{oiWF}$</td>
</tr>
<tr>
<td>Water Saturation at Switch to WF</td>
<td>$S_{wiWF}$</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Oil</td>
<td>$k_{ro}^o$</td>
</tr>
<tr>
<td>Endpoint Relative Permeability for Water</td>
<td>$k_{rw}^o$</td>
</tr>
<tr>
<td>Oil Corey Exponent</td>
<td>$m$</td>
</tr>
<tr>
<td>Water Corey Exponent</td>
<td>$n$</td>
</tr>
<tr>
<td>Water Viscosity</td>
<td>$\mu_w$</td>
</tr>
<tr>
<td>Oil Viscosity at Bubble Point</td>
<td>$\mu_{oB}$</td>
</tr>
<tr>
<td>Endpoint Mobility Ratio</td>
<td>$M^o$</td>
</tr>
</tbody>
</table>

Based on these parameters, the relative permeability curves are calculated in Figure 7-26. These curves are then used to create the waterflood fractional flow
construction in Figure 7-27. For this case, the $S_{oWF}$ as predicted by fractional flow is 0.547 whereas Mollaei’s correlation estimates it to be 0.456.

Figure 7-26: Relative permeability curves for oil-wet carbonate
Evolver is used to perform a deterministic optimization of the oil-wet carbonate reservoir’s NPV with the same constraints described in the previous section and assuming a constant WTI price of $79.15. The results of this optimization are summarized in Table 7-22; they are similar to those for the water-wet sandstone reservoir. Here the optimal solution suggests even fewer wells and the maximum oil flow rate for the constant rate period of primary recovery. The optimal solution here still suggests eschewing the waterflood by minimizing the time spent on it. It also recommends maximizing the injection pressure and WAG ratio for the CO₂ flood.
Table 7-22: Optimal variables determined by Evolver, oil-wet carbonate

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>$t_{\text{life}2}$</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{\text{pat,side}}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{\text{o,max}}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{\text{wi(WF)}}$</td>
</tr>
<tr>
<td>CO$_2$ Flood Injection Pressure</td>
<td>$p_{\text{wi(CO}_2\text{)}}$</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

Other Key Parameters

<table>
<thead>
<tr>
<th>Value Associated with Evolver's Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
</tr>
<tr>
<td>Length of CO$_2$ Flood</td>
</tr>
<tr>
<td>Length of Project</td>
</tr>
<tr>
<td>NPV of Project</td>
</tr>
</tbody>
</table>

Figure 7-28 illustrates the accumulated value of the oil-wet reservoir and recovery efficiency versus time; again, CO$_2$ flooding delivers most of the field’s value. The recovery efficiency here is generally lower than that for the water-wet sandstone, which is reflected in a lower NPV for the field. This result intuitively makes sense as the oil tends to adhere to the rock more in this case. Such physical intuition also likely explains why the life of the project is about 15 years longer than it is in the water wet case. While most of the value of the field accrues within the first 15 years of production, the CDCF does steadily increase afterwards, reflecting the steady improvement in recovery efficiency over longer time scales for oil wet reservoirs. The drop in the CDCF near the
beginning of the field’s life is due to the CAPEX associated from switching from primary to tertiary recovery.

![Graph showing CDCF and recovery efficiency for optimal solution, oil wet carbonate](image)

Figure 7-28: CDCF and recovery efficiency for optimal solution, oil wet carbonate

### 7.5.3 Water-Wet Sandstone With Smaller $S_{or}$

Returning back to the reservoir characteristics considered in section 7.5.1, the residual oil saturation $S_{or}$ is reduced from 0.4 to 0.3.$^{63}$ This change results in a smaller oil recovery target for CO$_2$ flooding post-waterflood. Maximizing the resulting system’s NPV with Evolver yields the results shown in Table 7-23.

---

$^{63}$ For $S_{or}$ values much lower than 0.3, Mollaei’s regression predicts $S_{or}$ values that do not allow for feasible CO$_2$ fractional flow constructions.
Table 7-23: Optimal variables determined by Evolver, water wet sandstone with lower $S_{or}$

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>$t_{life2}$</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{pat,side}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{o,max}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{wi}$ (WF)</td>
</tr>
<tr>
<td>CO₂ Flood Injection Pressure</td>
<td>$p_{wi}$ (CO₂)</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

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<tr>
<th>Other Key Parameters</th>
<th>Value Associated with Evolver’s Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
<td>$t_{life1}$</td>
</tr>
<tr>
<td>Length of CO₂ Flood</td>
<td>$t_{life3}$</td>
</tr>
<tr>
<td>Length of Project</td>
<td>$t_{project}$</td>
</tr>
<tr>
<td>NPV of Project</td>
<td>$1,974,311,271$</td>
</tr>
</tbody>
</table>

The optimal solution for the lower residual oil saturation case is quite similar to that of the original, higher $S_{or}$ case in section 7.5.1. In this case, the waterflood injection pressure is reduced to the bottomhole pressure of the producers, effectively shutting in the wells for the month of the waterflood. For this solution, the producer again skips the waterflood to start the CO₂ flood as soon as possible after primary recovery. Figure 7-29 depicts a profile of CDCF and recovery efficiency versus time similar to those of the original water wet case with a higher $S_{or}$. 
7.5.4 Higher CO₂ Purchase Costs

For this case, all the original reservoir characteristics (e.g. water wet sandstone and $S_{or} = 0.4$) from the original case in section 7.5.1 are used again, but now the cost per Mscf of CO₂ is quadrupled from 1 percent of the oil price to 4 percent of the oil price. This alteration to the cost structure of tertiary recovery is chosen to see if it might lead to a substantial change in the optimal solution such that CO₂ flooding is not favored so heavily in place of waterflooding. Evolver’s optimal solution is provided in Table 7-24. Despite a sharply higher operational cost of CO₂ flooding, the optimal solution is characterized by the same general pattern of using a relatively small number of wells, producing at a high rate per well for primary recovery, skipping the waterflood, injecting and producing at high rates for the CO₂ flood, and using a high WAG ratio. It appears the higher CO₂ costs have shortened the length of the project substantially, though.
Table 7-24: Optimal variables determined by Evolver, water wet sandstone with quadrupled CO₂ purchase costs

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
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</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{pat,side}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{o,max}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{wi} ,(WF)$</td>
</tr>
<tr>
<td>CO₂ Flood Injection Pressure</td>
<td>$p_{wi} ,(CO₂)$</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

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<tr>
<th>Other Key Parameters</th>
<th>Value Associated with Evolver’s Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
<td>$t_{life1}$</td>
</tr>
<tr>
<td>Length of CO₂ Flood</td>
<td>$t_{life3}$</td>
</tr>
<tr>
<td>Length of Project</td>
<td>$t_{project}$</td>
</tr>
<tr>
<td>NPV of Project</td>
<td>$1,592,667,839$</td>
</tr>
</tbody>
</table>

Figure 7-30 shows similar trends to those in Figure 7-23 for CDCF and recovery efficiency, but CDCF declines faster after it has reached its maximum value, reflecting the higher cost of CO₂.
7.5.5 Oil Price Modeled Using a Logistic Equation

In chapter three, a logistic model for the mean oil price was proposed to adapt the mean–reverting model to price forecasts and long-term economic constraints that would likely limit the growth of prices. This model was described by Equation 7-9.

$$p_{\text{mean}}(t) = \frac{75}{1 + e^{-0.3(t-10)}} + 75$$

Equation 7-9

Capital and operating costs are assumed to be correlated with $p_{\text{mean}}$ as they were with the logistically-modeled mean price in chapter three:

$$\text{CAPEX}_{\text{index,real}} = 1.32(p_{\text{mean}}) + 98.759$$

$$\text{OPEX}_{\text{index,real}} = 0.7448(p_{\text{mean}}) + 117.02$$

Equation 7-10
These CAPEX and OPEX indices are used to scale the capital and operating costs described earlier in this chapter, with the exception of the portion of the CO₂ cost that is linked directly to the oil price.

Evolver is used to determine the decision variable values that maximize the reservoir NPV assuming the oil price is described by this equation. All other reservoir characteristics are described in section 7.5.1. Table 7-25 summarizes the optimal decision variables as determined by Evolver. Compared to the solution for the case where the mean oil price is assumed to be constant, as seen in Table 7-20, this solution involves a much slower production rate during primary recovery. Otherwise, the optimal decision variables are the same, with the exception of the waterflood injection pressure at its minimum possible value.
Table 7-25: Optimal variables determined by Evolver, water wet sandstone with logistic model for oil price

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>$t_{life2}$</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{pat,side}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{o,max}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{wi,WF}$</td>
</tr>
<tr>
<td>CO₂ Flood Injection Pressure</td>
<td>$p_{wi,CO₂}$</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Key Parameters</th>
<th>Value Associated with Evolver’s Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
<td>$t_{life1}$                             13.7 years</td>
</tr>
<tr>
<td>Length of CO₂ Flood</td>
<td>$t_{life3}$                             79.2 years</td>
</tr>
<tr>
<td>Length of Project</td>
<td>$t_{project}$                           92.9 years</td>
</tr>
<tr>
<td>NPV of Project</td>
<td>$3,385,178,394$</td>
</tr>
</tbody>
</table>

The rationale for the changes in the optimal solution becomes more apparent when examining Figure 7-31. The operator produces at the slowest possible rate during primary recovery to delay the production boosts from switching to waterflooding and then CO₂ flooding until the oil price is close to its maximum value. This merely illustrates the obvious guideline that the operator should strive to produce more oil while the price is relatively high. Given operator expectations of increasing oil prices that ultimately plateau, the reservoir development strategy that maximizes NPV entails delaying the large production increases from secondary and/or tertiary recovery until the oil prices are high.
Figure 7-31: CDCF and recovery efficiency for optimal solution, water wet sandstone and logistic model of oil price

7.6 GEOLOGIC/TECHNICAL PARAMETER UNCERTAINTY VERSUS OIL PRICE UNCERTAINTY

The geologic-parameter-based model offers the opportunity to approximate how particular sources of uncertainty for model input variables contribute to uncertainty regarding the NPV of the project. The contributions of geologic/technical uncertainty and the oil price uncertainty are considered here. The optimal values for decision variables $t_{\text{life}}$, $n_{\text{pat,side}}$, $q_{\text{o,max}}$, $p_{\text{wi (WF)}}$, $p_{\text{wi (CO}_2\text{)}}$, and $W_R$ are chosen using a deterministic optimization assuming the most likely or mean values of the parameters in Table 7-26 and a constant oil price of $79.15. All other reservoir characteristics are taken from the base case of a water wet reservoir described in section 7.5.1.
Table 7-26: Uncertain geologic/technical parameters – distributions

