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Predicting and Optimizing the Performance of the Expanding Solvent Steam Assisted Gravity Drainage (ES-SAGD) Process Using an Improved Semi-Analytical Proxy Model

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Predicting and Optimizing the Performance of the Expanding Solvent Steam Assisted Gravity Drainage (ES-SAGD) Process Using an Improved Semi-Analytical Proxy Model

by

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Dedication

To my entire family and soon to be my new family, in India and in USA.
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Abstract

Predicting and Optimizing the Performance of the Expanding Solvent Steam Assisted Gravity Drainage (ES-SAGD) Process Using an Improved Semi-Analytical Proxy Model

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Steam Assisted Gravity Drainage (SAGD) is a commonly used EOR/IOR method for improving recovery in heavy oil reservoirs. However, continued research for a more energy efficient method has led to the development of an improved version called Expanding Solvent (ES)-SAGD, which has the potential to replace conventional SAGD method for production from some heavy oil reservoirs. This thesis provides some insights into determination of the reservoir performance of ES-SAGD process using an improved semi-analytical method. This model is then used for optimizing the solvent requirement while minimizing the steam injected.

The semi-analytical model is determined by combining Butler’s oil drainage analytical model and solvent dilution effect of VAPEX process. The predictive ability of this model was improved by accounting for concentration and viscosity dependent solvent diffusion process. Results from this extended model in terms of solvent injection, oil production and Cumulative Steam to Oil Ratio (cSOR) were compared with that of
reservoir simulation at various levels of grid resolution. Furthermore, the results from simulation were analyzed using response surface methodology including gradient based optimization technique to determine optimum operating conditions, which was then compared with more robust multi-objective optimization based on Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Pareto-optimality. Both the optimization techniques were used within the improved semi-analytical formulation to come up with optimized operational parameters.

Modeling solvent diffusivity as a function of solvent concentration gives better results than those obtained using a constant value for diffusivity. Moreover, results for some key performance factors are in good agreement between the semi-analytical model and the numerical simulation, rendering this model suitable for performing solvents-screening studies. The multi-objective optimization framework within the semi-analytical model is demonstrated to be a feasible option for determining optimum ranges of key operating parameters that would result in success of the project. Intermediate values of solvent fraction ranging 0.1 to 0.2 for almost the entire range of injection pressures result in high bitumen recoveries and relatively low cSOR. The results indicate that higher values of solvent fraction at low operating pressures and lower values of solvent fraction at high operating pressures lead to optimized oil recovery rate and lower steam-oil-ratio. The multi-objective optimization process results in several combinations of control parameters that yield solutions along the Pareto-optimum front. These combinations are all viable solutions to the optimization problem.
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Chapter 1: Introduction

Based on the current survey of heavy oil resources worldwide, it has been well established that unconventional oil production globally would become a promising alternative to the dwindling supply from conventional resources. Current estimates of heavy oil and bitumen around the world, per the U.S. Geological Survey, are on the order of 3 billion barrels of original oil in place (OOIP) and 5 billion barrels OOIP, respectively (Meyer et al., 2007).

Heavy oil and bitumen have been frequently referenced as two different entities in the classification scheme of hydrocarbon resources. Although a clear distinction between them has not been well established, it has been proposed that the difference in API gravity between heavy oil and bitumen renders them a characteristic distinction. Heavy oil has API gravity between 100 API and 200 API inclusive and a viscosity greater than 100cp while bitumen has API gravity less than 100 API and a viscosity greater than 10,000cp (Meyer et al., 2007). The high viscosity of heavy oil and bitumen resources at reservoir conditions is the main obstacle to producing oil. Therefore, development of expensive and complex techniques for profitable production becomes inevitable.

The content of this work focuses on thermal recovery of bitumen from subsurface reservoirs using a relatively new method of producing bitumen called as Expanding Solvent - Steam Assisted Gravity Drainage (ES-SAGD). The ES-SAGD is mostly a thermal enhanced oil recovery (EOR) method with the additional solvent providing further reduction in oil viscosity. ES-SAGD, similar to the Steam Assisted Gravity Drainage (SAGD) process, involves the use of horizontally drilled injector and producer pair, in which the injector is placed typically 3 – 5 m above the producer. In ES-SAGD, a single solvent or a mixture of solvents is co-injected with steam into the reservoir using
the upper injector and subsequently, the heated and diluted bitumen/oil is drained to the bottom producer, as shown by Figure 1.1, under the influence of gravity. The solvents typically comprise of light hydrocarbons primarily from C4 to C7 and are selected based on their predicted performance under the prevailing reservoir conditions.

![Figure 1.1: Schematic of a field scale application of the ES-SAGD process with the general physics displayed on the front view by the right (courtesy of JAPEX).](image)

The injected steam with vaporised solvent rises and forms a chamber above the well pair. Bitumen, whose viscosity is lowered by the combined heat and solvent dilution, drains along the walls of the steam chamber towards the producer well. The added advantage of ES-SAGD process over SAGD process is the lower steam requirement due to additional presence of solvent in the porous media. Moreover, increased volumes of solvent injection cause more solvent dilution into oil resulting in reduction of viscosity of the oil, which in turn leads to higher oil recoveries.
Analytical models for predicting recovery in the ES-SAGD process were unavailable until few years back. ES-SAGD, involving interplay of complex physical processes, was understood through experimental analysis and simulation studies before development of proxy models representing the process physics. Quite recently, Gupta and Gittins (2012) came up with a semi-analytical method in a quasi-steady state framework to determine the solvent requirement during the ES-SAGD process. The semi-analytical model was developed by combining Butler’s oil drainage analytical models (Butler et al., 1981, 1985) and VAPEX solvent dilution effect (Gupta et al., 2010). With continued in-depth understanding of ES-SAGD process, there is a need to modify the analytical models to account for process complexity.

1.1 Research Motivation

ES-SAGD is a relatively new process in the realm of thermal recovery and needs to be well understood to realize its potential as an energy efficient process. The research work done so far in ES-SAGD in terms of analytical or semi-analytical modeling is still at the beginning stages. Modeling and optimization of this hybrid process has been done through numerical simulation studies. But, numerical simulation of a short length-scale process like solvent transport at elevated temperatures may yield less accurate result prediction due to inherent numerical dispersion issues. High grid resolution down to 10\textsuperscript{th} or 100\textsuperscript{th} of a meter is required for accurate results and that results in long runtime of models. Similar studies on scale appropriate behaviour on reservoir altering processes have been performed (Singh and Srinivasan, 2014; Singh, 2014). Few analytical or semi-analytical models exist that can be used to evaluate the sensitivity of the process to various operational parameters and to predict results quickly and accurately.
This thesis has evolved with the intent of developing a robust semi-analytical mathematical model that would provide insight into the physics of the ES-SAGD processes and yield accurate estimates of production rates and thermal efficiency in a reasonable computation time. The starting point of our proxy is the semi-analytical model developed by Gupta and Gittins (2012). This model is modified to capture the details of the temperature dependent solvent diffusion process. Results from this extended model in terms of solvent injection, oil production and Cumulative Steam to Oil Ratio (cSOR) are compared with that of reservoir simulation at various levels of grid resolution. As an alternate, the results from simulation are analysed using response surface methodology including gradient based optimization technique to determine optimum operating conditions. Both the extended semi-analytical formulation as well as the response surface-based proxy is used within a multi-objective optimization framework in order to come up with optimized injection schedule and other operational parameters. This model could thus be a useful tool for performing preliminary recovery predictions and solvent optimization studies during the initial phase of the ES-SAGD project.

1.2 Thesis Roadmap

This thesis aims for better understanding of ES-SAGD process through simple models and aims to open research avenues for future work. The chapters of this thesis address the main themes discussed in the previous section and are divided accordingly.

Chapter 2 provides an extensive literature review on evolution of proxy models for ES-SAGD process, modeling of phenomenon of solvent diffusion into oil and previous work done on optimization of ES-SAGD process.

Chapter 3 is dedicated to the semi-analytical model developed by extending and modifying the Gupta and Gittins’ (2012) model for determining optimum solvent
requirement in ES-SAGD process. This approach gives a preliminary understanding of the key parameters that affect the recovery using the ES-SAGD process.

In Chapter 4, our effort to validate the semi-analytical model against simple 2D numerical simulation model will be detailed. A special focus on dynamic refining at the moving steam-solvent-oil interface is presented to address the issue of numerical dispersions in a gridded model.

Chapter 5 presents the Response Surface Analysis technique used for representing the effect of reservoir and operational parameters on process performance is presented. This is followed by the application of the calibrated response surface to perform optimization of the ES-SAGD process. A gradient based optimization technique is used with Response surface models to achieve optimum conditions. Then, similar optimization using the extended semi-analytical model is explored in Chapter 6.

The last chapter, Chapter 7, summarizes the findings of this research and provides recommendations on the way forward.
Chapter 2: Review of Relevant Literature

2.1 An Overview

This chapter presents relevant literature and research conducted on solvent aided SAGD or ES-SAGD process with an emphasis on various proxy models that have been developed to understand this process. These models are primarily organized into analytical, semi-analytical or heuristic approaches for this process. The mechanism of solvent diffusion into bitumen marks the main contrasting feature in ES-SAGD when compared with SAGD process. A dedicated review of efforts to model the solvent diffusion process will help further our understanding of the ES-SAGD process. Therefore, a small portion of this chapter is devoted to a review of models for the solvent dilution process. At the end, we will review various techniques that have been implemented for optimization of process operating parameters specifically, but not limited to, solvent requirement.

The SAGD process involves injection of steam through a horizontal injector, which is placed approximately 5 meters above a horizontal producer, and production of heated bitumen though the lower producer. In a homogeneous reservoir, the heated steam rises to form an “inverted cone” shaped steam chamber above the injector. The heat from the steam lowers the viscosity of bitumen, which consequently drains along the edges of the rising steam chamber under the influence of gravity.

ES-SAGD, initially conceived by Nasr et al. (2003), is an improved version of SAGD, where a single solvent (hydrocarbon) or a combination of light hydrocarbons (mainly from C4 to C7) is/are co-injected with steam to recover additional bitumen while minimizing the steam requirement. The solvent injected in vapor phase expands along with steam and mixes with bitumen at the edge of the steam chamber, thereby reducing bitumen viscosity further. In addition, the volume of steam injected reduces because of
the partial volume occupied by the solvent. At the steam chamber edge, the solvent partitions into a liquid-phase based on the equilibrium ratio ($k_i$) that in turn depends on the temperature at the edge of the steam chamber. The solvent in the liquid-phase then mixes with the bitumen in order to result in viscosity reduction. The depth of solvent penetration into the bitumen is governed by its diffusivity.

The performance of a process like ES-SAGD is predominantly controlled by mass and heat transfer that occurs over a short length scale within a very narrow region around the steam chamber interface. Because it is difficult to accurately represent this physics using a numerical scheme employing some level of discretization, it is important to develop analytical or semi-analytical models for the processes at the steam-bitumen interface. Also, analytical or semi-analytical models could establish a base for solvent screening studies which can be furthered by efficient modelling and simulation.

2.2 Modelling Steam Assisted Gravity Drainage (SAGD) Process

The steam chamber growth in any steam assisted recover process typically occurs in three phases. First, the injected steam rises vertically from the injector – referred to as the vertical growth phase. Once the rising steam intercepts the overburden or seal, it grows laterally and expands horizontally – this phase is termed as horizontal growth period (steam chamber development). Depletion phase (Llaguno et al., 2002) is the last phase, where steam chamber collapses and production is reduced to an economical limit. Most of the studies on SAGD have been focused on the more stable horizontal growth of steam chamber than the unstable rising steam chamber phase. Prior to the evolution of prototypes for solvent assisted SAGD processes, models were developed for SAGD process to predict bitumen rates, which is the basic parameter of interest in any recovery process. Butler et al., 1981 were the first ones to derive the classic model for single phase
flow SAGD process by coupling Darcy’s law and heat transfer. They assumed a 1-D quasi-steady state temperature distribution with heat conduction ahead of steam chamber edge as the primary heat transfer mechanism. Butler’s (Butler et al., 1981) expression for bitumen rates, valid only for the horizontal growth phase of steam chamber until the steam reaches the reservoir boundaries, is given by:

\[
q = \sqrt{\frac{2\phi \Delta S_o k g a H}{m \nu_{os}}} \quad (2.1)
\]

where,

\[
m = \left[ \nu_{os} \int_{T_s}^{T_R} \left( \frac{1}{\nu_o} - \frac{1}{\nu_{or}} \right) \frac{dT}{(T-T_R)} \right]^{-1} \quad (2.2)
\]

Equation 2.1 states that the bitumen drainage rates are directly proportional to the square root of porosity ($\phi$), mobile oil saturation ($\Delta S_o$), reservoir permeability ($k$), acceleration due to gravity ($g$), thermal diffusivity ($\alpha$), and the thickness of the reservoir or bitumen column ($h$), and inversely proportional to the square root of the Butler parameter ($m$) and the bitumen kinematic viscosity at steam temperature ($\nu_{os}$). Butler’s parameter “$m$” (Equation 2.2) relates the variation of bitumen viscosity to temperature and its values vary from 3-5 for typical heavy oils. Higher the value of “$m$” attributes to less flexible viscosity-temperature curve from reservoir to steam temperature. In other words, the gradient of viscosity with temperature is small, from reservoir to steam temperature, for heavy oils having high values, typically ranging from 4-5, of “$m$”. Bitumen, typically, has such high values of “$m$”.

This model has been derived for a snapshot of the mass transfer process at a particular instant in time and is therefore invariant with respect to time. It is evident that the above model provides a simple method to improve bitumen flow rates – by increasing the residual oil saturation, reservoir permeability or by decreasing the oil viscosity. Butler
and Stephens (1981) further modified the above model, incorporating the concept of tangential drainage or “TANDRAIN”, by replacing the 2 in above Equation 2.1 with 1.5 to account for the extra head needed to move the draining bitumen sideways to the producer. Butler also obtained another equation which replaces the 2 in (2.1) with 1.3 and was called the Linear Drainage (LINDRAIN) approximation. These two modifications improved the predictions for oil rate and yielded more accurate estimates for oil recovery. Over the years, many more amendments were done to Butler’s equation for example by Reis (1992; 1993) who incorporated an empirical constant in the above equation (2.1) to account for changing interface velocity. In addition, there were few other models developed later to understand bitumen flow in an unsteady state framework (Alali et al, 2009).

### 2.3 Modelling Expanding Solvent- Steam Assisted Gravity Drainage (ES-SAGD) Process

The notion of adding solvent for extraction of viscous oil was proposed during 1970’s through the work done by Allen (1973), Brown et al. (1977) and Nenniger (1979). Their inventions primarily focused on introducing solvent in gaseous state at reservoir conditions and its consequent dissolution in bitumen resulting in flow of less viscous bitumen towards the producer. Butler and Mokrys, 1991 were the pioneers in implementing this idea through experiments using large scaled physical model with Tangleflag heavy oil and propane as solvent. Heavy oil recovery process that uses only vaporized solvent, without steam, came to be known as VAPEX. Butler and Mokrys (1991) also derived the first mathematical model, to predict bitumen recovery rate (Q), from combination of Darcy’s law and Fick’s law of diffusion under steady-state conditions and constant interface velocity. Their expression for oil rate was:

\[
Q = \frac{1}{u} \int_{Z_o}^{Z_{max}} V (1 - C_s) dZ \tag{2.3}
\]
where, $C_s$ is solvent concentration obtained by the Fick’s Law of diffusion for specific solvent, $Z$ is the thickness of the bitumen column, $U$ is velocity of steam chamber perpendicular to chamber wall and $V$ is velocity in x-direction.

Modeling of VAPEX (VApour EXtraction) process analytically was still in incipient phase until late 1990s while numerous analytical models were still being developed for SAGD process. However, this process of using solvent for heavy oil recovery was studied through extensive numerical models and experiments. For instance, Das and Butler (1998) conducted experiments using a Hele-Shaw cell. They found that a higher driving force for gravity drainage could be achieved by injecting solvent in vapor phase instead of liquid phase. Moreover, reduction in solvent solubility into bitumen at higher temperatures was also observed through experiments (Butler and Mokrys, 1991, Das and Butler, 1995, Jiang et al., 2000). Therefore, determining an optimum steam-solvent-bitumen interface temperature, which is typically lower than steam injection temperature, plays an important role in achieving maximum solvent dilution into bitumen.

2.3.1 Brief Review on Simulation and Experimental Studies on ES-SAGD

As the studies on addition of solvent into bitumen progressed, researchers started exploring the idea of co-injecting solvent with steam for improved viscous oil recovery and to alleviate some of the limitations of traditional SAGD process. These limitations include high heat loss to the overburden and low thermal efficiencies. A new process called Steam Alternating Solvent (SAS) process (Zhao et al., 2005) was studied by performing lab experiments, the results from which were matched using numerical simulations employing fine scale gridding. They concluded that the addition of solvent to steam yields higher oil recovery than the steam only process and also economizes on
steam requirement. Eventually, numerous experimental studies were performed for better understanding of ES-SAGD process. Importantly, Ayodele et al., 2009 observed, through a set of experiments, that low pressure multi-component ES-SAGD process was more competitive than typical SAGD or high pressure ES-SAGD processes. This observation could prove useful in designing solvent aided SAGD process in low pressure reservoirs similar to the ones found in Athabasca.

Govind et al., 2008 performed sensitivities on impact of key factors such as solvent fraction, injection pressure, injection strategy and solvent type on oil recovery and Steam Oil Ratio (SOR). Butane, having higher vapour pressure, facilitated better solvent dilution in bitumen and consequently lowered the bitumen viscosity more. A higher operating pressure with butane as solvent was found to be optimum for higher oil recovery. Many other simulation studies have been conducted on ES-SAGD process giving a better understanding of the process (Zhao et al., 2005, Ardali et al., 2010, Jha et al., 2012).

2.3.2 Analytical and Semi-Analytical Models for ES-SAGD

It can be well perceived that most of the research work on ES-SAGD process is based on experimental studies was validated using numerical models. However, a few attempts have been made to develop analytical or semi-analytical model for ES-SAGD process. But, such models are far less in number when compared to several existing models for SAGD or experimental studies for ES-SAGD. It is well established that analytical models of the important process physics could provide useful insights into the process sensitivities. Sharma et al., 2011 developed a mathematical model to study the effects of methane co-injection as non-condensable gas on the rate of heat and mass transfer during SAGD process. They determined the fraction of methane in vapour phase
along the mobile zone ahead of the steam chamber edge at steady state conditions. In order to accomplish that, they found the equilibrium concentration at the interface using Dalton’s law of partial pressures, the rates of methane accumulation near the interface and its counter current diffusion into the steam chamber (Sharma et al., 2011). Instead of using constant oil saturation as in Butler’s first model (Butler et al., 1981), a linear oil saturation-temperature profile was assumed along the mobile oil zone and used to compute oil recovery rate. Although Butler’s expression was simple as he did not intend to model multi-phase flow, Sharma’s (2011) model does consider multi-phase flow effects by employing the linearly changing oil saturation condition, although the linear assumption is not fully representative of the actual process physics. The methane mole fraction, as shown in Figure 2.1, in bitumen shows a non-monotonic profile – first increasing until a maximum value at finite distance from chamber edge and then decreasing away from interface. Oil phase mobility is highest at the highest concentration of methane in bitumen. Similar behavior could be expected with other solvents as well (Perez-Perez et al., 2014). Sharma et al. (2011), however, concluded that methane co-injection deteriorated the drainage rate, when compared to SAGD case, due to methane’s property of insulating heat transfer to the bitumen ahead.

Figure 2.1: Methane mole fraction in bitumen and phase mobility with distance (Sharma et al., 2011).
Azom and Srinivasan (2012) proposed a semi-analytical model that couples mass, energy and solvent composition balance to derive an expression for oil rate. This model also takes into account the temperature dependent capillary effect for phase distribution within the mobile zone ahead of steam chamber edge. This capillary effect, a classical phenomenon known as Marangoni effect (Nield, 1998, Castor and Somerton, 1977), results in a lowering of the interfacial tension between bitumen and condensate at elevated temperature. Azom and Srinivasan (2012) accounted for this thermo-capillary effect using a dimensionless ratio called the thermo-capillary number, which is approximately the ratio of thermal diffusivity to temperature dependent capillary diffusivity. They derived functional relationship between oil rate and solvent to oil viscosity ratio, Lewis number (ratio of thermal diffusivity “α” to solvent diffusivity “D”), and thermo-capillary number. They showed sensitivity of dimensionless oil rate to each parameter and concluded that there is a threshold value for thermo-capillary number, below which ES-SAGD does not show enhancement over SAGD. This model was an innovative approach towards modelling the complicated temperature-dependent multiphase flow effects that occur during thermal recovery process. However, to date, there is no concrete evidence or experimental analysis that verifies the presence of such thermo-capillary effect in heavy oil reservoirs.

Most recently, Rabiei et al., 2014 employed an exponential form of Heat Integral Method (HIM) to find unsteady state temperature distribution and solvent concentration ahead of steam chamber. This was achieved by incorporating solvent diffusion to the existing Butler’s (Butler et al., 1981) semi-analytical model for SAGD. Solvent dilution was modeled by assuming transverse dispersion and concentration dependent molecular diffusion of solvent transport with steam. They then determined the solvent concentration and solvent penetration depth, which were used in Butler’s oil rate equation to predict
recovery from the entire mobile zone ahead of steam chamber. They also used Butler’s (1981) viscosity temperature function for both solvent and oil and applied logarithmic mixing rule to simplify the viscosity function. Moreover, they assumed equilibrium state for temperature and solvent solubility at the interface throughout the process and used simple procedure to compute equilibrium conditions. A multiplication factor, as a history match parameter, was introduced to account for multi-phase effects. From the plots presented in their publication, it can be concluded that similar results can be obtained by working within a quasi-steady state framework. This is because the length scale of mass transfer is so small compared to that of heat transfer that transients during mass transfer have very minimal impact on such a coupled solution. However, solution obtained in unsteady state framework is indeed quite useful to model complex heat-mass transfer problem.

2.3.3 Semi-analytical Model by Gupta and Gittins (2012)

Recently, Gupta and Gittins (2012) came up with a semi-analytical method in a quasi-steady state framework to determine the optimum solvent required for injecting with steam in the ES-SAGD process. The semi-analytical model was developed by combining Butler’s oil drainage analytical models (Butler et al., 1981; Butler, 1985) and VAPEX solvent dilution effect (Gupta et al., 2010).

2.3.3.1 Theory

The semi-analytical model calculates the variation of oil viscosity with temperature and solvent concentration from the steam chamber interface to some finite extent into the reservoir in a 2D framework. Conceptually, the model determines the temperature at which the solvent completely co-exists in liquid phase with bitumen at the steam chamber edge (Gupta and Gittins, 2012). This temperature is termed as the
interface temperature and is lower than steam temperature. At the interface temperature, the dew point of the solvent is achieved and mass transfer between phases occurs in liquid-liquid mixture. At this condition, it is assumed in the model that there is no solvent in the steam phase. This ensures existence of maximum solvent in contact with the bitumen thereby providing additional lowering of bitumen viscosity, unlike in conventional SAGD process where only thermal effects serve to lower the oil viscosity.

From the combination of models for solvent dilution into bitumen and viscosity reduction of bitumen due to heat and solvent addition, expressions for steam injection rate and bitumen production rate can be derived. This model is only applicable during the horizontal growth phase of the steam chamber and uses an expression similar to Butler’s expression Equation 2.1 for bitumen rate, which is independent of time.

**2.3.3.2 Model Equations**

The model uses fundamental transport equations in a moving reference frame, in which the steam chamber is expanding in a quasi-steady state manner (Butler, 1991). Figure 2.2 illustrates this assumption conceptually. The steam-solvent-bitumen interface moves with frontal velocity denoted “U” with a gradient operator “dξ” in the direction normal to the interface. With this setup as basis, the gradient for temperature and solvent concentration from the interface is derived for a moving steam-solvent-bitumen interface.
This model, with input fluid PVT properties, determines the interface temperature ($T_i$) at the steam chamber operating pressure ($P_t$). As mentioned previously, at this interface temperature the solvent completely co-exists in liquid phase with bitumen at the steam chamber edge and all other parameters like mass rate of solvent required, bitumen production rate and steam injection rate are determined at this temperature. As an initial step, an arbitrary interface temperature is assumed. A random guess value that is less than steam temperature can be assumed as an initial step. At this temperature, solvent concentration ($C_i$) in bitumen phase is found using the following Equation 2.4 and Equation 2.5.

$$y_{sol} = 1 - \frac{P_{wsat}(T_i)}{P_t} \quad \text{and} \quad x_{sol} = \frac{y_{sol}}{k_i} \quad (2.4)$$

$$C_i = \frac{x_{sol}M_{sol}}{\rho_{sol}} + \frac{(1-x_{sol})M_{oil}}{\rho_{oil}} \quad (2.5)$$

where, $y_{sol}$ is mole fraction of solvent in vapour phase, $P_{wsat}$ is saturation pressure of steam, $x_{sol}$ is mole fraction of solvent in liquid phase, $k_i$ is equilibrium ratio of solvent at
interface temperature ($T_i$) and operating Pressure ($P_t$), $\rho_{\text{oil}}$ and $\rho_{\text{sol}}$ are bitumen and solvent densities respectively, $M_{\text{sol}}$ and $M_{\text{oil}}$ are molecular weights of solvent and oil respectively.

Total oil flow ($q_{\text{oil}}$) through the element $d\xi$, in the direction parallel to the inclining walls of steam chamber, is given by Equation 2.6. This equation is derived from material balance of oil in a small element $d\xi$. This derivation is briefly explained in Chapter 3.

\[
q_{\text{oil}} = kgsin\theta \int_0^\infty \left( \frac{1}{\nu_{\text{oil}}} - \frac{1}{\nu_{\text{oil}}R} \right) d\xi
\]  

(2.6)

where, $k$ is reservoir permeability, $g$ is acceleration due to gravity, $\nu_{\text{oil}}$ is kinematic viscosity of heated bitumen, $\nu_{\text{oil}}R$ is the kinematic viscosity of bitumen at original reservoir conditions and $\theta$ is angle made by steam chamber with horizontal.

The oil volumetric flow and solvent mass flow at the assumed interface temperature ($T_i$) and given chamber operating pressure ($P_t$) is evaluated over a finite length into the reservoir using the integrals $I$ and $I_{\text{sol}}$ described in Equation 2.7 and Equation 2.8. These integrals are evaluated numerically for small steps of velocity of interface in $\xi$ direction “U”, for a practical range from 1 to 15 cm/day (Gupta and Gittins, 2012). As the solvent condenses into bitumen phase at the interface temperature, concentration of oil in that finite element $d\xi$ decreases. The integral “I” is thus obtained from Equation 2.6 by considering the portion of element $d\xi$ that is occupied by bitumen. Similarly, the solvent flow $I_{\text{sol}}$ is obtained from Equation 2.6 by evaluating the mass flow rate in the portion of element volume occupied by solvent. The density of solvent $\rho_{\text{sol}}$ is taken within the integral sign for $I_{\text{sol}}$ as it varies along the distance with concentration, $C$ and temperature, $T$. The term “gsin\theta” accounts for the gravity effects during the oil and solvent drainage along the chamber walls.

\[
I = kg \sin\theta \int_0^\infty (1 - C) \left( \frac{1}{\nu_{\text{oil}}} - \frac{1}{\nu_{\text{oil}}R} \right) d\xi , \text{ where } C = C_R + (C_i - C_R)e^{-\frac{U\xi}{D}}
\]  

(2.7)
\[ I_{sol} = k g s i n \theta \int_{0}^{\infty} (\rho_{sol} C) \left( \frac{1}{v_{oil}} - \frac{1}{v_{oilR}} \right) d\xi \] (2.8)

where, \( C_R \) is solvent concentration at original reservoir condition, \( C_i \) is solvent concentration at moving steam-solvent-bitumen interface and D is solvent diffusivity, which is constant in this model.

From material balance consideration (Gupta and Gittins, 2012), \( \frac{\sin \theta}{U} \) is written as:

\[ \frac{\sin \theta}{U} = \phi \Delta S_o \frac{dy}{dq_{oil}} \] (2.9)

where, \( \phi \) is porosity of the reservoir, \( \Delta S_o \) is mobile oil saturation and \( y \) is distance in \( y \)-direction from reservoir bottom. Combining Equations 2.7 and 2.9 and integrating for the entire reservoir thickness \( H \), bitumen flow rate as given in Equation 2.10 is arrived at (Gupta and Gittins, 2012). The bitumen production rate \( m_{oil} \) and solvent injection rate \( m_{sol} \) are then computed using the Equations 2.11 and 2.12.

\[ q_{oil} = \sqrt{1.5 \phi \Delta S_o H I U} \] (2.10)

\[ m_{oil} = q_{oil} \rho_{oil} \] (2.11)

\[ m_{sol} = q_{oil} \frac{I_{sol}}{T} \] (2.12)

Rate of steam to be injected is then obtained from basic energy conservation equations. These equations have been formulated from an assumption of 1D conductive heating of a semi-infinite slab as illustrated in the following Figure 2.3.