<table>
<thead>
<tr>
<th>Variables with Triangular Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Formation Parting Pressure (psi)</td>
</tr>
<tr>
<td>Dykstra-Parsons Coefficient</td>
</tr>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>Endpoint Relative Permeability to Oil</td>
</tr>
<tr>
<td>Endpoint Relative Permeability to Water</td>
</tr>
<tr>
<td>Oil Corey Exponent</td>
</tr>
<tr>
<td>Water Corey Exponent</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables with Lognormal Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Skin</td>
</tr>
<tr>
<td>Average Permeability (mD)</td>
</tr>
</tbody>
</table>

Table 7-27 summarizes the results of the deterministic optimization, which are quite similar to those described in Table 7-20. Now \( V_{DP} \) is 0.7 instead of 0.6.
Table 7-27: Optimal variables determined by Evolver ($V_{DP} = 0.7$)

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value Determined by Evolver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Waterflood</td>
<td>$t_{life2}$</td>
</tr>
<tr>
<td>Square Root of the Number of Patterns for Primary Recovery</td>
<td>$n_{pat, side}$</td>
</tr>
<tr>
<td>Production Rate for Each Well in Constant Rate Period of Primary Production</td>
<td>$q_{o,max}$</td>
</tr>
<tr>
<td>Waterflood Injection Pressure</td>
<td>$p_{wi} (WF)$</td>
</tr>
<tr>
<td>CO₂ Flood Injection Pressure</td>
<td>$p_{wi} (CO₂)$</td>
</tr>
<tr>
<td>WAG Ratio</td>
<td>$W_R$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Key Parameters</th>
<th>Value Associated with Evolver’s Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Primary Recovery</td>
<td>$t_{life1}$ $1.2$ years</td>
</tr>
<tr>
<td>Length of CO₂ Flood</td>
<td>$t_{life3}$ $36.7$ years</td>
</tr>
<tr>
<td>Length of Project</td>
<td>$t_{project}$ $37.9$ years</td>
</tr>
<tr>
<td>NPV of Project</td>
<td>$$1,622,489,222$</td>
</tr>
</tbody>
</table>

Three Monte Carlo simulations are conducted; the first assumes that geologic parameters are uncertain with distributions described in Table 7-26. These distributions are selected to be representative of the uncertainty surrounding key characteristics of the reservoir before primary recovery begins. Here, other parameters that strongly influence oil recovery like $S_{oF}$ and Koval factors are calculated using Mollaei’s regressions. Some variables in Table 7-26 are assumed to have triangular distributions because they need to be bounded within specific ranges to yield feasible values. Others are assumed to be lognormally distributed; they do not have an upper bound per se, but must be nonnegative. Lognormal distributions are central to characterizing permeability variation in a reservoir through the Dykstra-Parsons coefficient (Lake 1989), so this type of
distribution was also used here to model uncertainty regarding the mean permeability of the reservoir. The injection pressure for the CO$_2$ flood is assumed to be described by the same distribution as is the formation parting pressure, since it was found in previous sections that the Evolver algorithm always suggested maximizing this injection pressure. All other decision variables are assumed to be static throughout each simulation at their optimal values given in Table 7-27.

The second Monte Carlo simulation assumes that geologic parameters are constant but that the oil price is uncertain and follows the mean-reverting process with a constant logarithm of the mean price and described by the parameters shaded in gray in Table 2-2. Half the fitted volatility is assumed based on judgment, as this volatility seems to be a more realistic estimate of how the oil price might behave than is the fitted volatility. The latter reflects recent relatively extreme price behavior by historical standards. Finally, the third Monte Carlo simulation combines the uncertainties of both cases, accounting both for geologic/technical parameter uncertainty and oil price uncertainty simultaneously.

The distributions of NPV outcomes that result from each of these three simulations are shown in histograms in Figures 7-32, 7-33 and 7-34. Vertical axes depict the relative frequencies of each bucket of outcomes. The geologic/technical parameter uncertainty appears to represent more of a downside risk for NPV, while the oil price uncertainty represents more of an upside risk for NPV. This observation is based on sizes of the distribution tails in the first two figures. Combining these two sources of
uncertainty results in a NPV outcome distribution that is skewed positively and therefore has more upside risk.

Figure 7-32: Case 1: NPV outcomes - technical/geologic parameter uncertainty; constant WTI price of $79.15
Figure 7-33: Case 2: NPV outcomes - oil price uncertainty (mean-reverting price model with half fitted volatility); geologic parameters known

Figure 7-34: Case 3: NPV outcomes - both geologic/technical uncertainty and oil price uncertainty
Table 7-28 summarizes metrics for the distributions corresponding to each of the Monte Carlo simulation results. The coefficient of variation is a dimensionless measure of the standard deviation of a distribution; it is simply the standard deviation divided by the mean. This measure allows for comparison of the variation associated with distributions that have different means and standard deviations. The coefficient of variation of the case with just geologic/technical uncertainty alone is 89 percent of the coefficient of variation of the case with both price and geologic/technical uncertainty combined. By contrast, the coefficient of variation of the case with just oil price uncertainty alone is 65 percent of the coefficient of variation of the case with both price and geologic/technical uncertainty combined. Moreover, the case with the geologic/technical uncertainty does not include uncertainty regarding the dimensions of the reservoir, which could add substantial uncertainty to the NPV outcome distribution in Figure 7-32. These results indicate that the geologic/technical uncertainty contributes a greater proportion of the total uncertainty regarding the project’s NPV than does uncertainty regarding oil prices.
Table 7-28: Parameters of distributions from Monte Carlo simulation results

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean NPV ($MM)</th>
<th>Standard Deviation of NPV ($MM)</th>
<th>Coefficient of Variation for Simulated Reservoir NPV (Dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Geologic/Technical Parameters Uncertain)</td>
<td>1,442</td>
<td>261</td>
<td>0.1812</td>
</tr>
<tr>
<td>2 (Oil Price Realization Uncertain)</td>
<td>1,784</td>
<td>236</td>
<td>0.1322</td>
</tr>
<tr>
<td>3 (Geologic/Technical Parameters and Oil Price Realization Uncertain)</td>
<td>1,691</td>
<td>343</td>
<td>0.2031</td>
</tr>
</tbody>
</table>

7.7 CONCLUSIONS

The results of the various cases in the previous section suggest a few general insights regarding reservoir life cycle optimization, assuming a solution gas drive reservoir that is suitable for CO\textsubscript{2} flooding:

- Overall, production is accelerated so that most recoverable oil is produced within the first 30 to 40 years of the project.\textsuperscript{64}
- This acceleration is achieved through a relatively small amount of drilling and completions, as this CAPEX is expensive. The suggested well spacing from the example in this chapter is 1650 feet, which is quite wide.
- For primary recovery, the constant flow rate is relatively high and essentially maximized for many cases considered here. These results suggest the operator should accelerate production by producing oil quickly in primary recovery. A notable exception is when the operator expects the mean oil price to increase in

\textsuperscript{64} This time scale is likely heavily dependent on the discount rate and here applies for a 7 percent real annual rate. Relative operational costs also appear to impact this time scale.
the future; in this case, he produces oil as slowly as possible in primary recovery to schedule the majority of production for the period where higher prices are expected.

- In general, an operator should schedule recovery so that he produces while the oil price is high, within reason. If the price is expected to increase permanently by $20 per barrel in 100 years, he probably should not change his behavior because the effects of discounting mean these changes are not very significant from an economic perspective today. On the other hand, if the price is expected to increase permanently by $50 per barrel in 10 years, an operator might want to delay the large increases in production from displacement methods until this price gain is realized.

- For the optimal cases considered here, acceleration is achieved for displacement methods, in particular CO₂ flooding, through injecting at the maximum feasible rate. Here, this maximum rate occurs when the injection pressure is pegged to the estimate for formation parting pressure.

- Waterflooding is essentially bypassed here for CO₂ flooding with a high WAG ratio. According to the model used in this chapter, the higher cost structure of CO₂ flooding is outweighed by the benefits of more complete oil recovery.

- Estimates of the contributions of sources of uncertainty to the realized NPV of the reservoir were computed. These estimates indicate that uncertainty in geologic parameters contributes more to the uncertainty in a reservoir’s NPV than does uncertainty regarding the oil price.

These results should be interpreted cautiously, as the theory of merging fronts in the reservoir because of early switching from waterflooding to CO₂ flooding should be verified with actual field data to confirm its accuracy. Moreover, in each case considered
in the previous section, only changes in individual assumptions are considered; changes in multiple variables could have interaction effects that are not captured by this single-variable sensitivity analysis. Plus, the cost structure for a particular location may differ enough from the one considered here that the optimal development strategy would diverge from the one recommended above.

On the other hand, the model does provide substantial evidence that pursuing CO₂ flooding at a relatively early stage of a reservoir’s development cycle could be a viable option for generating more value from the field.
Chapter 8: Conclusion

8.1 SUMMARY

This study considered approaches to maximizing the value of a reservoir over its entire life cycle, focusing on the implications of mean-reverting price models on optimal solutions suggested by decline curve models and then building a geologic-parameter-based model that estimates recovery and other technical parameters for all stages of production. This model was then used to determine values for decision variables such as well spacing, injection pressures, and the switching time from waterflooding to CO₂ flooding that maximize the NPV of representative reservoirs.

The geologic-parameter-based model uses a variant of a tank model for primary recovery that accounts for changes in reservoir fluid properties. For waterflooding and CO₂ flooding, a simplified model proposed by Mollaei (2011) that relies on Koval factors, fractional flow, and the concept of flow capacity and storage capacity is used to predict recovery. His model was updated here to account for cases where the waterflood front has not fully swept the reservoir, which has proven to be a likely scenario with life cycle optimization. To evaluate this model in terms of actual time rather than dimensionless time, which it uses to specify recovery calculations, analytical expressions for estimating fluid injection rates were derived. The expressions used to estimate injection rates are based on theory that approximates fluid flow in a homogeneous reservoir as a series of banks of constant composition; they are parameterized in terms of storage capacity and the heterogeneity factor of the reservoir to be consistent with Mollaei’s method.

The oil price was modeled using mean-reverting processes; a variation of this type of process was proposed to account for the possibility of an increasing mean price. A logistic growth model was also proposed to reflect expectations by some forecasters of an
increasing mean price and economic constraints that would likely limit appreciation over the long term. Production costs were found to be correlated with the oil price based on past data, but the relationship is quite complex and appears to change depending on the time period considered.