![Figure 2.3: One Dimensional conductive heating of a semi-infinite slab](image)
Here, initially solid is at uniform $T_R$ (say reservoir temperature) at time $t = 0$ and the surface $x = 0$ is raised to a temperature of $T_s$ (say steam temperature) and that this temperature is maintained at the surface (Carslaw and Jaeger, 1959). The heat flux into the surface of infinite solid is given by Equation 2.13:

$$\frac{Q}{A} = -K \left( \frac{\partial T}{\partial x} \right)_{x=0}$$ \hspace{1cm} (2.13)

where, $Q$ is heat flux, $A$ is area of the heated surface, $T$ is temperature distribution, $K$ is thermal conductivity of the solid. The temperature distribution ($T_{heat}$) for heat flow from steam temperature $T_S$ to reservoir temperature $T_R$ given by Equation 2.14

$$T_{heat} = T_R + (T_S - T_R) \text{erf} \left( \frac{x}{2\sqrt{\alpha t}} \right)$$ \hspace{1cm} (2.14)

It should be noted that temperature at any point on the heated surface is proportional to distance from the heat source $x$, time, $t$ and thermal diffusivity $\alpha$ Equations 2.13 and 2.14 form the starting points for all the heat calculations as described below in the following.

The rate of heat supplied to the flowing bitumen by the advancing interface with average velocity (“$V_{av}$”) of the interface over height (h) of the reservoir, is given by $Q_c$ and computed using Equation 2.15. The average velocity $V_{av}$ refers to average rate of advancing steam chamber interface in $x$-direction as shown in Figure 2.2.

$$Q_c = h V_{av} (C_{pR} \rho_R) (T_c - T_R)$$ \hspace{1cm} (2.15)

where, $T_R$ is initial reservoir temperature, $T_c$ is average temperature attained by steam chamber as a whole as steam front progresses in the chamber (approximated as average of $T_i$ and $T_R$) and $C_{pR}$ is specific heat of fully saturated reservoir, which is computed as saturation weighted specific heat given in Equation 2.16.

$$C_{pR} = C_{pRock} (1 - \phi) + \phi \ast S_o \ast C_{pbitumen} + \phi \ast (1 - S_o) \ast C_{pwater}$$ \hspace{1cm} (2.16)

where, $C_{pRock}$, $C_{pbitumen}$, $C_{pwater}$ are heat capacities of reservoir rock, bitumen and water respectively and $S_o$ is movable oil saturation.
Heat lost to the overburden is a time dependent quantity and relies upon the total duration of operation \((t)\). The heat loss rate increases as the time progresses. This heat loss “\(Q_{loss}\)” to the overburden due to the injected steam front moving with an average velocity “\(V_{av}\)” is calculated from Eq. 2.17 using thermal conductivity \((K_{ob})\) of overburden.

\[
Q_{loss} = 4V_{av}K_{ob}\sqrt{\frac{t}{\pi\alpha_{ob}}} (T_c - T_R) \tag{2.17}
\]

where, \(V_{av} = \frac{1.5tU}{\sqrt{\Delta S_{oh} \phi}} \tag{2.18}\)

The Equation 2.17 is found by evaluating cumulative heat flux as given in the Equation 2.13 through a particular elemental area in the direction normal to the surface when the temperature is raised from \(T_R\) to \(T_s\). Here the elemental area is expressed in terms of average velocity \((V_{av})\) of the expanding steam chamber front.

Heat stored ahead of the steam-bitumen-solvent interface (“\(Q_{res}\)” ) by the advancing interface with average velocity “\(U_{av}\)” in the direction perpendicular to the interface also contributes to the total heat expended by steam. “\(U_{av}\)” is the speed of the heat front perpendicular to the chamber edge (See Figure 2.2). This parameter “\(Q_{res}\)” , is a time dependent quantity, computed using Equation 2.19. The average perpendicular velocity “\(U_{av}\)” is numerically computed using Eq. 2.20 by integrating the velocity profile \(U_{y,t}\) over height of the reservoir.

\[
Q_{res} = -HK_{ob}U_{av}^2 \frac{d(U_{av})}{dt} (T_c - T_R) \tag{2.19}
\]

Where,

\[
U_{av} = \frac{\int_0^H u_{y,t}dy}{\int_0^H dy} \tag{2.20}
\]

In general \(U\), which is the component of velocity vector parallel to the negative direction of oil flow, is the function of height \(y\) and time \(t\) and is given by

\[
U = -\cos\theta \left(\frac{dy}{dt}\right)_x = -\cos\theta \left(\frac{dx}{dt}\right)_y \left(\frac{dy}{dx}\right)_t \tag{2.21}
\]
From the material balance considerations for a thin vertical element $d\xi$ (See Figure 2.2), oil rate $dq$ flowing parallel to steam chamber surface and normal to direction $\xi$ is given by mass balance equation as:

$$\left(\frac{dq}{dy}\right)_t = \phi \Delta S_o \left(\frac{dx}{dt}\right)_y$$  \hspace{1cm} (2.22)

Expanding and solving for all the terms on RHS of Equation 2.21 further and using Equation 2.22, we get

$$U_{y,t} = \sqrt{\frac{1.5 kglU}{2 \phi \Delta S_o (H-y)}} \frac{d^2}{1 + (dy/dx)^2} \frac{d^2}{(dy/dx)^2}$$  \hspace{1cm} (2.23)

$$(dy/dx) = \frac{4(H-y)^{3/2} \phi \Delta S_o}{\sqrt{1.5 kglU \lambda}}$$ \hspace{1cm} (2.24)

The detailed derivation for the relation between oil flow rate and velocities of interface has been provided in Butler, 1991.

Similarly, the expression for $V_{av}$, given in Equation 2.18, is obtained from the Equation 2.25 given below, where numerator is substituted by the material balance equation given and evaluated over height of reservoir.

$$V_{av} = \frac{\int_0^H (\frac{dx}{dy}) dy}{\int_0^H dy}$$  \hspace{1cm} (2.25)

With all the heat equations as discussed above, the mass of steam required to be injected is computed from Equation 2.26

$$m_{steam} = \frac{Q_c + Q_{loss} + Q_{res}}{\lambda}$$  \hspace{1cm} (2.26)

where, $\lambda$ is latent heat of steam.

Consequently, Cumulative Steam to Oil Ratio (cSOR) and Cumulative Solvent to Steam (cSSR) Ratio, which are critical parameters to assess the economic viability of the process, can be calculated knowing the flow rates of solvent, bitumen and steam.

The entire procedure is then repeated for a new lower value of interface temperature ($T_i$) and the iteration is continued until a minimum temperature is reached.
where the mole fraction of solvent in vapour phase becomes “0”. At this condition maximum dilution of solvent into bitumen occurs. This assumption is a theoretical bound to calculate just the sufficient amount of solvent required in this process. Validation of this model will be discussed in Chapter 4. The chief assumptions in this model are:

- The semi-analytical approach assumed a sharp interface between vapor and oil zone and uses constant solvent diffusivity. However, experimental observations show an existence of diffused layer where solvent concentration increases monotonically to a maximum concentration and decreases to a minimum value at the start of undrained oil zone (Sharma and Gates, 2010; Das and Butler, 1996). Moreover, solvent diffusivity also varies with concentration (Oballa and Butler, 1989; Das and Butler, 1996). This type of solvent diffusion, along with thermal effects, has a significant impact on volume of oil drainage (Sharma and Gates, 2010). Hence, changing diffusivity needs to be considered to improve the performance of the model.

- Frontal velocity $U$ is assumed to range between 1 to 15 cm/day. However, there is still limited hard evidence for this parameter, but, Gotwala and Gates, 2008 came up with values of 0.2 to 2 cm/day for highly viscous bitumen with Butler’s viscosity-temperature parameter “m” ranging from 4 to 5.

- This approach is not derived for multi-phase flow and hence the results can be used as approximation only when comparing with actual field scale or numerical simulation results.

  The estimation of key performance predictors like cSSR (Cumulative Solvent Steam Ration) and cSOR (Cumulative Steam Oil Ratio) from this approach helps in evaluating the technical feasibility of an ES-SAGD project and serves as a starting point for screening and simulation studies.
In this thesis, some modifications and improvements to this approach are suggested and the results from the improved approach are validated against 2D reservoir simulation. The modifications address few of the assumptions discussed above and help improve the model predictions. The results are discussed and recommendations for further study are provided.

2.3.4 Modelling Diffusion in ES-SAGD

In steam-solvent processes, additional lowering of oil viscosity due to solvent dilution into oil phase is one of chief phenomenon leading to improved recovery. This means that the solvent type, injected volume, pressure, and temperature at the walls of the depletion chamber is important because they determine the amount of solvent dissolved in the oil phase which in turn affects its viscosity. The variation of viscosity of solvent-bitumen mixtures with temperature and pressure has been analyzed in the context of the ES-SAGD process by Gates (2007). The primary conclusion was that the behavior is complex and must be understood in order to design efficient recovery process. Figure 2.4, taken from their publication, shows the pressure-temperature-viscosity (P-T-µ) diagram for mixtures of propane and Athabasca bitumen. Maximum dilution of solvent into bitumen is governed by the operating conditions in the reservoir. High pressures and low temperatures favor heavier solvents like hexane, heptane, while low pressures and higher temperatures suit lighter solvents to achieve maximum dilution.
Figure 2.4: Pressure temperature viscosity diagram for mixtures of propane and Athabasca bitumen and hexane and Athabasca bitumen (Sharma and Gates, 2010)

It is observed that the diluted oil drains along the chamber edge, where solvent diffusion and heat transfer from steam take place (Das and Butler, 1996). The dynamics at the steam chamber edge, especially with respect to solvent transport and heat transfer, is still not clear. However, researchers have attempted to understand such phenomenon through simple numerical and analytical models. Schmidt et al., 1982 were the first to publish work pertaining to gaseous CO₂ diffusivity in Athabasca bitumen. Following them Oballa and Butler, 1989 studied the interaction of bitumen-toluene system by measuring diffusion coefficients of toluene into bitumen at various concentrations. The overall diffusion coefficient was found to vary by an order of magnitude over the concentration range (0 to 100%). It increased from low values around 1.0 x10⁻⁶ cm²/s at 0 and 0.5 x10⁻⁶ cm²/s at 100% bitumen concentration to a maximum 4.8 x10⁻⁶ cm²/s at about 54 volume % bitumen. This is due to the presence of bulk flow during diffusion of molecules of different mass and size. For example, during diffusion of toluene into bitumen, the small toluene molecules tend to diffuse faster than the large molecules.
which are contained in bitumen. The intrusion of toluene molecules into bitumen causes a build-up of pressure in the bitumen section (Oballa and Butler, 1989). The resulting pressure gradient causes a bulk flow of bitumen solution toward the solvent-rich section. This type of behaviour is also observed in solvent-bitumen systems as well. Hence, the overall diffusion coefficient varies with rate of transfer of components across a fixed section. This behaviour was modeled by making overall diffusion coefficient as a variant with concentration of each participating species and the mixture viscosity.

Half a decade later, Das and Butler (1996) found that solvent diffusivity is also a strong function of the mixture viscosity, which in turn is the function of temperature and concentration. Empirical correlations, as the function of mixture viscosity, were developed for diffusivity of propane and butane into Peace River bitumen with the help of few simple experiments. They, then, came up with the following two relations (Equation 2.27 and Equation 2.28) for diffusivity of propane and butane into bitumen as a function of the mixture viscosity.

For propane-bitumen system,

\[ D_s = 1.306 \times 10^{-9} \mu^{-0.46} \]  
(2.27)

For butane-bitumen system,

\[ D_s = 4.13 \times 10^{-10} \mu^{-0.46} \]  
(2.28)

where, \( D_s \) is the intrinsic diffusivity of solvent in bitumen in m²/s and \( \mu \) is mixture viscosity in Pa.sec. The coefficient for butane - bitumen system indicates that butane diffuses at a slower rate than propane.

For a typical Athabasca oil sand, overall solvent diffusivity (Das and Butler, 1996; Etminan et al., 2009) is 1000 times smaller than thermal diffusivity (Butler et al., 1985). Therefore, assuming the steam front propagation speed of 0.2 to 2 cm/day, the length over which mass transfers would be approximately close to 0.001 to 0.2 m as
compared to length over which heat transfers, which would range between 0.5 and 10 m (Sharma and Gates, 2010). Due to the different length scales of heat conduction and mass diffusion, there exist two boundary layers in the mobile zone. The first layer (\(\delta C\)) exists near the steam chamber edge, where both solvent and steam have effect on oil flow. The second layer (\(\delta T\)) formed ahead of first layer, is high temperature zone, where only heat affects the oil viscosity (Sharma and Gates, 2010). The existence of two layers is illustrated using a conceptual diagram presented in Figure 2.5.

![Conceptual Diagram](image)

**Figure 2.5:** A cross-sectional view of mobile zone with thermal and solute boundary layers during lateral growth of steam chamber (Rabiei et al., 2014)

These short length processes strongly dictate the rate of the ES-SAGD process. Sharma and Gates, 2010 came up with a new theory to study the extent to which solvent mass transfer and heat transfer affect the oil mobility at the edge of steam chamber. They assumed that flow of solvent through porous media would create asymmetrical mixing zones due to mechanical dispersion (smearing of fluids) and unequal mass transfer. They proposed that the longitudinal dispersion coefficient would approximately be proportional to the first power of the average fluid velocity. They developed a simple equation (Equation 2.29) to determine overall diffusion coefficient (D) that couples the
solvent intrinsic diffusion coefficient and mechanical dispersion. Solvent intrinsic diffusion coefficient \( D_{so}^0 \) was obtained using Wilke-Chang correlation (Wilke and Chang, 1955), which describes the relation between solvent and bitumen diffusion coefficients and their respective viscosities in the mobile zone ahead of steam chamber edge.

\[
D = (D_{os}^0)^{x_s}(D_{so}^0)^{1-x_s} + \alpha U
\]  

(2.29)

where, \( D_{os}^0 \) is diffusivity of bitumen into solvent at infinite dilution, \( D_{so}^0 \) is diffusivity of solvent into bitumen at infinite dilution, \( x_s \) is solvent concentration at the interface, \( \alpha \) is thermal dispersivity in m\(^2\)/s and \( U \) is steam chamber frontal velocity measured normal to chamber edge, m/s. This equation (2.29) was used in the following nonlinear differential equation (Equation 2.30) derived from Fick’s’ Law under quasi-steady state assumptions.

\[
U \frac{\partial C}{\partial \xi} + \left[ (D_{os}^0)^{x_s}(D_{so}^0)^{1-x_s} + \alpha U \right] \frac{\partial^2 C}{\partial^2 \xi} = 0
\]  

(2.30)

Sharma and Gates’ (2010) model for determining changing diffusivity with concentration is a good starting point for studying steam-solvent-oil interaction that occurs during the ES-SAGD process. However, the length scale of instabilities caused due to mechanical dispersion is not expected to be significant compared to the reservoir thickness since gravity acts as a strong stabilizing force (Lake, 1996). Hence, the model for solvent diffusion without including additional mechanical dispersion should explain the solvent diffusion mechanism with reasonable accuracy.