Examining an operator’s optimal choices regarding switching times under various economic assumptions and using a decline curve model for a reservoir in Wyoming yielded some insights maximizing the NPV of a reservoir:

- The results of a life cycle optimization suggests if the reservoir were developed today, the maximum NPV would be obtained if the operator chose to switch recovery methods sooner versus the actual switching decisions for the field.
- The assumption of positive correlation of costs with the oil price was found to have a minor effect on the optimal switching times.
- Simple estimates of the value of information regarding the oil price for the purposes of reservoir development were performed assuming a mean-reverting oil price. The results of these analyses suggests that an operator who knows the production characteristics of a field \textit{a priori} and seeks to maximize its value gains a relatively small increase in the reservoir’s NPV by knowing the realization of the uncertain future oil price at the outset of production. For realistic parameters of the mean-reverting model, the expected improvement in NPV by knowing the price realization is estimated to be on the order of 5 to 10 percent for the reservoir considered.
- However, the larger the oil price volatility, the more value the decline curve model suggests there is to be gained by adjusting the optimal recovery phase switching times based on price fluctuations. A real options approach could provide direction on how to maximize value for the dynamic optimization
problem for a prospective reservoir in the face of uncertain mean-reverting prices (Hahn et al. 2007). Alternatively, an operator could mitigate this price risk by purchasing futures.

Optimal solutions found using the geologic-parameter-based model and a representative reservoir suggest that accelerating production by switching to CO₂ flooding at an early stage of a reservoir’s development cycle could be used to maximize the value of the field. This conclusion was found to be valid even for alternate assumptions regarding physical and economic parameters such as relative permeability, the residual oil saturation to waterflooding and the cost of CO₂. The pattern these results exhibit suggests the following strategies to maximize NPV:

- At an annual real discount rate of 7 percent, and assuming a constant oil price⁶⁵ around $80 per barrel, most recoverable oil from the reservoir should be extracted roughly within the first four decades of production. This strategy is achieved generally by maximizing the production rate during primary recovery and using the maximum possible pressure difference across the reservoir for waterflooding and CO₂ flooding. The latter entails injecting at pressures near the formation parting pressure. This strategy also avoids the large costs of an extensive drilling campaign, as suggested injector-producer distance for the example reservoir considered is 1650 feet. This distance translates to 125-acre spacing for primary recovery.

- For the cost structure and other assumptions considered in this study, the operator who maximizes NPV essentially eschews waterflooding in favor of CO₂ flooding with a large WAG ratio of 5. According to model here, the higher cost structure

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⁶⁵ Similar conclusions would likely apply for a mean-reverting price with a constant mean.
of CO$_2$ flooding is outweighed by the benefits of a lower final oil saturation post-
CO$_2$ flood.

- The oil price was alternatively modeled as increasing in the future using a
  particular logistic equation proposed in chapter three; production costs were
  assumed to be correlated to the price. With these economic assumptions, the
  operator’s optimal strategy shifts so that primary production occurs at the slowest
  possible rate. This strategy involves allocating the increase in production
  provided by displacement techniques to times when the oil price plateaus around
  its highest expected value.

- In essence, there is a base case optimal solution associated with a constant oil
  price and known geologic parameters. If the oil price is expected to vary over
  time so that it is relatively higher in later periods, the optimizing operator
  exercises his option to schedule the boost in production from displacement
  processes during this later period. The guiding principle, which is quite intuitive,
  is to produce when the price is high.

These conclusions apply for the representative reservoir and cost structure
considered; assumptions based on information for royalties, taxes and costs in Wyoming
were used in this study. Other combinations of reservoirs and geographies could lead to
different conclusions.

Moreover, a downside to the acceleration strategy would be relative paucity of
information about the reservoir that could inform the CO$_2$ flood’s design at an early stage
of production. For an actual reservoir, this issue could be mitigated by drilling some
extra wells during primary recovery to create a smaller pilot pattern to test the feasibility
of the CO$_2$ flood before converting the rest of the field.
Monte Carlo simulations were also performed to estimate the contributions of different types of uncertainty in input parameters to uncertainty in the simulated reservoir NPV. In particular, the contribution of oil price uncertainty was compared to that of geologic/technical parameter uncertainty. Table 8-1 summarizes the results of this analysis. The dimensionless variation in NPV resulting solely from geologic/technical uncertainty, expressed as a proportion of the dimensionless variation in NPV resulting from both geologic/technical and oil price uncertainty, is estimated to be larger than a comparable metric for uncertainty solely from oil prices. The estimate for geologic/technical uncertainty does not account for possible variation in the size of the reservoir, so this analysis likely underestimates the relative contribution of geologic/technical uncertainty.

Table 8-1: Comparing sources of uncertainty for reservoir NPV

<table>
<thead>
<tr>
<th>Case</th>
<th>Coefficient of Variation for Simulated Reservoir NPV (Dimensionless)</th>
<th>Coefficient of Variation Expressed as a Percentage of that for Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Geologic/Technical Parameters Uncertain)</td>
<td>0.1812</td>
<td>89</td>
</tr>
<tr>
<td>2 (Oil Price Realization Uncertain)</td>
<td>0.1322</td>
<td>65</td>
</tr>
<tr>
<td>3 (Geologic/Technical Parameters and Oil Price Realization Uncertain)</td>
<td>0.2031</td>
<td></td>
</tr>
</tbody>
</table>
8.2 RECOMMENDATIONS FOR FUTURE WORK

As the geologic-parameter-based model covers recovery processes whose dynamics are quite complicated, there are some refinements that can be made to improve predictions for the purposes of life cycle optimization. Some suggested additional research and some potential extensions to the analysis are proposed here:

- Mollaei proposed multivariate regressions obtained by applying a response surface modeling technique to a set of simulation results. He recommended using these regressions to estimate key parameters for his model for predicting oil recovery from displacement methods, such as the Koval factors and $S_{oF}$. As the simulations he used for these regressions assumed that the reservoir is uniformly saturated at the beginning of the CO$_2$ flood, it is suggested that this response surface modeling technique is applied to additional simulation results that assume partial waterflood front sweep of the reservoir at CO$_2$ flood initiation.

- Moreover, the simulations Mollaei used for his response surface modeling exercise appear to have assumed generic parameters for relative permeability curves and residual oil saturation. This design could potentially limit the applicability of the regressions he proposed. Additional regressions could be performed to account for variations in these and other relevant reservoir parameters.

- The equations derived for injection rates in chapter six are useful analytical expressions, but it would also be worthwhile to compare their predictions to field data and/or simulation results.
For the hypothetical reservoir cases considered in chapter seven, production cost estimates are based on average data for onshore fields in a particular region of the United States. While these estimates are fine for a generic reservoir, actual field costs will vary depending on the nature of the reservoir and its geography, which could alter the optimal development strategy. Therefore possible future uses of the model should employ more specific cost estimates for actual reservoirs.

Moreover, for an actual reservoir, a more thorough cost-benefit analysis of various artificial lift techniques would be worthwhile as it could lead to an optimal solution that favors waterflooding over CO₂ flooding. While the bottomhole pressure of producers for the CO₂ flood is constrained by the MMP, in theory the producer bottomhole pressure for the waterflood could be much smaller. The injection pressure could also be higher, which suggests more detailed modeling of the formation parting pressure.

An operator’s decisions in practice are made in a dynamic context with evolving information that affects the valuation of a reservoir. In this study, it is generally assumed that the optimization of the reservoir’s value is based on geologic and technical parameters taken to be known with certainty. At the outset of a project, much of the information regarding a reservoir is uncertain, and as time progresses this uncertainty declines as more data become available. For some decision variables, such as well spacing, the reservoir information at the beginning of the project is more relevant since spacing becomes constrained by choices made at this time. On the other hand, decision variables such as injection pressures and
the time at which the operator switches from waterflooding to CO₂ flooding are made when more information is known about the reservoir. One extension of the model would be to apply a real options approach to valuing a project, as suggested by Hahn et al. (2007). Such an approach would capture more flexibility of decision timing inherent in an actual project.

- The modeling of primary recovery in chapter four assumes a solution gas drive reservoir. This method could be extended to other reservoir drive mechanisms.

- Finally, the geologic-parameter-based model considered here assumes that the operator chooses CO₂ flooding for EOR. Depending on the reservoir characteristics, other EOR techniques may be applicable and could offer better economics. An extension of this work could consider adapting Mollaei’s model for other isothermal EOR processes to life cycle optimization.
Appendix A: Oil Prices and Cost Indices

Table A1: Oil prices and Upstream Capital Cost Index (GDP deflator from BEA, nominal WTI from EIA/Thompson Reuters, and nominal UCCI from IHS 2013)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Reindexed GDP Deflator (2013 Base Year)</th>
<th>Average Nominal WTI (Dollars)</th>
<th>Average Real WTI (2013 Dollars)</th>
<th>Nominal UCCI</th>
<th>Real UCCI</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>76.2</td>
<td>30.4</td>
<td>39.7</td>
<td>100.0</td>
<td>131.2</td>
</tr>
<tr>
<td>2001</td>
<td>77.9</td>
<td>26.0</td>
<td>33.3</td>
<td>101.8</td>
<td>130.6</td>
</tr>
<tr>
<td>2002</td>
<td>79.2</td>
<td>26.2</td>
<td>33.0</td>
<td>103.9</td>
<td>131.2</td>
</tr>
<tr>
<td>2003</td>
<td>80.8</td>
<td>31.1</td>
<td>38.3</td>
<td>105.8</td>
<td>130.9</td>
</tr>
<tr>
<td>2004</td>
<td>83.1</td>
<td>41.5</td>
<td>49.7</td>
<td>109.5</td>
<td>131.7</td>
</tr>
<tr>
<td>Q1 2005</td>
<td>84.8</td>
<td>49.9</td>
<td>58.6</td>
<td>114.6</td>
<td>135.1</td>
</tr>
<tr>
<td>Q3 2005</td>
<td>86.3</td>
<td>63.4</td>
<td>73.2</td>
<td>126.0</td>
<td>146.1</td>
</tr>
<tr>
<td>Q1 2006</td>
<td>87.6</td>
<td>63.3</td>
<td>71.9</td>
<td>148.0</td>
<td>168.8</td>
</tr>
<tr>
<td>Q3 2006</td>
<td>89.1</td>
<td>70.3</td>
<td>78.8</td>
<td>167.4</td>
<td>187.9</td>
</tr>
<tr>
<td>Q1 2007</td>
<td>90.5</td>
<td>58.2</td>
<td>64.1</td>
<td>179.2</td>
<td>198.1</td>
</tr>
<tr>
<td>Q3 2007</td>
<td>91.4</td>
<td>75.3</td>
<td>82.2</td>
<td>197.8</td>
<td>216.4</td>
</tr>
<tr>
<td>Q1 2008</td>
<td>92.4</td>
<td>97.9</td>
<td>105.6</td>
<td>210.5</td>
<td>227.8</td>
</tr>
<tr>
<td>Q3 2008</td>
<td>93.8</td>
<td>117.6</td>
<td>125.4</td>
<td>229.9</td>
<td>245.2</td>
</tr>
<tr>
<td>Q4 2008</td>
<td>93.9</td>
<td>57.9</td>
<td>61.6</td>
<td>221.0</td>
<td>235.4</td>
</tr>
<tr>
<td>Q1 2009</td>
<td>94.1</td>
<td>43.2</td>
<td>46.0</td>
<td>210.0</td>
<td>223.2</td>
</tr>
<tr>
<td>Q2 2009</td>
<td>93.9</td>
<td>59.9</td>
<td>63.8</td>
<td>205.2</td>
<td>218.6</td>
</tr>
<tr>
<td>Q3 2009</td>
<td>94.0</td>
<td>68.2</td>
<td>72.4</td>
<td>202.0</td>
<td>214.9</td>
</tr>
<tr>
<td>Q4 2009</td>
<td>94.3</td>
<td>76.2</td>
<td>80.6</td>
<td>201.0</td>
<td>213.2</td>
</tr>
<tr>
<td>Q1 2010</td>
<td>94.7</td>
<td>78.9</td>
<td>83.2</td>
<td>201.5</td>
<td>212.9</td>
</tr>
<tr>
<td>Q2 2010</td>
<td>95.1</td>
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<td>Average Real WTI (2013 Dollars)</td>
<td>Nominal UOCI</td>
<td>Real UOCI</td>
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Appendix B: Decline Curve Model with Mean-Reverting Oil Price (Half Volatility and Uncorrelated Costs Case)