Recently, Rabiei et al., 2012, used steam-bitumen linear stability analysis to investigate the effect of various solvents on convective mixing at the edge of steam chamber. An assumption of buoyancy driven convective mixing due to solute and temperature gradient in porous media formed the basis of their study. However, it is still not clear whether small differential pressure between injector and producer, typically less
than 200 kPa, would cause any appreciable convective fluid flux ahead of steam chamber interface.

2.3.5 Optimization of ES-SAGD process

ES-SAGD is clearly one of the emerging hybrid thermal-solvent recovery technologies. The benefit of ES-SAGD is being realised through field and pilot scale projects such as Solvent Aided Process (SAP) at Christina Lake operated by Cenovus Energy (Gupta and Gittins, 2006). The rate of oil production, amount of solvent and steam required to be injected, scheduling of solvent co-injection with steam are major factors influencing the project economics of an ES-SAGD process. Improved oil recovery with minimum environmental foot print should be the objective of the optimization procedure. For thermal-solvent processes, the three major physical phenomena that control oil production are: gravity and viscous forces, heat conduction, and mass diffusion/ dispersion. This means that first sufficient heat and solvent must be added at sufficient rates to the native oil to make it mobile so that gravity can assist its flow towards the production well. Lighter hydrocarbons are typically expensive due to the complex extraction methods needed to get these fractions in their purest forms and the high demand for them for energy production and as feedstock for petrochemical industries. Previous studies show that higher amounts of solvents lead to higher oil rates (Gupta et al., 2005). But it should be kept in mind that as steam has better capability to deliver latent heat directly to the oil, excess solvent concentration at the chamber edges would make the heat transfer to oil less efficient. However, it remains unclear what the optimal solvent concentration in the injected stream should be.

Existing research work has focused mostly on identifying or screening the solvent that can result in best performance in terms of recovery at prevailing reservoir conditions
and there appears to be very limited work done on optimizing solvent injection schemes. For instance, Gates and Chakrabarty (2006) used Simulation Annealing (SA) technique to optimize two operating parameters namely injection pressure and solvent fraction in the injected stream by minimizing the cost function associated with an ES-SAGD project. The Simulated Annealing (SA) method, a random search technique, is based on the search algorithm proposed by Metropolis et al. (1953) to find the equilibrium configuration of a number of atoms (parameters) at a given temperature. The algorithm conducts a random search that attempts to lower the value of the cost function by proposing perturbations to the model parameters and adopting a probability function for accepting or rejecting those perturbations. Simulation of the ES-SAGD process using CMG-STARS™ was performed corresponding to the perturbations of the operational parameters till the optimum was attained. Gates and Chakrabarty (2006) used a measure of energy efficiency of process as a cost function. The cost function shown in Equation 2.31, includes the difference between the simulated cumulative injected energy to oil ratio (cEOR) at the end of each year and the corresponding target values. The quantity cEOR was defined as the sum of enthalpy of the injected steam and heating value of the net solvent injected (difference between injected and produced solvent) per unit volume of oil produced. In this formulation (Equation 2.31), they used cEOR of 1GJ/m³ as the target value.

\[ CF = [(cEORYEAR_1 - cEORTARGET)^2 + (cEORYEAR_2 - cEORTARGET)^2 + ...]^{1/2} \] (2.31)

An optimum injection scheme for 7 years, as shown in Figure 2.6 was obtained that gave better energy efficiency as compared to the pure SAGD process. The results of this study demonstrated that solvent addition to SAGD can yield lower cumulative energy-to-oil ratios than only SAGD process and therefore can be energetically more efficient than SAGD.
Edmunds et al., 2009 published an approach that uses Genetic Algorithm (GA) to tailor the rate and the composition of the solvent injected. GA compares the results obtained from a reservoir simulation against pre-determined economic criteria. Similar to Gates’ (Gates and Chakrabarty, 2006) method, deviations from the economic criteria create a new set of input variables for future simulations and thus an optimal solution is achieved eventually in successive generations. The main drawback of GA is that there is no absolute assurance of finding global optimum.

Al-Gosayir et al., 2012 proposed an optimization technique that uses orthogonal arrays for experimental design of input parameters and implements a response surface methodology for evaluating a fitness function. Their fitness function was a non-dimensionless number based on Cumulative Steam to Oil Ratio (cSOR) and Recovery Factor (RF). Linear and non-linear proxy models for fitness functions were obtained from three reservoir models for a homogenous reservoir and calibrated against output from
initial coarse-gridded full-physics simulations. With the intent to alleviate computational time Nearly Orthogonal Array (NOA) was implemented to generate a significant set of input parameters for the initial run of full-physics flow simulations. Using these calibrated models, fitness function was evaluated using GA to find optimum conditions, namely the injection maximum BHP, solvent fraction and producer maximum BHP, of the process. The optimum conditions using the nonlinear proxy model achieved a close match to the flow simulation results with the same input conditions in the case of a homogenous reservoir. However, the same check for the case of heterogeneous reservoir did not give good match. It is not clear how the proxy models were calibrated against the results from flow simulations. Recovery factor and cSOR probably exhibit non-linearity with respect to variations in input parameters. Hence, combining both these entities into one fitness function may not work especially if the underlying reservoir heterogeneity further enhances the non-linear characteristics of the flow and transport processes.

In all the optimization techniques discussed above, full-physics reservoir flow simulations have been implemented based on the evaluation of objective function. However, in order to obtain accurate results when short length scale flow and transport processes dominate; the use of finely gridded models becomes a necessity. Moreover, the objective function in all these approach pertains to some form of economic evaluation. With the inherent limitations of such models in terms of high computation time and possibility of numerical dispersion, a simple proxy that captures essential physics of the process as well as optimizes the steam and solvent requirement becomes essential.

2.4 Chapter Summary

The need for analytical models to study ES-SAGD process is apparent and researchers have made several attempts to realise this need in terms of developing fit-for-
purpose proxy models. In this chapter, we have discussed the evolution of analytical models from those for the SAGD process to those for the more complex ES-SAGD process. We have also explored several approaches to optimize the process in order to come up with the optimum solvent required to be injected with steam. This work will expand the analytical model derived by Gupta and Gittins (2012) to accommodate effects of concentration dependent solvent diffusion and a suitable optimization scheme.
Chapter 3: The Improved Semi-Analytical Model

The Expanding-Solvent Steam-Assisted Gravity Drainage (ES-SAGD) process is a variant of the SAGD process (Nasr et al., 2003) that involves the injection of hydrocarbon solvent together with steam in the vapor phase to improve bitumen recovery. In the ES-SAGD process, once the vaporized solvent reaches its dew point pressure and temperature, the solvent condenses out of the vapor phase into the bitumen phase thereby diluting it. This diluted bitumen has much lower viscosity than that observed in the SAGD process. The key to a successful ES-SAGD process is the effective partitioning of solvent into the liquid phase, which facilitates mixing and consequent lowering of bitumen viscosity. The amount of solvent partitioning into the liquid phase is dependent on equilibrium constant ($K_i$) at the dew point conditions. The lower the viscosity of the solvent, the better the recovery of bitumen (due to solvent mixing), but, in terms of thermodynamics, lower viscosity solvents generally partition less into bitumen. This then creates an optimization problem with the key variables being the solvent partitioning coefficient and its viscosity. Gupta and Gittins’ (2012) semi-analytical model is centered primarily on maximizing solvent partition into bitumen and thereby predicting the optimum amount of solvent injected.

The ES-SAGD process has been studied extensively experimentally and numerically since it was invented, but there exist very few quantitative descriptions and analyses of the ES-SAGD process within an analytical or semi-analytical framework. The benefit of developing such numerical or semi-analytical models is that they can be used to design field implementations of the ES-SAGD process such as optimizing the spacing of well pairs, amount of steam and solvent required, injection schedule etc. Heat and mass transfer length scales differ significantly as discussed in previous chapter. Hence,
there is a need to develop and test comprehensive theories that operate at different scales, a key strength of analytical approaches. In this chapter, we will describe the improved semi-analytical model extending Gupta and Gittins’ (2012) model, using butane (C\textsubscript{4}H\textsubscript{10}) as solvent, to capture temperature and concentration dependent solvent diffusion. The inclusion of superposition theorem to accommodate variable injection pressure scheme is elaborated in Chapter 6 along with the discussion pertaining to optimization of the ES-SAGD process. This model encompasses key physical phenomenon of solvent partitioning, diffusion and bitumen dilution through mathematical equations and will provide bitumen production, steam and solvent requirement as quantifiable outputs. We will also explore the application of this model to investigate physical characteristics of the ES-SAGD process.

3.1 MODEL DEVELOPMENT

In the ES-SAGD process, the amount of solvent absorbed into oil phase is determined by the phase-equilibrium conditions that are controlled by the prevailing temperature and pressure conditions at the steam-solvent-bitumen interface. Additional lowering of viscosity due to solvent dilution provides incremental recovery over viscosity lowering due to heat alone as in the SAGD process. This forms the basis for the semi-analytical model developed by Gupta and Gittins (2012). This semi-analytical approach computes viscosity lowering due to heat using Butler’s classic oil drainage model (Equation 2.1 in Chapter 2) and due to solvent dilution using the VAPEX solvent dilution model. The two models are combined into one and evaluated to obtain bitumen rate, amount of injected solvent and steam.
The mathematical framework for the improved semi-analytical model is the same as discussed in Chapter 2. However, key modifications and improvements to Gupta and Gittins’ (2012) semi analytical model are elaborated in this section.

The conceptual diagram in Figure 3.1 represents a 2D unconfined, homogenous reservoir with height “H”, permeability “k” and unit length in direction perpendicular to the wall of steam chamber. Figure 3.1 will be used as the representative diagram for this model development. A small element of length “dξ” measured perpendicular to the draining surface is extracted as shown in the figure. This element makes an angle “θ” with the horizontal plane.

![Figure 3.1: Conceptual diagram of oil drainage geometry at inclined surface (Gupta and Gittins, 2012)](image)

The main driver for the bitumen flow is gravity and therefore, flow due to pressure differential between the wells is assumed to be negligible in this process. The interactions with reservoir pressure are not modelled in the semi-analytical model and perhaps, the calculations are done under an assumption of a pure steam-solvent chamber environment, specifically modeling the physical processes at the steam-solvent-bitumen interface. The
equations are derived by integrating the small element of the interface, which travels with “U” velocity perpendicular to the draining surface, over entire thickness of the pay zone “H”. In thermal recovery processes, kinematic viscosity of bitumen is the function of temperature. The kinematic viscosity of the bitumen “νoil” at any arbitrary temperature is calculated using Butler’s equation (Butler, 1991), as given below in Equation 3.1, which is in terms of the reservoir temperature (TR), interface temperature (Ti) and Butler’s viscosity constant “m”.

\[
\frac{ν_i}{ν_{oil}} = \left(\frac{T - T_R}{T_i - T_R}\right)^m
\]  

(3.1)

The temperature distribution (T) in Equation 3.1 for the heat front with thermal diffusivity “α” advancing at constant speed of “U” is given by Equation 3.2 (Butler et al., 1981). This equation depicts an exponential decay of temperature from the steam temperature (Ts) to the native reservoir temperature (TR).

\[
T = T_R + (T_i - T_R)e^{-\frac{U\xi}{\alpha}}
\]  

(3.2)

Oil flow (qoil) through the element of width dξ is given by Darcy’s equation:

\[
dq_{oil} = \frac{kΔρg\sinθ}{μ}dξ
\]  

(3.3)

where, k is formation permeability, Δρ is density difference between bitumen and water, μ is dynamic oil viscosity and θ is angle made by inclining chamber walls with horizontal. The μ/Δρ in Equation 3.2 can be substituted with kinematic viscosity “ν”.

Substituting the expression for kinematic viscosity from Equation 3.1 into Equation 3.3, the oil rate in the heated zone, qoil can be found by integrating the resulting equation from the well pair to an infinite extent into the reservoir.

\[
q_{oil} = kg\sinθ \int_0^\infty \left(\frac{1}{ν_{mix}} - \frac{1}{ν_R}\right) dξ
\]  

(3.4)

In the above integral (Equation 3.4), “νmix” represents the kinematic viscosity of the solvent-bitumen mixture at an arbitrary location from the steam chamber edge and is a
function of temperature and solvent concentration. The kinematic mixture viscosity \( \nu_{\text{mix}} \) found by dividing dynamic mixture viscosity by mixture density.

\[
\nu_{\text{mix}} = \frac{\mu_{\text{mix}}}{\rho_{\text{mix}}}
\]  

(3.5)

In this improved approach, the dynamic mixture viscosity \( (\mu_{\text{mix}}) \), therefore, is modeled using mixing rule (Govind et al, 2008), based on the mole fractions of solvent and oil in the liquid phase at the interface (Equation 3.6).

\[
\ln(\mu_{\text{mix}}) = x_{\text{sol}} \ln(\mu_{\text{sol}}) + (1 - x_{\text{sol}}) \ln(\mu_{\text{oil}})
\]  

(3.6)

where,

\[
\rho_{\text{mix}} = \sum_{i=1}^{2} w_i \rho_i
\]

\( x_{\text{sol}} \) is solvent mole fraction at any location from steam chamber edge, \( \nu_{\text{sol}} \) and \( \nu_{\text{oil}} \) are viscosities of solvent and bitumen, respectively, at that location, \( w_i \) and \( \rho_i \) are mass fractions and density of each component of the mixture.

Combining the solvent dilution process and thermal effects, the bitumen rate is computed as in Gupta and Gittins’ (2012) approach using Equation 3.7.

\[
q_{\text{oil}} = kgsin\theta \int_0^\alpha (1 - C)(\frac{1}{\nu_{\text{mix}}} - \frac{1}{\nu_{R}})d\xi
\]  

(3.7)

The Equation 3.7 is also derived from Darcy’s law given in Equation 3.3. Integrating the portion of total volume contained by bitumen in an oil-solvent mixture with solvent concentration “C” and over an infinite length into reservoir, Equation 3.3 becomes:

\[
q_{\text{oil}} = kgsin\theta \int_0^\alpha (1 - C)\Delta \rho (\frac{1}{\mu})d\xi
\]  

(3.8)

The Equation 3.8 then can be written in terms of kinematic oil viscosity, and the resulting expression in form of Equation 3.7 is obtained.

If the overall diffusion coefficient of solvent is taken to be \( D \) and thermal diffusivity is taken to \( \alpha \), the variation of solvent concentration from interface \( C_i \) to \( C_R \) deep into the native reservoir can be modeled using Equation 3.9.

\[
C = C_R + (C_i - C_R)T^\frac{\alpha}{\beta}
\]  

(3.9)

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The oil volumetric flow and solvent mass flow at the assumed interface temperature \((T_i)\) is evaluated over an infinite length into the reservoir using the integrals \(I\) and \(I_{sol}\) described by Equations 3.10 and 3.11 below. These integrals are evaluated numerically for small steps of velocity of interface in \(\xi\) direction “\(U\)”, for a practical range from 0.2 to 2 cm/day (Gotwala and Gates, 2008). The integral “\(I\)” is same as the term within the integral sign of Equation 3.7. The expression for \(I_{sol}\) is also derived in the similar way as Equation 3.7. It is obtained from Equation 3.3 by integrating the flow rate in the portion of element volume, in a solvent-oil mixture at the interface, occupied by solvent with concentration \(C\). The density of solvent \(\rho_{sol}\) also varies along the distance with concentration, \(C\) and temperature, \(T\). The term “\(g\sin\theta\)” accounts for the gravity effects during the oil and solvent drainage along the chamber walls.

\[
I = \int_0^\infty (1 - C) \left(\frac{1}{\nu_{mix}} - \frac{1}{\nu_{oil}}\right) d\xi \quad (3.10)
\]

\[
I_{sol} = k g \sin \theta \int_0^\infty (\rho_{sol} C) \left(\frac{1}{\nu_{mix}} - \frac{1}{\nu_{oil}}\right) d\xi \quad (3.11)
\]

The assumptions of a sharp interface between steam chamber edge and native cold bitumen zone and a constant solvent diffusivity in Gupta and Gittins’ (2012) semi-analytical model are not valid for a process like ES-SAGD, where solvent concentration does not remain invariant with changing phase equilibrium conditions. In fact, experimental observations show an existence of diffused layer where solvent concentration varies from \(C_i\), the interfacial concentration (solubility), to the minimum concentration, \(C_m\), (Das and Butler, 1996) as shown in Figure 3.2. Solvent diffusivity, a function of solvent concentration and viscosity also varies from the value at the interface to that at minimum solvent concentration (Oballa and Butler, 1989; Das and Butler, 1996). In our formulation for the improved semi-analytical model, this phenomenon has
been modeled using concentration dependent correlation (Equation 3.12) similar to that used in Sharma and Gates, 2010 as described in Chapter 2.

\[ D = (D_{os}^0)^{x_s}(D_{so}^0)^{1-x_s} \]  

where, D is the total solvent diffusivity in m²/s, \( D_{os}^0 \) is diffusivity of bitumen into solvent at infinite dilution, \( D_{so}^0 \) is diffusivity of solvent into bitumen at infinite dilution, \( x_s \) is mole fraction of solvent. Solvent concentration \( (x_s) \) and diffusion coefficients form implicit equations, both of which are dependent on changing temperature and pressure conditions in the mobile zone. The implementation of this correlation is elaborated in the following text.

Figure 3.2: An illustration showing variation of solvent concentration from interface concentration \( C_i \) to minimum concentration \( C_m \) at the end of mobile zone (Das and Butler, 1996).

The model presented by Gupta and Gittins (2012) aims at determining the minimum interface temperature at which maximum solvent dilution occurs in bitumen phase. At this temperature and a given operating pressure \( (P_t) \), solvent concentration \( (C_i) \) in bitumen phase is found using the following Equation 2.4 and Equation 2.5. For
maximum dilution, the solvent has to partition completely into the liquid phase. The mobile zone is a narrow zone beyond the steam condensation front where multiphase flow of oil, gas and water takes place (Sharma et al., 2011). In this zone both solvent dilution and heating occur together and it extends further from the location of interface temperature towards the native cold bitumen region. It is assumed that the length of the mobile will extend from the interface until the point where temperature of the heat front decays to the value of original reservoir temperature. Therefore, the approximate length of the mobile zone ($\xi$) from the assumed interface temperature is found using the temperature distribution in Equation 3.5. However only a small portion of this mobile zone will undergo both solvent dilution and heating and the remaining part is dominated by heating effect only. This variation is due to the small value solvent diffusivity, which is generally an order or two magnitudes lesser than thermal diffusivity. At every iterative step for interface temperature ($T_i$) and at a given operating pressure ($P_i$), the solvent concentration and diffusivity are iteratively computed and integrated across the mobile zone from the interface. The mole fractions of solvent in the liquid phase along the mobile zone are determined by the phase equilibrium conditions. From the interface, where solvent dilution attains its maximum, the solvent diffusivity and solvent concentrations decrease along the length of this mobile zone as shown in Figure 3.3 below.
The governing equations for such variations along the length of mobile zone are given below from Equations 3.13 through 3.15 (Wilke and Chang, 1955; Sharma and Gates, 2010). These equations are computed for an elemental increment of length ($\Delta x$) and integrated over the entire mobile zone.

\[
D = (D_{os}^0) x_s (D_{so}^0)^{1-x_s} \tag{3.13}
\]

\[
D_{so}^0 = \frac{7.4 \times 10^{-12} (\phi M_o)^{0.5} T}{\mu_s V_s^{0.5}} \tag{3.14}
\]

\[
D_{os}^0 (T^\circ C) = K \left[ \frac{T}{T_20} \right] \left[ \frac{\mu_s \text{ at } 20^\circ C}{\mu_s \text{ at } T^\circ C} \right] \tag{3.15}
\]

where, $D$ is the overall diffusion coefficient in m$^2$/s, $D_{os}^0$ is diffusivity of bitumen into solvent at infinite dilution, $D_{so}^0$ is diffusivity of solvent into bitumen at infinite dilution, $x_s$ is mole fraction of solvent, $\phi$ is molecular association constant, which is taken as 1 for butane (Wilke-Chang, 1955), $M_o$ is molecular weight of solvent, $V_s$ is specific volume of solvent at NTP, $K$ is constant value assumed to be equal to diffusivity of bitumen at 20$^\circ$C, $\mu_s$ is viscosity of solvent at $T^\circ$C. In Equation 3.14, temperature of 20$^\circ$C is used as
reference temperature for calculating the diffusion of bitumen into solvent. The intrinsic diffusion coefficient of bitumen at infinite dilution at 20°C is found to be in the order of 1E-10 m²/sec (Oballa and Butler, 1989). Therefore this value is used in Equation 3.14 as the constant “K” for calculating bitumen diffusivity into solvent at various temperatures. Once, overall diffusion coefficient (D) is determined, the solvent concentration C in bitumen phase is determined using equations 3.16 and 3.17. Equation 3.16 gives normalized the temperature distribution \((T^*)\) in terms of fraction of temperature difference between interface temperature and reservoir temperature.

\[
T^* = \frac{T - T_R}{T_i - T_R} \quad \text{(3.16)}
\]

\[
C = C_R + (C_i - C_R)(T^*)^{\frac{a}{D}} \quad \text{(3.17)}
\]

where, \(\alpha\) is thermal diffusivity in m²/sec and D is solvent diffusivity in m²/s, together \(\alpha/D\) is called Lewis number. The solvent mole fraction \(x_{sol}\) along the mobile zone can be computed using Equation 3.18 once solvent concentration C is known.

\[
x_{sol} = \frac{\frac{C_{sol}}{\rho_{sol}}}{\frac{(1-C)M_{sol}}{\rho_{sol}} + \frac{C_{oil}}{\rho_{oil}}} \quad \text{(3.18)}
\]

Knowing solvent mole fraction at any location in the mobile zone mixture viscosity \(\nu_{mix}\) can be computed using Equation 3.5 and Equation 3.6. The properties of bitumen and solvent at that location are computed based on correlations such as the one presented in Table 3.2.

Subsequently, the bitumen volumetric rate \(q_{oil}\), bitumen mass rate \(m_{oil}\), solvent injection rate \(m_{sol}\) and steam injection rate \(m_{steam}\) are then computed, as previously discussed, using the Integrals for oil flow and solvent flow and Equations 3.19, 3.20, 3.21 and 3.22.

\[
q_{oil} = \sqrt{1.5kg\phi\Delta S_p HIU} \quad \text{(3.19)}
\]
\[
\begin{align*}
m_{oil} &= q_{oil} \rho_{oil} \\
m_{sol} &= q_{oil} \frac{l_{sol}}{l} \\
m_{steam} &= \frac{Q_c + Q_{loss} + Q_{res}}{\lambda}
\end{align*}
\] (3.20) (3.21) (3.22)

where, \(k\) is reservoir permeability, \(\Delta S_o\) is mobile oil saturation and \(H\) is reservoir thickness or bitumen column thickness, \(Q_c\) is rate of heat supplied to the draining oil by heat front, \(Q_{loss}\) is heat loss to overburden and \(Q_{res}\) is heat stored ahead of the steam-bitumen-solvent interface. The basis and descriptions for heat loss and heat transfer equations are elaborated in preceding Chapter 2. Important ratios such as Cumulative Steam Oil Ratio (cSOR) and Cumulative Solvent Steam Ratio (cSSR) are key performance indicators of ES-SAGD process and can be evaluated using this semi-analytical approach. cSOR is obtained by finding ratio of cumulative steam injected to cumulative bitumen produced and cSSR is found by taking ratio of cumulative solvent injected to cumulative steam injected. Hence, as discussed in Chapter 2, entire calculations are repeated iteratively for lower interface temperatures \((T_i)\) at a given chamber operating pressure \((P_i)\) until the condition of maximum solvent dilution at the interface is achieved.

The entire procedure is condensed in form of the work flow as described below and has been implemented in MATALB programming environment. In the work flow, \(x_n\) represents the distance from interface along the mobile zone and increases from \(x_n = 0\) at \(n = 0\) to value of length of mobile zone in \(\Delta x\) steps. In MATALB, the “integral” function is used, where the step size \(\Delta x\) is determined by global adaptive quadrature method.
Figure 3.4: A condensed work flow explaining calculation procedure for improved semi-analytical model
The method and workflow presented above, allow us to treat the steam-solvent-bitumen contact zone as one where the concentration of the solvent undergoes a gradual change from the injection concentration to the minimum concentration within the bitumen phase. In addition, the oil and solvent rates are found at every interface temperature (T_i) during the iteration. This is in contrast to the sharp interface assumption employed by Gupta and Gittins (2012) in their semi-analytical model.

Key modifications and improvements to Gupta and Gittins’ (2012) model are briefly summarized in the following:

1. Mixing rule, as given in Equation 3.6, is implemented to model solvent-bitumen mixture viscosity. The mixture kinematic viscosity is then used in the oil flow and solvent flow integrals given in Equations 3.10 and 3.11. In the mobile zone, the solvent concentration decreases from interface concentration, C_i to minimum concentration C_min in the cold bitumen zone. By implementing this viscosity mixing rule, the mixture viscosity becomes a variant with concentration, and therefore is minimum at the interface and increases towards cold bitumen zone.

2. Variation of solvent concentration from interface to native reservoir is modelled using a relation with Lewis Number, α/D as given in Equation 3.17. As discussed, the solvent diffusivity is couple of orders of magnitude less than thermal diffusivity and therefore, the extent of solvent penetration will be lesser than that of heat penetration. To accommodate this difference, the variation of solvent concentration is modelled in terms of the relative effect of solvent diffusion given thermal diffusion.

3. The concentration and viscosity dependent overall solvent diffusion coefficient, as given in Equation 3.13, is modelled using Wilke - Chang (1955) correlation and is employed in the model as described previously from Equations 3.13 through 3.19.
This correlation results in decreasing overall diffusion coefficient “D” with an increase in mixture viscosity and decrease in solvent concentrations. Therefore, the solvent diffusivity exhibits a declining trend from steam-solvent-bitumen interface towards cold bitumen zone.

All the modifications, as discussed above, results in relatively less optimistic bitumen recoveries than the case with constant solvent diffusivity.