Figure B1: Distribution of NPV outcomes with mean-reverting oil price and uncorrelated costs – half fitted volatility

Table B1: Time spent on recovery phases for maximized mean NPV (years) – uncorrelated costs, imperfect information, and half fitted volatility

| $t_{Life1}$ | 14 |
| $t_{Life2per}$ | 13 |
| $t_{Life2pat}$ | 13 |
| $t_{Life3}$ | 110 |
| **Mean NPV** | **$572,575,163$** |
Figure B2: Distribution of maximized NPVs with perfect information (uncorrelated costs, half fitted volatility)

Figure B3: Distribution of NPVs with imperfect information and optimal $t_{life}$ values (uncorrelated costs, half fitted volatility)
Figure B4: Distribution of $t_{life1}$ values for Monte Carlo simulation with perfect information (uncorrelated costs, half fitted volatility)

The mean is 14.44 years and standard deviation 4.28 years for the $t_{life1}$ distribution.
Figure B5: Average oil prices for 10 years after optimal of $t_{life1}$ with perfect information minus average oil prices for 10 years after optimal of $t_{life1}$ for imperfect information (uncorrelated costs, half fitted volatility)

The mean of the distribution is $5.61$. 
Appendix C: Visual Basic Code for Fractional Flow Calculations

These functions find the saturations associated with key points on the CO₂ fractional flow construction diagram, as discussed in chapter five.

```
Public Function Find_Swf_starM(Sw_star_o As Double, tolerance As Double) As Double
    Application.Volatile (True)
    Dim Sw_star_prev As Double, Sw_star_next As Double, kro¼f As Double, krw¼f As Double, m As Double, n As Double, A As Double, B As Double, Sorf As Double, Swif As Double
    Sw_star_prev = Sw_star_o
    kro¼f = Worksheets("Waterflood (M)").Range("E1")
    krw¼f = Worksheets("Waterflood (M)").Range("E2")
    m = Worksheets("Waterflood (M)").Range("E3")
    n = Worksheets("Waterflood (M)").Range("E4")
    Sorf = Worksheets("Waterflood (M)").Range("C1")
    Swif = Worksheets("Waterflood (M)").Range("C3")
    A = kro¼f * Worksheets("Waterflood (M)").Range("J1") / (krw¼f * Worksheets("Waterflood (M)").Range("J2")
    B = 1 / (1 - Sorf - Swif)
    Dim counter As Integer
    counter = 0
    Dim Sw_diff As Double
    Do
        Sw_star_next = Sw_star_prev - Tangent_chord_func(Sw_star_prev, A, B, n, m) / Tangent_chord_func_deriv(Sw_star_prev, A, B, n, m)
        Sw_diff = Sw_star_next - Sw_star_prev
        Sw_star_prev = Sw_star_next
        counter = counter + 1
    Loop Until (Abs(Sw_diff) < tolerance Or counter > 50000)
    Find_Swf_starM = Sw_star_next
End Function

Public Function Tangent_chord_func(Sw_star As Double, A As Double, B As Double, n As Double, m As Double) As Double
    Tangent_chord_func = Sw_star ^ n + (A - n * A) * (1 - Sw_star) ^ m - (m * A) * Sw_star * (1 - Sw_star) ^ (m - 1)
End Function

Public Function Tangent_chord_func_deriv(Sw_star As Double, A As Double, B As Double, n As Double, m As Double) As Double
```

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Tangent_chord_func_deriv = n * Sw_star ^ (n - 1) - m * (A - n * A) * (1 - Sw_star) ^ (m - 1) - (m * A) * ((1 - Sw_star) ^ (m - 1) - (m - 1) * Sw_star * (1 - Sw_star) ^ (m - 2))
End Function

Public Function Find_fwj_intersect(fwj As Double, Ssrf As Double, Swif As Double, mu_solv As Double, mu_water As Double, krs¼f As Double, krw¼f As Double, m As Double, n As Double, tolerance As Double) As Double
Application.Volatile (True)
Dim Sw_star_prev As Double, Sw_star_next As Double, A As Double, B As Double
' Guess the intersect to be somewhere between residual solvent and water saturations
Sw_star_prev = (Ssrf - 0.1 * (Ssrf - Swif))
'A is the water-solvent endpoint mobility ratio
A = krs¼f * mu_water / (krw¼f * mu_solv)
B = 1 / (1 - Ssrf - Swif)
Dim counter As Integer
counter = 0
Dim Sw_diff As Double
Do
Sw_star_next = Sw_star_prev - theta(Sw_star_prev, fwj, A, B, n, m) / Theta_prime(Sw_star_prev, fwj, A, B, n, m)
Sw_diff = Sw_star_next - Sw_star_prev
Sw_star_prev = Sw_star_next
counter = counter + 1
Loop Until (Abs(Sw_diff) < tolerance Or counter > 50000)
'Function returns the water saturation of the intersect of solvent velocity line with solvent-water fractional flow curve
Find_fwj_intersect = Sw_star_next * (1 / B) + Swif
End Function

Public Function theta(Sw_star As Double, fwj As Double, A As Double, B As Double, n As Double, m As Double) As Double
theta = (fwj - 1) * (Sw_star) ^ n + A * fwj * (1 - Sw_star) ^ m
End Function

Public Function Theta_prime(Sw_star As Double, fwj As Double, A As Double, B As Double, n As Double, m As Double) As Double
Theta_prime = B * (n * (fwj - 1) * (Sw_star) ^ (n - 1) - m * A * fwj * (1 - Sw_star) ^ (m - 1))
Public Function Find_SwB(Vs As Double, fwj As Double, Swj As Double, Sorf As Double, Swf As Double, mu_oil As Double, mu_water As Double, kro#f As Double, krw##f As Double, m As Double, n As Double, tolerance As Double) As Double
    Application.Volatile (True)
    Dim Sw_star_prev As Double, Sw_star_next As Double, Swj_star As Double, A As Double, B As Double
    ' Guess the intersect to be somewhere between residual oil and water saturations
    Sw_star_prev = 0.8
    Swj_star = (Swj - Swf) / (1 - Sorf - Swf)
    'A is the water-oil endpoint mobility ratio
    A = kro#f * mu_water / (krw##f * mu_oil)
    B = 1 / (1 - Sorf - Swf)
    Dim counter As Integer
    counter = 0
    Dim Sw_diff As Double
    Do
        Sw_star_next = Sw_star_prev - Beta(Sw_star_prev, Vs, fwj, Swj_star, A, B, n, m) / Beta_prime(Sw_star_prev, Vs, fwj, Swj_star, A, B, n, m)
        Sw_diff = Sw_star_next - Sw_star_prev
        Sw_star_prev = Sw_star_next
        counter = counter + 1
    Loop Until (Abs(Sw_diff) < tolerance Or counter > 50000)
    'Function returns the water saturation of the intersect of solvent velocity line with solvent-water fractional flow curve
    Find_SwB = Sw_star_next * (1 / B) + Swf
End Function

Public Function Beta(Sw_star As Double, Vs As Double, fwj As Double, Swj_star As Double, A As Double, B As Double, n As Double, m As Double) As Double
    Beta = (fwj - 1) * (Sw_star) ^ n + A * fwj * (1 - Sw_star) ^ m - (Vs / B) * (Swj_star - Sw_star) * ((Sw_star) ^ n + A * (1 - Sw_star) ^ m)
End Function
Public Function Beta_prime(Sw_star As Double, Vs As Double, fwj As Double, Swj_star As Double, A As Double, B As Double, n As Double, m As Double) As Double
    Beta_prime = B * (n * (fwj - 1) * (Sw_star) ^ (n - 1) - m * A * fwj * (1 - Sw_star) ^ (m - 1)) + Vs * ((Sw_star) ^ n + A * (1 - Sw_star) ^ m) - Vs * (Swj_star - Sw_star) * (n * (Sw_star) ^ (n - 1) - m * A * (1 - Sw_star) ^ (m - 1))
End Function

Appendix D: Determining Koval Factors, $S_{oWF}$ and $S_{oF}$

Estimating $\tilde{S}_o(t_D)$, as outlined in chapter five, requires the use of three Koval factors: $K_W$, $K_B$ and $K_C$. In the case of solvent floods, a value of $S_{oF}$ is also required for fractional flow analysis. For a reservoir with an existing production history, these values are obtained through a history matching process, whereas for forecasting the future performance without any relevant history for a particular displacement method, a different process is used. Mollaei (2011) matched his model to the production histories of several reservoirs by adjusting the Koval factors until the differences between actual production and that predicted by the model were minimized. An example of the results of one of these history matches for solvent flooding is depicted in Figure D1, which is for a four-pattern section of the SACROC project.
Mollaei finds that history matching using his model yielded $R^2$ values of greater than 0.8, often greater than 0.95, indicating that his model can obtain a strong statistical fit to actual field data.