3.2 APPLICATION OF IMPROVED SEMI-ANALYTICAL MODEL

A synthetic reservoir case is put together to demonstrate the application of the improved semi-analytical formulation. Heavy oil with a viscosity approximately in the range 1,000,000 cp at reservoir conditions, similar to bitumen found in Athabasca reservoirs, is considered for this case study. The relevant input data for this case are summarized in Tables 3.1 – 3.2. The steam front velocities, U were found to vary from 0.2 to 2 cm/day for Athabasca bitumen (Gotwala and Gates, 2008). Therefore, an intermediate value of 0.6 cm/day is used as a value for steam frontal velocities U in this application. The input data for key reservoir properties presented in Table 3.1 are similar to the ones used by Gupta and Gittins (2012) with minor changes. In this application, butane is used as the solvent due to its high vapor pressure and solubility in the pressure and temperature range for the study (Govind et al, 2008). Therefore, addition of butane with steam will result in better solvent dilution than with any other light hydrocarbons. The variation of solvent and bitumen properties such as bitumen viscosity, density, solvent viscosity, solvent phase equilibrium constant (K_i), solvent density with temperature are modeled using available correlations, that are presented in Table 3.2.
Table 3.1: Input data for reservoir and operating parameters used for demonstrating the improved semi-analytical model for ES-SAGD.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>API degree of reservoir crude</td>
<td>8</td>
<td>°API</td>
</tr>
<tr>
<td>Reservoir height (H)</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>Porosity (φ)</td>
<td>0.33</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Effective permeability (k)</td>
<td>4.2</td>
<td>Darcy</td>
</tr>
<tr>
<td>Initial oil saturation (S_{ini})</td>
<td>0.85</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Residual oil saturation (S_{or})</td>
<td>0.1</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Molecular weight of oil (M_o)</td>
<td>581</td>
<td>kg/kmole</td>
</tr>
<tr>
<td>Viscosity exponent (m)</td>
<td>4.2</td>
<td>-</td>
</tr>
<tr>
<td>Molecular weight of water phase (M_w)</td>
<td>18</td>
<td>kg/kmole</td>
</tr>
<tr>
<td>Thermal conductivity of reservoir rock (K_{rock})</td>
<td>2.85</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Thermal conductivity of reservoir water (K_{water})</td>
<td>0.60</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Thermal conductivity of bitumen (K_{oil})</td>
<td>0.1</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Thermal diffusivity of rock (α)</td>
<td>6.80E-07</td>
<td>m²/s</td>
</tr>
<tr>
<td>Thermal diffusivity of overburden/underburden</td>
<td>7.50E-07</td>
<td>m²/s</td>
</tr>
<tr>
<td>Initial reservoir temperature (T_r)</td>
<td>283</td>
<td>K</td>
</tr>
<tr>
<td>Initial reservoir solvent concentration (C_{ini})</td>
<td>0.00001</td>
<td>V/V</td>
</tr>
<tr>
<td>Operating chamber pressure (P_c)</td>
<td>2500</td>
<td>kPa</td>
</tr>
<tr>
<td>Injected Steam Temperature (T_s)</td>
<td>480</td>
<td>K</td>
</tr>
<tr>
<td>Steam tables</td>
<td>Standard tables</td>
<td></td>
</tr>
</tbody>
</table>
Table 3.2: Correlations used to evaluate solvent and bitumen properties.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlations</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bitumen viscosity (cp)</td>
<td>[ \ln \mu = \exp[-3.5784 \ln T + 22.8379 + 0.0052 \left( \frac{P}{1000} - 0.10325 \right)] ] where, T is in K, P is in kPa, ( \mu ) is in cp.</td>
<td>Butler, 1981</td>
</tr>
<tr>
<td>Bitumen density (kg/m(^3))</td>
<td>[ \rho = \rho_{15} (1 - \frac{0.063(T-15)}{100}) + \frac{141.5}{100} ] where, T is in °C and ( \rho ) is kg/m(^3) [ \rho_{15} \text{deg} = \frac{100}{131.5 + API} \times 999 ] where, T is in °C</td>
<td>Butler, 1981</td>
</tr>
<tr>
<td>Solvent viscosity (cp)</td>
<td>[ \mu_s = 0.0219066 \times e^{\frac{612.12}{T}} ] where, T is in K, ( \mu_s ) is in cp</td>
<td>CMG Manual 2013</td>
</tr>
<tr>
<td>Solvent density (kg/m(^3))</td>
<td>[ \rho_s = 0.2283 \times 0.2724 \times (-1 - \frac{T}{T_c})^{0.2863} ] where, T is in Kelvin, ( T_c ) is critical temperature for solvent - for Butane it is 425.13 K</td>
<td>CMG Manual 2013</td>
</tr>
<tr>
<td>Solvent equilibrium constant (K(_i))</td>
<td>[ K_i = \frac{3049.597}{P} \times e^{\frac{-789.549}{T}} ] where, T is in °C, ( P ) is in kPa</td>
<td>Govind et al., 2008</td>
</tr>
<tr>
<td>Specific heat of reservoir rock</td>
<td>[ C_{prock} = 0.715 + 0.0017077T - 1.908 \times 10^{-6}T^2 ] where, T is °C, ( C_{prock} ) is kJ/kg °C</td>
<td>Butler, 1991</td>
</tr>
<tr>
<td>Specific heat of oil</td>
<td>[ C_{poil} = 1.605 + 0.004361T - 4.046 \times 10^{-6}T^2 ] where, T is °C, ( C_{poil} ) is kJ/kg °C</td>
<td>Butler, 1991</td>
</tr>
<tr>
<td>Specific heat of water</td>
<td>[ C_{water} = 4.182 - 1.5 \times 10^{-3}T + 3.44 \times 10^{-7}T^2 + 4.26 \times 10^{-8}T^3 ] where, T is °C, ( C_{water} ) is kJ/kg °C</td>
<td>Butler, 1991</td>
</tr>
<tr>
<td>Specific heat of fluid saturated rock</td>
<td>[ C_{pmix} = (1-\phi)C_{prock} + \phi(\Delta S_o)C_{poil} + \phi S_w C_{water} ] where ( \phi ) is porosity of reservoir rock, ( \Delta S_o ) is bitumen saturation, ( S_w ) is water saturation and ( C_{pmix} ) is kJ/kg °C</td>
<td>CMG Manual 2013</td>
</tr>
<tr>
<td>Thermal conductivity of fluid saturated rock</td>
<td>[ K_{mix} = (1-\phi)K_{rock} + \phi(\Delta S_o)K_{oil} + \phi S_w K_{water} ] where ( \phi ) is porosity of reservoir rock, ( \Delta S_o ) is bitumen saturation, ( S_w ) is water saturation and ( K_{mix} ) is J/kg</td>
<td>CMG Manual 2013</td>
</tr>
</tbody>
</table>
3.2.1 Results from Improved Semi-Analytical Model

Figure 3.5 shows the trends obtained for variables such as mass oil rate, mass solvent rate, mass steam rate, SSR and SOR from Gupta and Gittins’ (2012) semi-analytical model for butane as solvent and with solvent diffusion coefficient as 6E-07 m$^2$/s. All other variables, except, SOR, increase with decreasing interface temperature. The results obtained using the improved semi-analytical approach, discussed in this chapter, compares well with trends described by Gupta and Gittins (2012). The integrands of Equation 3.10 ($I_{oil}$) and Equation 3.11 ($I_{sol}$) compute variation of mixture viscosity with temperature and concentration with frontal velocities, U. Figure 3.6 showing increase in the value of integrals with increase in frontal velocity U. It can be perceived that as the frontal velocity “U” increases the drainage area per unit time increases which assists in improved lowering of bitumen viscosity. Therefore, the integrals, being proportional to inverse of viscosity, also increase with U. Figures 3.7 and 3.8 show the trends of SOR, SSR, and solvent rate and oil rate with interface temperature. The fluid rates and other important ratios like SSR, SOR show a nonlinear i.e. a curved profile process due to the combination of non-linear solvent dilution and steam-heating processes. The model computes all the output parameter until the minimum interface temperature, the condition when the mole fraction of solvent in liquid phase is 1 is attained. This condition is a theoretical bound in this model. Therefore, a temperature of 403 K is an estimate of interface temperature at which maximum solvent dilution in bitumen takes place. Moreover, the oil rate and SOR achieve their maximum and minimum value respectively at this temperature. The Solvent Steam Ratio (SSR) of 0.3 is the optimum to achieve maximum bitumen recovery for this demonstration example of a 2D homogenous reservoir. It can also be seen from Figure 3.9 that oil production increases with SSR and SOR decreases with increasing SSR. According to Butler’s first
model (Butler et al., 1981, Butler, 1991), during the horizontal growth of steam chamber, oil flow rate remains constant. However, it is useful note that Butler’s steam rate expressions (see section 2.3 in Chapter 2 for elaborated explanation) i.e. heat loss to overburden and heat stored in the reservoir beyond the heat front vary with time. Hence, SOR increases while SSR decreases with time (Figure 3.10) as the amount of steam injected increases with time while the bitumen and solvent rates remain constant. The mixture viscosity decreases with solvent concentration as seen in the Figure 3.11. Moreover, solvent diffusivity is inversely proportional to fluid viscosity. Therefore solvent diffusivity increases with decreasing mixture viscosity. This reflects the expected behavior from modeling of solvent diffusivity as a variant using the Wilke-Chang (1955) correlation. Also, a similar trend for viscosity and solvent diffusivity is obtained by Das and Butler, 1996. The lowering of interface temperature favors the solvent to attain lower values of equilibrium coefficient ($K_i$), which in turn facilitates increased solvent partitioning into bitumen phase. This phenomenon causes increase in solvent concentration with decreasing interface temperature. Higher solvent dilution at this minimum interface temperature favors lowering of solvent-bitumen mixture viscosity. Figure 3.12 shows the behavior of overall butane diffusion coefficient and solvent concentration at distances from the interface towards the cold bitumen i.e. across the mobile zone. As expected, the solvent effects decay away from the interface and reach minimum values at the end of mobile zone. Figure 3.13 gives an example of the comparison of variation of solvent concentration with distance from interface between the improved semi-analytical model and semi-analytical model by Gupta and Gittins (2012). With constant diffusivity of $6E-7 \text{ m}^2/\text{s}$ and a frontal velocity $U$ of 0.6 cm/sec, the rate of decline of solvent concentration is almost constant and gradual in Gupta and Gittins’ approach. However in case of the improved semi-analytical model, the rate of
decline in solvent concentration from the interface varies with changing solvent diffusivity and mixture viscosity. The solvent dilution is dominant until the length of 1-2 m from the interface, after which temperature effects dominate the process. The decrease in solvent diffusivity towards cold bitumen region, therefore, affects the concentration profile of the solvent.

![Graph](image1)

**Figure 3.5:** Variation of mass oil rate, mass solvent rate, mass steam rate, SSR and SOR with interface temperature as obtained from Gupta and Gittins’ semi-analytical model for solvent with constant solvent diffusivity of 6E-7 m²/s

![Graph](image2)

**Figure 3.6:** Variation of integrals $I_{\text{sol}}$ and $I_{\text{oil}}$ with steam frontal velocity ($U$).
Figure 3.7: Variation of oil rate and solvent injection rate with interface temperature.

Figure 3.8: Variation of Steam Oil Ratio (SOR) (left axis) and Steam Solvent Ratio (SSR) (right axis) with interface temperature.
Figure 3.9: Variation of oil rate (right axis) and SOR (left axis) with SSR

Figure 3.10: Variation of SOR and SSR with time at the minimum interface temperature.
Figure 3.11: Variation of mixture viscosity with solvent concentration

Figure 3.12: Variation of solvent concentration (left axis) and overall diffusion coefficient, D (right axis) with distance from the interface.
Chapter Summary

It is determined that variation of solvent PVT properties with temperature and concentration play significant role in the performance of the ES-SAGD process. Lowering the interface temperature causes increased solvent concentration at the steam chamber edge and higher solvent diffusivity, thereby increasing the bitumen recovery and decreasing the steam requirement.

The following are the conclusions based on the improved semi-analytical approach and its application to a 2D simple reservoir model as discussed in this chapter.

- Important physical processes such as solvent diffusion into the bitumen can be modeled using this approach thereby rendering the model simple but robust.
- The variation of overall solvent diffusion coefficient with concentration and viscosity can be modelled using this approach using simple correlations. This obviates the use of a constant overall solvent diffusion coefficient (D), which would give less accurate results.

- The ES-SAGD process performs best at the minimum interface temperature of the steam front, where bitumen recovery is highest and cSOR is the lowest.

  The results from this model with suitable inputs representative of the reservoir rock and fluid properties, solvent properties thermal properties specific to a particular case could be used to predict the performance for ES-SAGD process.
Chapter 4: Validation of semi-analytical model

Thus far, we have developed an improved semi-analytical model to understand the ES-SAGD process. The model utilizes analytical and numerical schemes to model the process that take place resulting in thermal and mass transfer between the injected steam-solvent and the bitumen. It is however necessary to compare the results from this model against numerical simulation results in order to verify if the process physics are being represented accurately in the model. In this chapter, we compare the results from the semi-analytical model with the output obtained from full physics flow simulations using the commercial reservoir simulator CMG-STARS™. The simulations use the same set of input parameters for reservoir rock and fluid properties that are used to develop the results in the previous chapter. In addition, the method of dynamic refinement at the moving steam-solvent-oil interface in CMG-STARS™ reservoir simulation is implemented to control the effect of numerical dispersion on simulation results.

4.1 Demonstration Example In Flow Simulator

To compare the results obtained using both the original semi-analytical model proposed by Gupta and Gittins (2012) and the improved semi-analytical approach, a 2D homogenous simulation model with same input parameters as the semi-analytical model is constructed. The simulation grid consists of 150 x 1 x 100 grid blocks with grid dimension of 1m by 1m by 0.6m. Since it is a 2D model, the dimension in the y direction is assumed to be of unit length. Figure 4.1 shows X-Z profile of the initial reservoir pressure specified for the simulation. This model, with butane as solvent, is simulated for 4 years with operational constraints of 2500 kPa for well injection BHP, 5 months of pre-heating period and a steam trap control of 2°C for producer. Steam trap control used to avoid production of steam from the well by specifying the maximum value of
temperature difference, in the producer grid block, between steam saturation temperature and water temperature. The initial reservoir pressure is set at 1500 kPa at a reference depth of 270 m. The average steam injection rate of 1 m$^3$/day and Solvent Steam Ratio (SSR) of 0.2 are used as additional constraints.

![Initial reservoir pressure distribution in a 2D gridded reservoir model used in CMG-STARS\textsuperscript{TM}.](image)

Figure 4.1: Initial reservoir pressure distribution in a 2D gridded reservoir model used in CMG-STARS\textsuperscript{TM}.

The relevant input data for variation of bitumen viscosity, butane viscosity, bitumen density and butane density with temperature is obtained from data used in Gupta and Gittins’ (2012) model with some modifications. To enable proper interpolation between the values for the input data, a polynomial curve-fit model is used for bitumen density and butane properties. Piecewise polynomial interpolation function “interp1(pp)”
is used in MATLAB, 2014 to interpolate values between the data for bitumen viscosity as its variation followed a complex curvic profile. Figures 4.2 and 4.3 show the trends in the bitumen and butane (solvent) properties with temperature and their respective fitted polynomial degree model. Figure 4.2 (left) shows that the bitumen viscosity follows a curvilinear trend with temperature.

![Graphical representation of variation of bitumen density (left) and bitumen viscosity (right) with temperature and curve fit model used in semi-analytical model.](image1)

Figure 4.2: Graphical representation of variation of bitumen density (left) and bitumen viscosity (right) with temperature and curve fit model used in semi-analytical model.

![Graphical representation of variation of butane density (left) and viscosity (right) with temperature and curve fit model used in semi-analytical model.](image2)

Figure 4.3: Graphical representation of variation of butane density (left) and viscosity (right) with temperature and curve fit model used in semi-analytical model.
Table 4.1 shows some constants used in CMG-STARS™ input file for computing equilibrium constant $K_i$ for butane. These constants are used within Equation 4.1 for evaluating equilibrium constant ($K_i$) for butane.

$$K = \left( \frac{K_V}{P} \right) \exp\left( \frac{K_V}{T - K_V S} \right)$$  \hspace{1cm} (4.1)

Table 4.1: Constants used to determine equilibrium constant ($K_i$) for butane

<table>
<thead>
<tr>
<th>K-Values</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>KV1</td>
<td>3049.597 kPa</td>
</tr>
<tr>
<td>KV4</td>
<td>-789.549°C</td>
</tr>
<tr>
<td>KV5</td>
<td>-111.372°C</td>
</tr>
</tbody>
</table>

Because the ultimate goal is to compare the results from the semi-analytical model against the numerical simulation results, the same Equation 4.1 is used in the semi-analytical calculations with the constants specified in Table 4.1.