For estimating the economic potential of an undeveloped reservoir, it is necessary to have a method to predict the Koval factors and $S_{OF}$ a priori. To make this prediction, Mollaei essentially fits a series of multivariate regressions to Koval factors and $S_{OF}$ values obtained by “history matching” his model to a sample of results from a numerical simulator. This regression relies on a few key physical parameters of a reservoir to estimate each variable.
To determine the key parameters used for each regression, Mollaei relies on the results of a Winding Stairs sensitivity analysis and/or reservoir engineering judgment. The former measures the variance in output of a model in response to changes in its inputs, capturing interactions between input variables. Mollaei applies this method to determine the reservoir properties with the most influence on the performance of chemical floods using the Chemical Flood Predictive Model and data from the Tertiary Oil Recovery Information System. For waterfloods and solvent floods, he resorts to judgment to choose the most influential reservoir properties. The chosen parameters for each of these processes as well as the ranges of values he considers are provided below in Table D1 and Table D2. Note that the range of dimensionless pressures in Table D2 should read “0.007-0.659”, which is consistent with the actual values Mollaei lists for his trails.

Table D1: Reservoir parameters varied for waterflood simulation trials (from Mollaei 2011)

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<tr>
<th>Recovery Process/Reservoir Variable</th>
<th>Range of Variation</th>
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<tr>
<td>$M^0$, dimensionless</td>
<td>0.5 - 50.0</td>
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<tr>
<td>$V_{DP}$, dimensionless</td>
<td>0.4 - 0.9</td>
</tr>
<tr>
<td>$\lambda$, dimensionless</td>
<td>0.5 - 10.0</td>
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Mollaei then simulates the performances of several trials of hypothetical reservoirs using the GEM and UTCHEM simulators (34 for waterflooding and 42 for solvent flooding). The parameters listed in the tables above are varied within specified ranges according to a process called “Experimental Design” to generate an optimal number of trials and combinations of values for individual trials. These inputs used in the trials are designed to capture a relatively wide range of potential model outputs.

All reservoirs modeled in the simulators are assumed to be simple rectangular blocks subdivided into 16,810 cells and developed with four five spot patterns. The spatial variation of permeability is modeled using the dimensionless correlation length and Dykstra Parsons coefficient. Simulations for waterfloods are conducted accounting for different possibilities for the endpoint mobility ratio, whereas this is not the case for his simulations of solvent floods. Moreover, for all simulations, Mollaei assumes an initial oil saturation of 0.7 and a residual oil saturation to waterflooding of 0.28. He assumes Corey-type relative permeability curves, and presumably consistent values for

<table>
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<th>EOR Process/ Reservoir Variable</th>
<th>Range of Variations</th>
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</thead>
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<tr>
<td>Water to gas injection ratio, $W_R$, dimensionless</td>
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<tr>
<td>$(P\text{-MMP})/\text{MMP}$, dimensionless</td>
<td>1.007 - 1.659</td>
</tr>
<tr>
<td>$V_{DP}$, dimensionless</td>
<td>0.4 - 0.9</td>
</tr>
<tr>
<td>$\lambda$, dimensionless</td>
<td>0.5 - 10.0</td>
</tr>
</tbody>
</table>
the exponents of these functions for all trials, but it is unclear what the assumed values of these exponents are. Thus, his simulations for waterfloods at least account for variation in the endpoints of these important fluid properties, but his simulations for solvent floods do not appear to account for the effects of varying them at all. None of the simulations appear to account for potential differences in the curvature of the relative permeability curves. Therefore the regressions to predict Koval factors resulting from his analysis may not necessarily be applicable for a reservoir with relative permeability characteristics and/or a residual oil saturation that vary significantly from those assumed.

For each simulation trial, Mollaei conducts a “history match” to fit appropriate values of Koval factors to match his model to the output of the simulator. He also obtains values of $S_{oWF}$ and $S_{oF}$ for each trial. This process then results in a data set of Koval factors and post-flood saturations associated with the input parameters for the trials.

Finally, Mollaei uses a “Response Surface Modeling Technique” to develop multivariate nonlinear regressions that individually model the Koval factors and final oil saturations $S_{oWF}$ and $S_{oF}$ as functions of the physical parameters listed in Table D1 and Table D2. These regressions are as follows:
\[
K_w = 8.52049 + 0.36214 \cdot (M^0) - 35.73891 \cdot (V_{DP}) - 0.24703 \cdot (\lambda_x) - 0.010454 \cdot (M^0)^2 + 56.16179 \cdot (V_{DP})^2 - 4.03542 \cdot (10)^{-3} \cdot (\lambda_x)^2 + 0.058956 \cdot (M^0) \cdot (V_{DP}) - 2.816 \cdot (10)^{-3} \cdot (M^0) \cdot (\lambda_x) + 1.04064 \cdot (V_{DP}) \cdot (\lambda_x) + 1.02571 \cdot (10)^{-4} \cdot (M^0)^3 - 25.81984 \cdot (V_{DP})^3 + 5.38014 \cdot (10)^{-4} \cdot (\lambda_x)^3 - 1.26106 \cdot (10)^{-3} \cdot (M^0)^2 \cdot (V_{DP}) + 1.1092 \cdot (10)^{-4} \cdot (M^0)^2 \cdot (\lambda_x) + 0.12849 \cdot (M^0) \cdot (V_{DP})^2 + 1.8436 \cdot (10)^{-4} \cdot (M^0) \cdot (\lambda_x)^2 - 0.89478 \cdot (V_{DP})^2 \cdot (\lambda_x) - 0.010023 \cdot (V_{DP}) \cdot (\lambda_x)^2 - 0.010158 \cdot (M^0) \cdot (V_{DP}) \cdot (\lambda_x)
\]

Equation D1

\[
K_c = -61.0743 - 48.59713 \cdot (\Delta P_D) - 8.16085 \cdot (W_R) + 475.83772 \cdot (V_{DP}) - 3.8842 \cdot (\lambda_x) + 9.37539 \cdot (\Delta P_D) \cdot (W_R) + 56.0324 \cdot (\Delta P_D) \cdot (V_{DP}) - 2.80602 \cdot (\Delta P_D) \cdot (\lambda_x) - 17.66087 \cdot (W_R) \cdot (V_{DP}) + 0.43156 \cdot (W_R) \cdot (\lambda_x) + 17.94027 \cdot (V_{DP}) \cdot (\lambda_x) + 98.11462 \cdot (\Delta P_D)^2 + 3.18925 \cdot (W_R)^2 - 887.98935 \cdot (V_{DP})^2 - 0.23176 \cdot (\lambda_x)^2 - 0.57408 \cdot (\Delta P_D) \cdot (W_R) \cdot (V_{DP}) + 0.18552 \cdot (\Delta P_D) \cdot (W_R) \cdot (\lambda_x) + 2.19911 \cdot (\Delta P_D) \cdot (V_{DP}) \cdot (\lambda_x) - 0.078609 \cdot (W_R) \cdot (V_{DP}) \cdot (\lambda_x) - 0.76257 \cdot (\Delta P_D)^2 \cdot (W_R) - 83.59955 \cdot (\Delta P_D)^2 \cdot (V_{DP}) + 3.23717 \cdot (\Delta P_D)^2 \cdot (\lambda_x) - 1.27811 \cdot (\Delta P_D) \cdot (W_R)^2 - 24.53074 \cdot (\Delta P_D) \cdot (V_{DP})^2 - 0.11428 \cdot (\Delta P_D) \cdot (\lambda_x)^2 + 0.7474 \cdot (W_R)^2 \cdot (V_{DP}) - 0.042527 \cdot (W_R)^2 \cdot (\lambda_x) + 10.11483 \cdot (W_R) \cdot (V_{DP})^2 - 0.02265 \cdot (W_R) \cdot (\lambda_x)^2 - 18.97506 \cdot (V_{DP})^2 \cdot (\lambda_x) + 0.11332 \cdot (V_{DP}) \cdot (\lambda_x)^2 - 62.71628 \cdot (\lambda_x)^3 - 0.30201 \cdot (W_R)^3 + 582.25048 \cdot (V_{DP})^3 + 0.018055 \cdot (\lambda_x)^3
\]

Equation D2

\[
K_B = -11.12464 - 7.17701 \cdot (\Delta P_D) - 1.90115 \cdot (W_R) + 92.10754 \cdot (V_{DP}) - 0.57894 \cdot (\lambda_x) + 2.10799 \cdot (\Delta P_D) \cdot (W_R) + 0.01005 \cdot (\Delta P_D) \cdot (V_{DP}) - 0.17195 \cdot (\Delta P_D) \cdot (\lambda_x) - 1.20389 \cdot (W_R) \cdot (V_{DP}) - 0.020569 \cdot (W_R) \cdot (\lambda_x) + 3.20691 \cdot (V_{DP}) \cdot (\lambda_x) + 12.14825 \cdot (\Delta P_D)^2 + 0.55807 \cdot (W_R)^2 - 166.99687 \cdot (V_{DP})^2 - 0.058069 \cdot (\lambda_x)^2 + 321
\]
\[
0.046645 \times (\Delta P_D) \times (W_R) \times (V_{DP}) + 0.062301 \times (\Delta P_D) \times (W_R) \times (\lambda_x) + 0.20637 \times (\Delta P_D) \times (V_{DP}) \times (\lambda_x) - 0.041953 \times (W_R) \times (V_{DP}) \times (\lambda_x) - 1.20248 \times (\Delta P_D)^2 \times (W_R) - 17.78302 \times (\Delta P_D)^2 \times (V_{DP}) + 0.37265 \times (\Delta P_D)^2 \times (\lambda_x) - 0.20701 \times (\Delta P_D) \times (W_R)^2 + 8.27642 \times (\Delta P_D) \times (V_{DP})^2 - 0.037474 \times (\Delta P_D) \times (\lambda_x)^2 - 0.47201 \times (W_R)^2 \times (V_{DP}) + 6.86096 \times (10)^{-3} \times (W_R)^2 \times (\lambda_x) + 3.21843 \times (W_R) \times (V_{DP})^2 - 2.40465 \times (10)^{-3} \times (W_R) \times (\lambda_x)^2 - 3.50423 \times (V_{DP})^2 \times (\lambda_x) + 0.03239 \times (V_{DP}) \times (\lambda_x)^2 - 0.92882 \times (\Delta P_D)^3 - 0.023072 \times (W_R)^3 + 102.31645 \times (V_{DP})^3 + 4.1412 \times (10)^{-3} \times (\lambda_x)^3
\]