### 4.2 Validation Against Initial Simulation Results

Here, we discuss the quality of match between the results obtained using Gupta and Gittins’ (2012) semi-analytical model and that of CMG simulation. Keeping the same input parameters as in the semi-analytical approach, the results obtained using the analytical approach and 2-D simulations are compared. Approximately 40% mismatch between results of CMG simulation and the semi-analytical approach is observed. The minimum temperature at the interface approached a value of 142°C using Gupta and Gittins’ (2012) semi-analytical approach whereas the simulation results exhibited a variation from 130-170°C at the steam chamber edge. Figure 4.4 shows the butane mole fraction in the grid at the end of simulation. The solvent resides exclusively along the chamber walls as expected. However, in such flow simulations, the maximum value of
solvent fraction varies along the edge of the steam chamber. It can be observed that the solvent mole fraction increases to a maximum of ~0.45 at approximately 18 m from the well pair in the tapering part of steam chamber and to a maximum of 0.24 at approximately 70 m from well pair in the upper broader region of steam chamber and falls towards the cold bitumen region (See Figures 4.6 and 4.7). Figure 4.5 shows the decline in reservoir temperature away from the well pair. It should be noted that in some locations on the tapering end of steam chamber, where the solvent mole fraction reaches the maximum, grid block temperature range from 130-170 degrees C (See Figure 4.6), which is intermediate between steam temperature and native reservoir temperature. However, close to top of the reservoir, grid block temperatures, at the maximum solvent fraction, are around 190°C. (See Figure 4.7) This sets the interface temperature in CMG simulation and here the solvent dilution into bitumen attains its maximum. Figure 4.8 shows the variation of bitumen viscosity with distance from well pair. It can be seen that bitumen viscosity is lower in the region of solvent dilution (dark blue band along the steam chamber edge) than its neighboring regions. The bitumen viscosity attains its lowest value where the mole fraction of solvent in liquid phase is the highest. Therefore, it can be concluded that solvent dilution in ES-SAGD process indeed induces improved lowering of bitumen viscosity than in SAGD process, where only heat effects are present. In SAGD process, the bitumen viscosity would monotonously decrease from the well-pair, which is the source for steam injection.
Figure 4.4: Mole fraction of butane in oil phase at end of simulation in CMG-STARS™

Figure 4.5: Grid temperature in degrees C at end of simulation in CMG-STARS™
Figure 4.6: Variation of mole fraction of butane in oil phase with distance from well-pair at tapering part of steam chamber

Figure 4.7: Variation of mole fraction of butane in oil phase with distance from well-pair at the top of steam chamber (broader region).
Figure 4.8: Grid bitumen viscosity in cp at end of simulation in CMG-STARS™

Figures 4.9 through 4.11 show comparison of variation of bitumen rate, cumulative bitumen produced and cSOR with time between Gupta and Gittins’ (2012) semi-analytical model and 2D CMG simulation respectively. The results obtained from the semi-analytical approach seem too optimistic when compared with the CMG simulation results.
Figure 4.9: Comparison of oil rate predicted by Gupta and Gittins’ (2012) semi-analytical approach and CMG-STARSTM simulation.

Figure 4.10: Comparison of cumulative oil produced predicted by Gupta and Gittins’ (2012) semi-analytical approach and CMG-STARSTM simulation.
Based on the results presented above, it can be concluded that the simulation results match the trends observed in the semi-analytical approach, although the actual numerical values for response variables such as Cumulative Steam Oil Ratio (cSOR), cumulative bitumen produced and oil rate do not match well. The key reasons that might cause such discrepancies in the results are as follows:

- Gupta and Gittins’ (2012) semi-analytical approach assumes a sharp interface between vapour and oil zone. However, it has been well iterated in the previous chapters that experimental observations show an existence of diffused layer where solvent concentration decreases monotonically from the interface up to the undrained oil zone (Butler and Das, 1996). CMG simulations also show this kind of variation in solvent mole fraction (Figure 4.4). Existence of this zone considerably affects solvent diffusion into bitumen.
• The semi-analytical method does not take into account multiphase flow parameters such as relative permeability and capillary effects, unlike in the 2D simulation model. As seen earlier, the expressions for cumulative production and cSOR etc. only take into account absolute permeability of the reservoir. This yields optimistic estimate of oil rate using the semi-analytical approach.

• Flow simulations are sensitive to grid resolution. Coarse grids may give rise to artificial numerical dispersion. Fine grid is necessary to represent the length scale over which the solvent-dilution process occurs.

• Flow simulation involves reservoir pressure and its interaction with injection process. However, this interaction is not captured in the semi-analytical approach.

4.3 Validation of Improved Semi-Analytical Model With Results From Reservoir Simulation

The semi-analytical model by Gupta and Gittins (2012) uses a constant value of 6E-07 m²/sec for solvent diffusivity. This value is selected to attain a good match with the field observed responses for bitumen production. However, matching the field results by tuning solvent diffusivity does not seem to be a reasonable method to model solvent-diffusion dominated mass transport phenomenon. Therefore, as one of the methods to model concentration dependent solvent diffusivity as well as to improve the match between the semi-analytical model and reservoir simulation, the variation of solvent diffusivity with solvent concentration in bitumen phase and mixture viscosity is introduced in the semi-analytical model. This is done, as discussed in Chapter 3, by considering solvent diffusion as a function of varying solvent concentration using the Wilke-Chang (1955) correlation. The mixture viscosity is modelled using the mixing rule, which relates the solvent concentration and viscosities of each solvent and bitumen. Since the mixture viscosity and solvent concentration decreases away from the steam
chamber interface, the solvent diffusion coefficient also decreases. This would result in lower bitumen recoveries than that from Gupta and Gittins’ (2012) semi-analytical model, which assumes a sharp interface with constant solvent diffusivity. Generally, the solvent diffusivity is tuned to match the field production history for ES-SAGD process. However, with the modifications discussed in Chapter 3, the solvent diffusivity need not be varied and therefore does not have to be included with other common and dominant history matching parameters like relative permeabilities, reservoir heterogeneities etc.

Now, we analyze and discuss the quality of match between the improved semi-analytical model and that of CMG-STARS™ simulation and validate whether introduction of solvent diffusivity as a variant proved helpful. Figures 4.12 through 4.14 show comparisons of bitumen rate, cumulative bitumen produced and cSOR, respectively, between the three models – Gupta and Gittins’ (2012) semi-analytical model, improved semi-analytical model and 2D CMG simulation. It is evident from the figures that predictions using the improved semi-analytical model are closer to that of reservoir simulation. Modeling the variation in diffusivity with concentration and viscosity gives slightly less optimistic result than that obtained using a constant value of 6E-7 m²/sec for diffusion coefficient. It is expected that at locations, where mixture viscosity is higher with lower solvent concentration in liquid phase, the solvent diffusion would be lower and vice-versa for lower mixture viscosity with higher solvent concentration. Therefore, it is understandable that the bitumen recovery rates would be affected by varying diffusion.
Figure 4.12: Comparison of oil rate predicted by Gupta and Gittins’ (2012) semi-analytical approach, improved semi-analytical model and CMG- STARS™ simulation

Figure 4.13: Comparison of cumulative oil production predicted by Gupta and Gittins’ (2012) semi-analytical approach, improved semi-analytical model and CMG- STARS™ simulation
4.4 Key Modifications to Reservoir Model in CMG-STARSTM Simulation

Other possible reasons for the mismatch between semi-analytical model and CMG results could be due to the inability of the semi-analytical model to represent multiphase flow and numerical dispersion arising due to gridding in a numerical simulator. In the following section, we describe our efforts to minimize the difference in the results between the improved semi-analytical model and those obtained using CMG-STARSTM.

4.4.1 Straight line relative permeability

The semi-analytical approach does not take into consideration multi-phase flow effects for example, relative permeability to oil during drainage. This could be one of the reasons for discrepancy between results obtained using the semi-analytical approach as compared to CMG-STARSTM simulation. The fluid draining along the chamber wall is essentially multiphase and the specification of parameters such as relative permeability regulates the velocity of different phase flowing along the chamber wall. In order to
minimize the effect of multiphase flow effects, straight line relative permeability models are used as input to CMG as shown in Figure 4.15. The same concept is used in model developed for Steam Alternating Solvent Process (SAS) by Zhao et al., 2005. The straight line relative permeabilities basically imply that the velocity of any of the phases is unimpeded by the presence of the other phase. Straight line relative permeability would therefore increase the bitumen recovery rate obtained in the CMG simulation and bring the results closer to the improved semi-analytical model.

Figure 4.15: Straight line relative permeability curves used in CMG-STATS™ reservoir model.

Figure 4.16 shows some slight improvement of the match between the improved semi-analytical model and CMG simulation with introduction of straight-line relative permeabilities. By minimizing the multi-phase flow effects through straight line relative permeabilities, bitumen recovery has been increased and injected steam volumes, in terms of cSOR, have decreased. Approximately 10% increase in bitumen recovery can be seen from the graphs.
4.4.2 Dynamic Gridding in CMG-STARS™

The solvent dilution into bitumen, governed by PVT equilibrium, mass and thermal diffusion is a short-scale process and depicting such a process using coarse grid blocks in a commercial simulator may result in numerical dispersion issues. Better predictions may be achieved if the process is modeled using a fine grid. However, even for simple, homogenous fine grid reservoir model, the computation time may be too long. Dynamic gridding based on dynamic refinement of grid at the steam chamber interface may reduce the computation time considerably. Two schemes for dynamic local refinement of grid based on steam chamber front temperature or solvent concentration are implemented to minimize the effect of numerical dispersion. One scheme is based on changes in temperature and other scheme is based on changes in solvent mole fraction across contiguous grid blocks. In the temperature-based approach, a tolerance value for grid temperature change is assigned. If during simulation, a group of adjacent grid blocks have temperature change (ΔT) between consecutive time steps beyond the specified tolerance, then those groups of grid blocks are refined. In the second approach, if during
simulation, a group of adjacent grid blocks have mole fraction change ($\Delta x_{\text{butane}}$) beyond the specified tolerance, then those groups of grid blocks are refined.

Very coarse grid may lead to inaccurate numerical computations and very fine grid result in numerical instabilities that in turn lead to unexpected termination of simulation. Therefore, optimum grid dimensions are selected for coarse grid and fine grid model, both of which form reference cases in this study against which the results obtained using dynamic gridding are compared. The reference grids comprise of 150 x 1 x 100 grid blocks each of size 0.5m by 1m by 0.5m and 75 x 1 x 50 grid blocks each with 1m by 1m by 1m in fine and coarse grid models respectively. For dynamic gridding, the grid blocks, the dimensions of the refined grid blocks are specified to be one by twenty fourth of the coarse grid dimensions at the locations where concentration or temperature tolerance is met. To be more precise, six refined scale blocks in x-direction and four refined scale blocks in y-direction correspond to one block in x and z direction in coarse scale dimension (1m by 1m by 1m).

4.4.2.1 Solvent mole fraction based dynamic gridding

Sensitivity runs are performed with different values of tolerances based on changes in mole fraction of solvent in the liquid phase for triggering dynamic gridding. The values for the solvent mole fraction changes are 0.1, 0.3, 0.5, 0.8 and 0.9. Lower the tolerance value, finer is the grid refinement starting from the coarse grid. Figure 4.17 and 4.18 shows the difference in the solvent mole fraction profile at a particular snapshot in time corresponding to the smallest (0.1) and largest value (0.9) of mole fraction tolerance. As expected, the figure shows that with smaller tolerance value, the interface between the solvent and bitumen is crisply resolved. As the tolerance is increased, the interface is represented by coarse grid blocks and significant numerical dispersion is observed.
Therefore, higher the tolerance value, the more diffused is the solvent front i.e. the mass transfer from the steam chamber to the surrounding bitumen is not crisply resolved. However, at lower tolerance values, solvent diffusion is gradual, slowly increasing and decreasing along the length normal to steam chamber edge. The rate of steam chamber expansion is however higher in case with lower tolerance values.

Figure 4.17: Concentration profile of butane in oil phase at end of simulation for dynamic gridding based on mole fraction tolerance of 0.1 in CMG-STARSTM
It is observed that results from dynamic gridding based on solvent mole fraction in liquid phase spanned between the coarse grid and fine grid results as shown in Figures 4.18 and 4.19. As the tolerance value of mole fraction of solvent in the liquid phase for dynamic gridding is increased (from 0.1 to 0.9), the Cumulative Steam Oil Ratio (cSOR) and cumulative oil production curves goes closer to coarse grid results. Coarse scale results are indicated by black dotted, bolded line and fine scale results are indicated by red bolded, dotted line in the both figures (Figure 4.19 and Figure 4.20).
Figure 4.19: Cumulative Steam Oil ratio (cSOR) profiles obtained by changing the tolerance value for change in solvent mole fraction for triggering dynamic gridding in CMG-STARSTM. The profiles corresponding to the reference cases with coarse and fine gridding are shown as bold, dotted lines.
4.4.2.2 Temperature based dynamic gridding

Similarly, sensitivity runs are performed with different tolerance values of minimum temperature change for dynamic gridding. The temperature tolerance values are specified to be 5, 10, 50, 100 degree Celsius. Figure 4.21 and Figure 4.22 show the difference in the temperature profile in the reservoir corresponding to the smallest (5°C) and largest value (100°C) of temperature change tolerance during dynamic gridding. As
the temperature change tolerance is increased, the simulation results are significantly affected by numerical dispersion and as a result the temperature profile is smeared and not as crisply resolved.

Figure 4.21: Temperature profile at the end of simulation based on temperature change tolerances of 5°C during dynamic gridding in CMG-STARS™.
Figure 4.22: Temperature profile at the end of simulation based on temperature change tolerances of 100°C during dynamic gridding in CMG-STARS™

From Figures 4.23 and 4.24, it can be seen that as the tolerance value for grid block temperature change to trigger dynamic gridding is increased from 5°C to 40°C, the cSOR (Cumulative Steam Oil Ratio) and cumulative oil produced stay close to fine grid results, hence, no significant difference is seen. However, if the tolerance value is increased beyond 40°C, the cSOR and cumulative oil produced now converge to the coarse grid simulation. This is because temperature increases are in small steps of 2-5°C while a tolerance value (ΔT) of 50°C or more is seldom achieved by any grid block group and its neighboring blocks and so the blocks remain coarse for any high threshold value. Therefore, we can conclude that if temperature tolerance is used to control dynamic gridding, small values of tolerance in the vicinity of 5 - 10°C should be selected.
Figure 4.23: Cumulative Steam Oil Ratio (cSOR) profiles obtained by changing the tolerance value for change temperature for triggering dynamic gridding in CMG-STARSTM. The profiles corresponding to the reference cases with coarse and fine gridding are shown as bold, dotted lines.

Figure 4.24: Cumulative Steam Oil Ratio (cSOR) corresponding to different temperature change tolerance values for triggering dynamic gridding in CMG-STARSTM.
Dynamic gridding based on mole fraction of solvent produces significant differences when the tolerance values are varied from 0.1 to 0.9. But dynamic gridding based on temperature, either show convergence to fine grid results if the tolerance is set to be small or to coarse scale results if tolerance is set high. In other words, it is difficult to fine tune the grid in order to obtain reasonably good results while saving significantly on computation cost because such a solution would require low temperature tolerance and in that case the entire region within the steam chamber would be finely meshed. Finer grid blocks at the steam chamber edge better represent the solvent diffusion process into the bitumen. Hence, dynamic gridding based on mole fraction of butane, with tolerance value of 0.1 is considered for this study.