Equation D3

\[
S_{df} = 0.20012 - 0.02557 \times (\Delta P_D) - 0.037532 \times (W_R) + 0.23188 \times (V_{DP}) + 4.53543 \times (10)^{-3} \times (\lambda_x) + 1.93914 \times (10)^{-3} \times (\Delta P_D) \times (W_R) - 0.097652 \times (\Delta P_D) \times (V_{DP}) + 2.70964 \times (10)^{-3} \times (\Delta P_D) \times (\lambda_x) + 0.048883 \times (W_R) \times (V_{DP}) - 1.68578 \times (10)^{-3} \times (W_R) \times (\lambda_x) + 6.42492 \times (10)^{-3} \times (V_{DP}) \times (\lambda_x) + 8.98656 \times (10)^{-4} \times (\Delta P_D)^2 + 7.24988 \times (10)^{-3} \times (W_R)^2 - 0.48553 \times (V_{DP})^2 - 1.11119 \times (10)^{-3} \times (\lambda_x)^2 - 3.68059 \times (10)^{-3} \times (\Delta P_D) \times (W_R) \times (V_{DP}) + 1.69708 \times (10)^{-4} \times (\Delta P_D) \times (W_R) \times (\lambda_x) - 8.84339 \times (10)^{-3} \times (\Delta P_D) \times (V_{DP}) \times (\lambda_x) + 4.79314 \times (10)^{-4} \times (W_R) \times (V_{DP}) \times (\lambda_x) - 2.36617 \times (10)^{-3} \times (\Delta P_D)^2 \times (W_R) - 0.1114 \times (\Delta P_D)^2 \times (V_{DP}) + 1.92949 \times (10)^{-3} \times (\Delta P_D)^2 \times (\lambda_x) + 3.57839 \times (10)^{-4} \times (\Delta P_D) \times (W_R)^2 + 0.22762 \times (\Delta P_D) \times (V_{DP})^2 - 1.58887 \times (10)^{-5} \times (\Delta P_D)^3 \times (\lambda_x)^2 - 5.4991 \times (10)^{-3} \times (W_R)^2 \times (\lambda_x)^2 - 0.014068 \times (W_R) \times (V_{DP})^2 + 2.6604 \times (10)^{-5} \times (W_R) \times (\lambda_x)^2 - 9.90097 \times (10)^{-3} \times (V_{DP})^2 \times (\lambda_x) + 1.88536 \times (10)^{-4} \times (V_{DP}) \times (\lambda_x)^2 + 0.073288 \times (\Delta P_D)^3 - 4.34448 \times (10)^{-4} \times (W_R)^3 + 0.38523 \times (V_{DP})^3 + 6.34764 \times (10)^{-5} \times (\lambda_x)^3
\]

Equation D4

where \( V_{DP} \) is the Dykstra Parsons coefficient, a measure of vertical heterogeneity of permeability in the reservoir, \( \lambda_x \) is the dimensionless correlation length, a measure of
horizontal heterogeneity of permeability, $M^o$ is the endpoint mobility ratio, a mobility ratio calculated using the measured endpoint relative permeabilities of oil and water for the reservoir, $W_R$ is the WAG ratio, $\Delta P_D$ is a dimensionless pressure difference comparing the pressure at which the producer wells operate and the minimum miscibility pressure (MMP) of the fluid in the reservoir, normalized by the MMP. These equations essentially capture the power of numerical simulation, albeit with the relatively narrow assumptions specified above, for use in a less computationally-intensive model. Indeed, for each regression, the independent variables are the very parameters that have been selected based on judgment to be significant in influencing the performance of the reservoir. There could many other reservoir characteristics that strongly influence parameters such as the Koval factors that are not included in these regressions.

For his simulation trials, Mollaei assumes that the thermodynamic MMP is 2110 psi. This will also be the assumed MMP for the fluids of the hypothetical reservoirs evaluated in this study. In reality, the MMP can vary significantly depending on the reservoir; this is another parameter that likely should be incorporated in more comprehensive regressions to estimate Koval factors and $S_{OF}$ similar to those above. Its value also impacts injection rates for a reservoir, as it places a lower bound on the pressure at which producer wells can operate while still maintaining miscible displacement of the oil in the reservoir. To determine the value of the MMP, an operator needs to do fluid analyses such as slim tube experiments. Factors such as the presence of impurities in the injected solvent can impact the ultimate value of this variable. Finally,
Mollaei also assumes that there is a uniform initial reservoir pressure of 15 psi above MMP for his simulations.

The final oil saturation after waterflooding $S_{oWF}$ can be estimated using fractional flow analysis as discussed in chapter five, although Mollaei provides a regression to estimate it from the endpoint mobility ratio, Dykstra Parsons coefficient and dimensionless correlation length. It is noteworthy that the value for $S_{oWF}$ predicted by this regression can differ substantially, on the order of 0.1, from its value as predicted by fractional flow analysis, depending on the curvature of relative permeability curves and heterogeneity parameters chosen. These differences are likely because Mollaei assumes “typical” values for initial oil saturation, residual oil saturation to waterflooding, and Corey exponents that do not vary across simulation trials. For this study, the $S_{oWF}$ determined by fractional flow analysis is used as this value is more likely to be more germane to the reservoir evaluated.

Moreover, Koval factors account for both the effects of heterogeneity and the mobility of displacing versus displaced fluid. For his simulations, Mollaei assumes that the reservoirs initially have uniform oil saturations. This assumption does not necessarily hold for the more complicated scenarios with multiple fronts present in reservoir early in a CO$_2$ flood due to incomplete waterflood front sweep. In these scenarios, Koval factors may differ because of the behavior of multiple step changes in fluid viscosity across a reservoir.

Given the concerns discussed above, Mollaei’s regressions are used to predict Koval factors and final oil saturations in this study for exposition purposes. Future
research is suggested to determine whether new regressions that account for relative permeability variables and other reservoir characteristics such as MMP are needed. Running regressions on Koval factors and $S_{or}$ values fitted to simulation trials with both waterflood and CO$_2$ flood-related fronts initially present in the reservoir is also suggested.

One could alternatively examine Koval factors for reservoirs with similar characteristics to the one to be developed to estimate the relevant factors for the prospective reservoir. A downside of this more statistics-oriented approach is that differences in reservoir parameters and desired development decisions may be so substantial that it is essentially impossible to find reservoirs that are comparable enough for reliable estimation.

**Appendix E: Visual Basic Code for Calculating CO$_2$ Density as a Function of Temperature and Pressure**

This function estimates the density of CO$_2$, given a temperature and pressure, from correlations given by Ouyang (2011) and tables from Jarrell et al. (2002).

```vbnet
Public Function CO2_Density(T As Double, P As Double)
    'takes T in deg F and P in psia, assumes that P > 100 psia and T >= 0 deg F'
    T_celsius = (T - 32) * (5 / 9)
    Dim rho As Double
    rho = 0
```

325
If (\(T_{\text{celsius}} > 40\) And \((P > 1100\) And \(P < 9000\)) Or \((T_{\text{celsius}} < 40\) And \(P > 3600\)) Then

'conditions to use Ouyang's correlation

If \(P < 3000\) Then
For i = 0 To 4
    Ai = 0
    For j = 0 To 4
        Ai = Ai + \((T_{\text{celsius}} ^ j) \times \)
        Worksheets("CO2_Tables").Range("B4").Offset(i, j)
    Next j
    rho = rho + Ai \times P ^ i
Next i
Else
For i = 0 To 4
    Ai = 0
    For j = 0 To 4
        Ai = Ai + \((T_{\text{celsius}} ^ j) \times \)
        Worksheets("CO2_Tables").Range("B11").Offset(i, j)
    Next j
    rho = rho + Ai \times P ^ i
Next i
End If

'convert density from kg/m^3 to lbm/ft^3
CO2_Density = rho * 0.0624279606

Else

'interpolate from Practical Aspects of CO2 Flooding data
lower_P = P - (P - (100 * (P \ 100)))
If lower_P = 3600 Then
    lower_P = 3500
End If

'shift references to cells to ensure the desired pressure is bracketed
row_offset = lower_P / 100 - 1
lower_T = T - (T - (10 * (T \ 10)))

'there are 38 rows between each temperature increment in the tables, offset by appropriate number of temp increments
row_offset = row_offset + (lower_T / 10) \times 38

rho_lower_T = WorksheetFunction.Forecast(P,
    Array(Worksheets("CO2_Data_Tables").Range("B10").Offset(row_offset, 0),
        Worksheets("CO2_Data_Tables").Range("B10").Offset(row_offset + 1, 0)),
    Array(Worksheets("CO2_Data_Tables").Range("A10").Offset(row_offset, 0),
        Worksheets("CO2_Data_Tables").Range("A10").Offset(row_offset + 1, 0)))
rho_upper_T = WorksheetFunction.Forecast(P,
Array(Worksheets("CO2_Data_Tables").Range("B10").Offset(row_offset + 38, 0),
Worksheets("CO2_Data_Tables").Range("B10").Offset(row_offset + 38 + 1, 0)),
Array(Worksheets("CO2_Data_Tables").Range("A10").Offset(row_offset + 38, 0),
Worksheets("CO2_Data_Tables").Range("A10").Offset(row_offset + 38 + 1, 0)))

CO2_Density = WorksheetFunction.Forecast(T, Array(rho_lower_T, 
rho_upper_T), Array(lower_T, lower_T + 10))

End If
End Function
## Glossary

<table>
<thead>
<tr>
<th>Symbol or Acronym</th>
<th>Definition</th>
<th>Units (if applicable – may be dimensionless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{S}_w$</td>
<td>Average water saturation in the reservoir (similarly defined for oil)</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{w}^\prime$</td>
<td>Dimensionless water saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$r_w^\prime$</td>
<td>Effective wellbore radius</td>
<td>feet</td>
</tr>
<tr>
<td>$\lambda_{ul}$</td>
<td>Mobility of unswept region of layer $l$</td>
<td>mD/cp</td>
</tr>
<tr>
<td>$\tilde{\sigma}_\epsilon$</td>
<td>Standard error from regression to fit mean-reverting parameters</td>
<td></td>
</tr>
<tr>
<td>$\bar{\cdot}$</td>
<td>Denotes a fitted parameter</td>
<td></td>
</tr>
<tr>
<td>$\bar{\cdot}$</td>
<td>Denotes the average value of a parameter</td>
<td></td>
</tr>
<tr>
<td>$h_l$</td>
<td>Thickness of layer $l$</td>
<td>feet</td>
</tr>
<tr>
<td>$\gamma_o$</td>
<td>Oil API gravity</td>
<td>°API</td>
</tr>
<tr>
<td>$A_{res}$</td>
<td>Total areal extent of the field</td>
<td>acres</td>
</tr>
<tr>
<td>$B_{CO_2}$</td>
<td>CO₂ formation volume factor</td>
<td>RB/STB</td>
</tr>
<tr>
<td>$B_o$</td>
<td>Oil formation volume factor</td>
<td>RB/STB</td>
</tr>
<tr>
<td>$B_{OB}$</td>
<td>Oil formation volume factor at the bubble point</td>
<td>RB/STB</td>
</tr>
<tr>
<td>$C_A$</td>
<td>Areal sweep efficiency</td>
<td>fraction</td>
</tr>
<tr>
<td>$E_R$</td>
<td>Recovery efficiency</td>
<td>% or proportion of $OPI$</td>
</tr>
<tr>
<td>$E_{R}^\infty$</td>
<td>Theoretical ultimate recovery efficiency</td>
<td>% or proportion of $OPI$</td>
</tr>
<tr>
<td>$F_B$</td>
<td>Flow capacity evaluated assuming $K_B$ for Koval factor (similar definition for subscripts W and C)</td>
<td>fraction</td>
</tr>
<tr>
<td>$H_k$</td>
<td>Heterogeneity factor</td>
<td></td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>$I_{CG}$</td>
<td>Total injection rate into a five-spot using an expression derived from Christman and Gorell’s (applied to a waterflood)</td>
<td>RB/day</td>
</tr>
<tr>
<td>$I_{avg}$</td>
<td>Average injection rate into a reservoir during a waterflood (derived using Caudle and Witte’s conductance ratio)</td>
<td>RB/day (= STB/day for water)</td>
</tr>
<tr>
<td>$I_{tot}$</td>
<td>Total injection rate into a reservoir during a waterflood (derived using Caudle and Witte’s conductance ratio)</td>
<td>RB/day (= STB/day for water)</td>
</tr>
<tr>
<td>$J_e$</td>
<td>Composite productivity index</td>
<td>RB/day/psi</td>
</tr>
<tr>
<td>$K_W$</td>
<td>Koval factor associated with the waterflood front (Koval factors associated with the oil bank front and solvent front are similarly defined with subscripts B and C)</td>
<td></td>
</tr>
<tr>
<td>$M_S$</td>
<td>Mobility ratio for estimating $\gamma$</td>
<td></td>
</tr>
<tr>
<td>$M_{w,o}$</td>
<td>Water-oil mobility ratio</td>
<td></td>
</tr>
<tr>
<td>$N_l$</td>
<td>Total number of layers in a reservoir</td>
<td></td>
</tr>
<tr>
<td>$N_p$</td>
<td>Cumulative oil produced</td>
<td>STB</td>
</tr>
<tr>
<td>$N_w$</td>
<td>Number of wells drilled</td>
<td></td>
</tr>
<tr>
<td>$OPEX_{index,real}$</td>
<td>Operating cost index, expressed in real terms</td>
<td></td>
</tr>
<tr>
<td>$P_{FPP}$</td>
<td>Formation parting pressure</td>
<td>psi</td>
</tr>
<tr>
<td>$Q_{CO_2,ic}$</td>
<td>Rate of CO$_2$ injection at bottomhole injection conditions</td>
<td>RB/day</td>
</tr>
<tr>
<td>$Q_{tot}$</td>
<td>Total injection rate into a field using expression derived from Christman and Gorell’s</td>
<td>RB/day</td>
</tr>
<tr>
<td>$R_{net}$</td>
<td>Net CO$_2$ recycle ratio</td>
<td></td>
</tr>
<tr>
<td>$R_p$</td>
<td>Produced gas-oil ratio</td>
<td>scf/STB</td>
</tr>
<tr>
<td>$R_{res}$</td>
<td>Reservoir CO$_2$ recycle ratio</td>
<td></td>
</tr>
<tr>
<td>$R_s$</td>
<td>Dissolved gas-oil ratio</td>
<td>scf/STB</td>
</tr>
<tr>
<td>$S_{OB}$</td>
<td>Oil saturation in the oil bank</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{OF}$</td>
<td>Final oil saturation after solvent front has swept the medium</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{oWF}$</td>
<td>Average oil saturation behind the shock front</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{or}$</td>
<td>Residual oil saturation to waterflooding</td>
<td>fraction</td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>$S_w$</td>
<td>Water saturation</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{WB}$</td>
<td>Water saturation in the oil bank</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{wJ}$</td>
<td>Water saturation where $f_{wJ}$ intersects the water-solvent fractional flow curve</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{WWF}$</td>
<td>Average water saturation behind the shock front</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{wf}$</td>
<td>Water saturation associated with the shock front in a waterflood</td>
<td>fraction</td>
</tr>
<tr>
<td>$S_{wl}$</td>
<td>Initial water saturation (here assumed to be at the start of the waterflood and equal to $1 - S_{WWF}$)</td>
<td>fraction</td>
</tr>
<tr>
<td>$T_{adval}$</td>
<td>Ad valorem tax rate</td>
<td>%</td>
</tr>
<tr>
<td>$T_{sev}$</td>
<td>Severance tax rate</td>
<td>%</td>
</tr>
<tr>
<td>$V_{DP}$</td>
<td>Dykstra Parsons coefficient</td>
<td>bbl</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Pore volume (initial)</td>
<td>bbl</td>
</tr>
<tr>
<td>$W_R$</td>
<td>WAG ratio</td>
<td></td>
</tr>
<tr>
<td>$c_o$</td>
<td>Oil compressibility</td>
<td>psi$^{-1}$</td>
</tr>
<tr>
<td>$c_t$</td>
<td>Total compressibility</td>
<td>psi$^{-1}$</td>
</tr>
<tr>
<td>$c_w$</td>
<td>Water compressibility</td>
<td>psi$^{-1}$</td>
</tr>
<tr>
<td>$f_w$</td>
<td>Fractional flow of water (similarly defined for oil)</td>
<td>fraction</td>
</tr>
<tr>
<td>$f_{WB}$</td>
<td>Water fractional flow in the oil bank</td>
<td>fraction</td>
</tr>
<tr>
<td>$f_{wJ}$</td>
<td>Water fractional flow of injected fluid in a CO$_2$ flood</td>
<td>fraction</td>
</tr>
<tr>
<td>$f_{WWF}$</td>
<td>Water fractional flow associated with $S_{WWF}$ (on water-oil fractional flow curve)</td>
<td>fraction</td>
</tr>
<tr>
<td>$f_{wf}$</td>
<td>Water fractional flow associated with the shock front in a waterflood</td>
<td>fraction</td>
</tr>
<tr>
<td>$i_b$</td>
<td>Base injection rate (assumes pre-waterflood saturation of fluids)</td>
<td>RB/day</td>
</tr>
<tr>
<td>$k_o$</td>
<td>Oil permeability</td>
<td>mD</td>
</tr>
<tr>
<td>$k_{rw}$</td>
<td>Water relative permeability (similarly defined for oil and solvent)</td>
<td></td>
</tr>
<tr>
<td>$k_{rw}^o$</td>
<td>Endpoint relative permeability for water (similarly defined for oil and solvent)</td>
<td></td>
</tr>
<tr>
<td>$n_{pat.side}$</td>
<td>Square root of total number of primary recovery patterns</td>
<td></td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>$n_{pat,tot}$</td>
<td>Total number of primary recovery patterns</td>
<td></td>
</tr>
<tr>
<td>$p_b$</td>
<td>Bubble point pressure</td>
<td>psi</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Initial reservoir pressure</td>
<td>psi</td>
</tr>
<tr>
<td>$p_{mean}$</td>
<td>Mean oil price</td>
<td>$</td>
</tr>
<tr>
<td>$p_{real,WTI}$</td>
<td>Real WTI price</td>
<td>$</td>
</tr>
<tr>
<td>$p_{wf}$</td>
<td>Flowing bottomhole pressure for primary recovery</td>
<td>psi</td>
</tr>
<tr>
<td>$p_{wi,CO_2}$</td>
<td>Injector bottomhole pressure for CO$_2$ flood</td>
<td>psi</td>
</tr>
<tr>
<td>$p_{wi,WF}$</td>
<td>Injector bottomhole pressure for waterflood</td>
<td>psi</td>
</tr>
<tr>
<td>$p_{wi}$</td>
<td>Injector bottomhole pressure</td>
<td>psi</td>
</tr>
<tr>
<td>$p_{wp}$</td>
<td>Producer bottomhole pressure</td>
<td>Psi</td>
</tr>
<tr>
<td>$q_{o,max}$</td>
<td>Constant initial oil production rate per well for primary recovery</td>
<td>STB/day</td>
</tr>
<tr>
<td>$q_{osc,of}$</td>
<td>Oil production rate of the field for primary recovery under “open flow” conditions (not choked back), measured at standard conditions</td>
<td>STB/day</td>
</tr>
<tr>
<td>$q_{osc}$</td>
<td>Oil production rate at standard conditions</td>
<td>STB/day</td>
</tr>
<tr>
<td>$q_{osci}$</td>
<td>Constant initial oil production rate of the field for primary recovery</td>
<td>STB/day</td>
</tr>
<tr>
<td>$r_{BD}$</td>
<td>Dimensionless radius of the oil bank front</td>
<td>fraction</td>
</tr>
<tr>
<td>$r_{CD}$</td>
<td>Dimensionless radius of the solvent front</td>
<td>fraction</td>
</tr>
<tr>
<td>$r_{WFD}$</td>
<td>Dimensionless radius of the waterflood front</td>
<td>fraction</td>
</tr>
<tr>
<td>$t_{D_{BT}}^W$</td>
<td>Waterflood front breakthrough time (similar times associated with the oil bank front and solvent front are defined with superscripts B and C)</td>
<td>pore volumes</td>
</tr>
<tr>
<td>$t_{D_{sw}}^W$</td>
<td>Waterflood front sweepout time (similar times associated with the oil bank front and solvent front are defined with superscripts B and C)</td>
<td>pore volumes</td>
</tr>
<tr>
<td>$t_{DCO_2}$</td>
<td>Dimensionless time since the</td>
<td>pore volumes</td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>$t_{D,ST}$</td>
<td>beginning of the CO$_2$ flood</td>
<td></td>
</tr>
<tr>
<td>$t_D$</td>
<td>Dimensionless time from waterflooding to CO$_2$ flood</td>
<td>pore volumes</td>
</tr>
<tr>
<td>$t_{Life1}$</td>
<td>Time spent on primary recovery (similarly defined for other phases)</td>
<td>months or years</td>
</tr>
<tr>
<td>$t_{Project}$</td>
<td>Lifespan of a project</td>
<td>months or years</td>
</tr>
<tr>
<td>$v_{Sw}$</td>
<td>Specific velocity of a water saturation</td>
<td></td>
</tr>
<tr>
<td>$v_{AS}$</td>
<td>Specific velocity of the shock front (waterflood front) in a waterflood</td>
<td></td>
</tr>
<tr>
<td>$v_h$</td>
<td>Specific velocity of a hypothetical shock between the oil bank and oil saturation prior to waterflooding</td>
<td></td>
</tr>
<tr>
<td>$v_c$</td>
<td>Specific velocity of the solvent front</td>
<td></td>
</tr>
<tr>
<td>$v_{OB}$ (also $v_B$)</td>
<td>Specific velocity of the oil bank</td>
<td></td>
</tr>
<tr>
<td>$x_D$</td>
<td>Dimensionless injector-producer distance</td>
<td>fraction</td>
</tr>
<tr>
<td>$\gamma_{gs}$</td>
<td>Gas gravity (assuming air takes a value of one) that would result from separator conditions of 100 psig</td>
<td></td>
</tr>
<tr>
<td>$\lambda_{RS}$</td>
<td>Relative mobility the region behind the solvent front</td>
<td>cp$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{RWF}$</td>
<td>Relative mobility of the region behind the waterflood front</td>
<td>cp$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{ROB}$</td>
<td>Relative mobility the oil bank</td>
<td>cp$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{RoiWF}$</td>
<td>Relative mobility of the pre-waterflood region</td>
<td>cp$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_{TM}$</td>
<td>Relative mobility of unswept region</td>
<td>cp$^{-1}$</td>
</tr>
<tr>
<td>$\lambda_x$</td>
<td>Dimensionless correlation length</td>
<td></td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>Oil viscosity</td>
<td>cp</td>
</tr>
<tr>
<td>$\mu_{OB}$</td>
<td>Oil viscosity at the bubble point</td>
<td>cp</td>
</tr>
<tr>
<td>$\mu_{OD}$</td>
<td>Dead oil viscosity</td>
<td>cp</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>Water viscosity</td>
<td>cp</td>
</tr>
<tr>
<td>$\rho_{CO_2,IC}$</td>
<td>Density of CO$_2$ at injection conditions</td>
<td>lbm/M ft$^3$</td>
</tr>
<tr>
<td>$\rho_{CO_2,SC}$</td>
<td>Density of CO$_2$ at standard conditions</td>
<td>lbm/M ft$^3$</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>Stationary increment of $x_t - x_{t-1}$</td>
<td></td>
</tr>
<tr>
<td>$\phi_{WF}$</td>
<td>Porosity at switch to waterflooding</td>
<td>fraction</td>
</tr>
<tr>
<td>$\phi_i$</td>
<td>Initial porosity</td>
<td>fraction</td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>BHP</td>
<td>Bottomhole pressure</td>
<td></td>
</tr>
<tr>
<td>BLS</td>
<td>Bureau of Labor Statistics</td>
<td></td>
</tr>
<tr>
<td>$C_A$</td>
<td>Shape factor</td>
<td></td>
</tr>
<tr>
<td>CAPEX</td>
<td>Capital expenditure</td>
<td>$</td>
</tr>
<tr>
<td>CDCDF</td>
<td>Cumulative discounted cash flow</td>
<td>$</td>
</tr>
<tr>
<td>CPI</td>
<td>Consumer price index</td>
<td></td>
</tr>
<tr>
<td>$D_{tbh}$</td>
<td>Injector tubing diameter</td>
<td>inches</td>
</tr>
<tr>
<td>EIA</td>
<td>Energy Information Administration</td>
<td></td>
</tr>
<tr>
<td>EOR</td>
<td>Enhanced oil recovery</td>
<td></td>
</tr>
<tr>
<td>EVPI</td>
<td>Expected value of perfect information</td>
<td>$</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross domestic product</td>
<td>%</td>
</tr>
<tr>
<td>IRR</td>
<td>Internal rate of return</td>
<td>%</td>
</tr>
<tr>
<td>$m$</td>
<td>Oil corey exponent</td>
<td></td>
</tr>
<tr>
<td>$M^o$</td>
<td>Endpoint mobility ratio</td>
<td>psi</td>
</tr>
<tr>
<td>MMP</td>
<td>Minimum miscibility pressure</td>
<td>psi</td>
</tr>
<tr>
<td>$n$</td>
<td>Water corey exponent</td>
<td></td>
</tr>
<tr>
<td>NPV</td>
<td>Net present value</td>
<td>$</td>
</tr>
<tr>
<td>OPEX</td>
<td>Operating expenditure</td>
<td>$</td>
</tr>
<tr>
<td>PPI</td>
<td>Producer Price Index</td>
<td></td>
</tr>
<tr>
<td>$Q^D$</td>
<td>Demand curve</td>
<td></td>
</tr>
<tr>
<td>$Q^S$</td>
<td>Supply curve</td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>Reservoir barrels</td>
<td></td>
</tr>
<tr>
<td>$r_w$</td>
<td>Well radius</td>
<td>feet</td>
</tr>
<tr>
<td>$S_{oi,discovery}$</td>
<td>Initial oil saturation at beginning of primary recovery</td>
<td></td>
</tr>
<tr>
<td>$S_{oi,WF}$</td>
<td>Initial oil saturation at switch to waterflooding</td>
<td>fraction</td>
</tr>
<tr>
<td>STB</td>
<td>Stock tank barrels</td>
<td></td>
</tr>
<tr>
<td>$S_{wi,WF}$</td>
<td>Initial water saturation at switch to waterflooding</td>
<td>fraction</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>months or years</td>
</tr>
<tr>
<td>$T_{surf}$</td>
<td>Surface temperature</td>
<td>°F</td>
</tr>
<tr>
<td>UCCI</td>
<td>Upstream Capital Cost Index</td>
<td></td>
</tr>
<tr>
<td>UOCI</td>
<td>Upstream Operating Cost Index</td>
<td></td>
</tr>
<tr>
<td>$V_{pwf}(also~V_{p,WF})$</td>
<td>Pore volume at switch to waterflooding</td>
<td>bbl</td>
</tr>
<tr>
<td>WTI</td>
<td>West Texas Intermediate (a benchmark oil price)</td>
<td></td>
</tr>
<tr>
<td>$x$</td>
<td>Natural logarithm of the oil price</td>
<td>ln($)</td>
</tr>
<tr>
<td>$\Delta P_D$</td>
<td>Dimensionless pressure deviation from</td>
<td></td>
</tr>
<tr>
<td>Symbol or Acronym</td>
<td>Definition</td>
<td>Units (if applicable – may be dimensionless)</td>
</tr>
<tr>
<td>------------------</td>
<td>------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>MMP at producers</td>
<td>Volatility of the oil price</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Drainage area of a well in primary recovery (equal to pattern area for secondary and tertiary recovery)</td>
<td>acres</td>
</tr>
<tr>
<td>$A, A_{pat}$</td>
<td>Storage capacity</td>
<td>fraction</td>
</tr>
<tr>
<td>$CF(t)$</td>
<td>Discount at which a local oil trades versus WTI</td>
<td>$</td>
</tr>
<tr>
<td>$Costs(t)$</td>
<td>Reservoir depth</td>
<td>feet</td>
</tr>
<tr>
<td>$D$</td>
<td>Effective viscosity ratio</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>Flow capacity</td>
<td>fraction</td>
</tr>
<tr>
<td>$H$</td>
<td>Total thickness of the reservoir</td>
<td>feet</td>
</tr>
<tr>
<td>$I$</td>
<td>Total injection rate into a five-spot during a waterflood (derived using Caudle and Witte’s conductance ratio)</td>
<td>RB/day (=STB/day for water)</td>
</tr>
<tr>
<td>$OOIP$</td>
<td>Original oil in place</td>
<td>STB or RB</td>
</tr>
<tr>
<td>$Opt(NPV_{ImperfectInfo})$</td>
<td>The NPV an operator realizes when he has chosen the optimal values of $t_{life}$ and has imperfect information on the oil price (does not know the exact realization but knows the mean-reverting parameters)</td>
<td>$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Total injection rate into a five-spot using an expression derived from Christman and Gorell’s</td>
<td>RB/day</td>
</tr>
<tr>
<td>$R$</td>
<td>Overall royalty rate for the project</td>
<td>%</td>
</tr>
<tr>
<td>$Revenues(t)$</td>
<td>Discount rate</td>
<td>%</td>
</tr>
<tr>
<td>$Roy_{total}$</td>
<td>Revenues realized at time $t$</td>
<td>$</td>
</tr>
<tr>
<td>$S$</td>
<td>Spot price of oil or saturation</td>
<td>$</td>
</tr>
<tr>
<td>$T$</td>
<td>Reservoir temperature</td>
<td>°F</td>
</tr>
<tr>
<td>$d$</td>
<td>Injection rate into a medium/layer of constant permeability for a waterflood (depends on distribution of fluids in medium)</td>
<td>RB/day</td>
</tr>
<tr>
<td>$i$</td>
<td>Permeability</td>
<td>mD</td>
</tr>
<tr>
<td>$p$</td>
<td>Reservoir pressure</td>
<td>psi</td>
</tr>
<tr>
<td>Symbol or Acronym</td>
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</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>$q$</td>
<td>Injection rate for a medium/layer of constant permeability using Christman and Gorell’s expression</td>
<td>RB/day</td>
</tr>
<tr>
<td>$s$</td>
<td>Skin factor</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Conductance ratio for waterflood injection (a subscript denotes this value applies to a particular layer, flow capacity or storage capacity)</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Random variable - normally distributed variable with a mean of zero and a standard deviation of one</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Mean reversion coefficient</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time constant for production</td>
<td>years</td>
</tr>
</tbody>
</table>
References


Hultzsch, P.A. Decision and Risk Analysis Through the Life of the Field. MS thesis, University of Texas at Austin, Austin, Texas (December 2005).


Mollaei, A. 2011. Forecasting of Isothermal Enhanced Oil Recovery (EOR) and Waterflood Processes. PhD Dissertation, University of Texas at Austin, Austin, Texas (December 2011).