4.5 Validation with Modified Reservoir Modeling in CMG-STARS™

With the above discussed modifications to the CMG reservoir model, the results of the improved semi-analytical model and 2D simulation model are compared against each other. The trends and magnitude of the results obtained using the semi-analytical model is in reasonably good agreement with the numerical simulation results, after modifications. Figures 4.25, 4.26 and 4.27 show the improvement of the match between the improved semi-analytical model and modified CMG simulation. The results using the improved analytical; formulation are better than those obtained using the original semi-analytical approach presented by Gupta and Gittins (2012).

As the results indicate, numerical simulation with dynamic gridding and straight line relative permeability produces results that are very close to the improved semi-analytical results. Dynamic gridding better represents the diffusion of solvent into the bitumen and the associated length-scales. It is also evident that modeling variations in
solvent concentration along the mobile bitumen zone improves the predictive capability of the semi-analytical model.

Figure 4.25: Comparison of bitumen rate predicted using Gupta and Gittins’ (2012) semi-analytical approach, improved semi-analytical model, CMG- STARS™ simulation before and after modification.

Figure 4.26: Comparison of cumulative bitumen predicted using Gupta and Gittins’ (2012) semi-analytical approach, improved semi-analytical model, and CMG-STARS™ simulation before and after modification.
Figure 4.27: Comparison of cSOR predicted using Gupta and Gittins’ (2012) semi-analytical approach, improved semi-analytical model, and CMG- STARS™ simulation before and after modification.

4.6 CHAPTER SUMMARY

Validating the semi-analytical model data against a full physics reservoir simulation model is crucial to test the behavior and validity of the model. Therefore, the key conclusions from the validation exercise describe in this chapter are:

1. The semi-analytical approach has been formulated for capturing the basic mass and heat transfer processes that take place during the ES-SAGD process. This approach can provide a preliminary estimate of the trends of fluid rates, cSOR and SSR for ES-SAGD project and thereby help in project screening studies.

2. The numerical dispersions that occur in numerical simulations tend to mask the actual mass-transfer mechanisms that take place at the steam chamber edge. Dynamic gridding based on solvent mole fraction change occurring at the steam chamber edge seems to be a feasible option to mitigate such numerical artifacts.
Finer grid blocks at the steam chamber edge gives more accurate results as compared to coarse grid simulation.

3. The results from the improved semi-analytical method are in reasonable agreement with 2D simulation results, after implementing modifications.
**Chapter 5: Optimization in ES-SAGD Process Using Response Surface Methodology (RSM)**

ES-SAGD is one of the emerging hybrid thermal-solvent recovery technologies. Like any other new technology, understanding the physical processes involved, designing the right operating conditions for a technically effective implementation that ensure sustainable economics of the project are essential for the success of an ES-SAGD process. In ES-SAGD, the bitumen is rendered more mobile by both thermal and solvent dilution mechanisms. However, it is not obvious what the optimal solvent concentration in the injected stream should be and how much heat is sufficient for minimizing energy loss. On one hand, it is necessary to minimize the steam requirement but on the other hand, injection of excessive solvent fraction is expensive and furthermore can result in adverse heat transfer at the steam-solvent-bitumen interface. Hence the need to formally implement optimization strategies, which would maximize the bitumen recovery and minimize the steam and solvent requirement simultaneously, resulting in a successful ES-SAGD project.

This chapter investigates two optimization methods based on a proxy model developed using Response Surface Methodology (RSM). The chapter starts with a brief background of the response function calibration process. The proxy model development process starts with the selection of important control variables, performing regression, testing the proxy model for accuracy and adequacy. Subsequently, the implementation of two optimization techniques namely gradient based approach for single objective optimization and hybrid Pareto-based multi-objective optimization is presented. Finally, the range of optimum operational parameters are presented and discussed.
5.1 THEORETICAL BACKGROUND

5.1.1 Response Surface Methodology (RSM)

Response surface methodology is one of the common and useful statistical techniques for empirical model building. It builds a functional relationship between response of the interest ($y_i$) and selected number of independent control variables ($\xi$), as indicated in Equation 5.1

$$y_i = \eta_i + e_i$$  
(5.1)

where, $\eta_i$ is a true value of response corresponding to any particular combination of the control variables ($\xi$), as given in Equation 5.2, in the absence of any error, experimental or mathematical, $e_i$ is the error that includes the effect of measurement and numerical modelling error on the response. This error will be treated as a statistical error, assuming distribution with mean zero and variance $\sigma^2$. The index “$i$” in the above equation denotes the number of observations or experiments.

The components of Response Surface Methodology (RSM) are:

1. The experimental design for exploring the space of independent control variables of the process.
2. A proxy model that is an empirical statistical model approximating the relationship between the response and the set of selected controlling variables similar to the Equation 5.1 above.
3. Optimization methods to determine the optimal setting of independent control variables that produce desirable (maximum or minimum) value of the response.

This part of RSM will be discussed in the next section 5.1.2
The response model can be linear, quadratic or polynomial. Linear or first order models are generally used as a preliminary step to select and screen control variables based on their impact on the process under study. A linear model can be written as:

$$\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k$$  \hspace{1cm} (5.3)

where, $\beta_0$ is a constant term, $\beta_k$, $k = 1$ to $n$ are regression coefficients that represent the rate of change of $\eta$ with respect to $x_j$, $x_k$ are normalised control variables and $\eta$ is the true response.

Quadratic models or second order models are commonly used as reasonably good approximations for the response function because of their flexibility and accuracy in predicting the true response ($\eta$). In addition, the regressor variables ($\beta_{11}, \beta_{12}, \ldots, \beta_{kk}$) can be estimated relatively easily. A quadratic model can be expressed as:

$$\eta = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k + \beta_{11} x_1^2 + \ldots + \beta_{kk} x_k^2 + \beta_{12} x_1 x_2 + \ldots + \beta_{k-1.k} x_{k-1} x_k$$  \hspace{1cm} (5.4)

where, $\beta_0$ is a constant term, $\beta_k$, $k = 1$ to $n$ are regression coefficients for individual control variables, $\beta_{kk}$, $k = 1$ to $n$ are regression coefficients for second order terms $x_k^2$ and $\beta_{k-1,k}$, $k = 1$ to $n$, are regression coefficients for interacting terms $x_{k-1} x_k$. Details on the basic terminologies and methods for computing the regressor coefficients can be found in Montgomery et al., 2006.

5.1.1.1 Experimental design

The determination of the regressor variables ($\beta_{11}, \beta_{22}, \ldots, \beta_{kk}$; $\beta_{12}, \beta_{23}, \ldots, \beta_{k-1.k}$), given in Equation 5.4, depends on experimental data. The number of experiments required is directly proportional to the size of the control variable space. Larger number of control variables will mandate a larger number of experiments to generate a robust response surface model. Sometimes, experiments in the form of computationally
intensive sub-surface flow simulations may make the RSM exercise impractical due to the \textit{CPU} expense.

Several methods for experimental design, like Full Factorial Design (FFD) Fractional Factorial Design, Central Composite Design (CCD), Box Benken Design (BBD), D-Optimal designs, have been developed to minimize the \textit{CPU} cost without compromising process characteristics. The conceptual understanding of all these designs has been well elucidated in Montgomery at al., 2006. However, for this study, we will focus more on the Full Factorial Design and Multi-level Full Factorial Design.

5.1.1.1.1 Full Factorial Design

A full factorial design requires $a^k$ experiments, where each of “$k$” control variables has “$a$” levels. This is considered as the most “fool proof” design as it contains all possible combinations of a set of control variables. But it is also expensive in terms of experimental resources needed, as the number of experiments increase exponentially with the number of control variables. To fit a second-order model, the least number of experiments required is $3\times k$ (Montgomery et al., 2006). On the other hand, only $2\times k$ experiments are needed to fit a first-order model. Figure 5.1 shows a Full Factorial Design with three control variables, A, B, C, two levels (-1,1) plus one center point (0,0,0). The number of experiments in this case would be $2^3 +1$(center point) = 9 (= 3×$k$, where $k=3$). Table 5.1 shows the typical full factorial design for 3 variables and 2 levels.
Figure 5.1: Simple illustration of three factor, two level and one center point for a Full Factorial design

Table 5.1: Design matrix for three factor Full Factorial Design

<table>
<thead>
<tr>
<th>Expt. No.</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>+</td>
<td>+</td>
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<tr>
<td>8</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

5.1.1.1.2 Multi-level Full Factorial Design

Multi-level Full Factorial Design is a relatively new concept and uses the same principle as the Full Factorial design. This design is used when one wants to
systematically vary experimental factors, assign each factor a discrete set of levels. In this
design, each control variable can have any number of levels, and the number of levels can
differ amongst them. The general notation for this design is \( \prod_{i=1}^{i=n} a_i \), where \( a_i \) is the
number of levels for each control variable in a class \( k_j \). For example multi-level factorial
design for two control variables each with 4 and 5 control levels will be given as \( 4^1 \times 5^1 \),
where the total number of experiments is \( 4^1 \times 5^1 = 20 \). The design matrix for a \( 2^1 \times 5^1 \)
multi-level full factorial design with two control variables A and B is given in the
following Table 5.2

Table 5.2: Design matrix for a \( 2^1 \times 5^1 \) Multi-level Full Factorial Design with two control
variables A and B.

<table>
<thead>
<tr>
<th>Runs</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-0.5</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>-0.5</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The features of these designs are:

- Every control variable can have as many control points as the analyst wishes. The
  number of observations/measurements required to make the inference is smaller
  because you can consider the number of levels for each control variable explicitly.
- They capture a lot of information about the main effects and interactions in a
  relatively few number of runs.
One can test whether non-linear terms are needed in the model, at least as far as the three-level factors are concerned. This form of experimental design will be used in this work to study the optimization in ES-SAGD process.

5.1.1.2 Determination of Regressor Variables (β)

The parameters in Equation 5.4 are determined by fitting the model to the experimental data which might be measured responses corresponding to a set of combinations of control levels. Thus, for a three variable model Equation 5.4 can be written in the general form as follows:

\[ Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{33} x_3^2 + \beta_{12} x_1 x_2 + \beta_{23} x_2 x_3 + \beta_{13} x_1 x_3 + e \] (5.5)

Equation 5.5 is called a multiple linear regression model as the regressors are established by minimizing the average square error. In order to estimate the parameters in multiple linear regression models, the method of least squares is typically used (Montgomery et al., 2006).

We can then write Equation 5.5 for k-experiments as:

\[ Y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_{11} x_{i1}^2 + \beta_{22} x_{i2}^2 + \beta_{33} x_{i3}^2 + \beta_{12} x_{i1} x_{i2} + \beta_{23} x_{i2} x_{i3} + \beta_{13} x_{i1} x_{i3} + e_i \], for i = 1 to k (5.6)

The least squares method identifies the values of β’s in Equation 5.6 as those that lead to a minimum sum of the squares of the errors \( e_i \).

The least squares function (E) is

\[ E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} [y_i - f(x_{i1}, x_{i2}, \ldots, x_{ik}; \beta_1, \ldots, \beta_k; \beta_{11}, \ldots, \beta_{kk}; \beta_{12}, \ldots, \beta_{k-1,k})]^2 \] (5.7)
The function $E$ is to be minimized with respect to $\beta_0, \beta_1, \ldots, \beta_k; \beta_{11}, \ldots, \beta_{kk}; \beta_{12}, \ldots, \beta_{k-1,k}$. Differentiation Equation 5.7 with respect to regressor coefficients $\beta_k$ and equating the obtained expression to zero gives $n$ equations which can be solved simultaneously to determine the $n$ regressor coefficients, $\beta_k$.

In this work, the responses of the ES-SAGD process to control variables are numerically evaluated using the reservoir simulator CMG-STARS™. Therefore, the term ‘experiment’ is referred to ‘numerical experiments’, unless otherwise specified, throughout the thesis.

### 5.1.1.3 Response Surface Model Adequacy

The validity of proxy model should be tested to check the overall adequacy of the model and each parameter contained in it that define the proxy model. In this section, we describe two such importance testing methods also called as hypothesis testing. These hypothesis tests assume that errors $e_i$ in the model are normally and independently distributed with zero mean and with variance $\sigma^2$, which is dependent on the sum of squares of error as described in Equation 5.14 (Montgomery et al., 2006) in the following text. The following tests are used in this work.

1. **Test for significance of the regression model**

   The total error $SS_T$ is partitioned into a sum of squares due to regression $SS_R$ (variance of the regressed values), and a sum of squares error $SS_E$ (error variance). The $SS_R$ measures the amount of variability in the measured responses $y_i$ accounted for by the regression line, and the $SS_E$ measures the amount of the deviation between the true data and the fit. A good response function has the mean of the set $SS_E$ equal to zero, or at least close to zero. The F-statistic, that follows the Fisher–Snedecor distribution, is then calculated, which is defined as
\[ F_0 = \frac{MSR}{MSE} = \frac{SSR/DF_R}{SSE/DF_E} = \frac{SSR/k}{SSE/(n-k-1)} \]  

(5.8)

where \(MS_R\) and \(MS_E\) are the regression mean square residuals and the mean square errors respectively. \(DF_R\) is the degrees of freedom of the regression, which is expressed as

\[ DF_R = p - 1 \]  

(5.9)

where \(p\) is the number of regression coefficients in a model including \(\beta_0\). \(DF_E\) is the degrees of freedom of the error.

\[ DF_E = n-p-k+1 \]  

(5.10)

\(k\) is the number of regressor variables.

From \(F_o\), a confidence level \(p\)-value can be determined, which is given by

\[ p\text{-value} = 1 - F_{cdf}(F_0, DF_E, DF_R) \]  

(5.11)

Here, \(F_{cdf}\) is the cumulative density function (cdf) value corresponding to \(F_o\) in \(F\) distribution with prescribed residual and error degrees of freedom. The \(F\) distribution is a right-skewed distribution used most commonly in Analysis of Variance. Figure 5.2 illustrates the \(F\)-distribution for given \(\alpha\), \(DF_E\), \(DF_R\). The \(p\)-value value shows how good the fit is likely to be in the entire space of control variables. The lower the \(p\)-value alludes to higher the confidence interval is. Also it has to be less than \(\alpha\) for the rejection of the null-hypothesis. We require 99% confidence interval; therefore, a \(p\)-value has to be 0.01 or less. In this work, models with \(F > F_{\alpha=0.99, \, n-k-1}\) and \(p\)-values < 0.01 will be considered as good approximation of the response function.

Figure 5.2: Conceptual diagram for \(F\)-statistic distribution.
The test to determine if there is a linear relationship between the response \( y \) and any of the regressor variables, \( x_1, x_2, \ldots, x_k \) is to test the hypothesis as given in Equation 5.12.

\[
H_0: \beta_1 = \beta_2 = \ldots = \beta_k = 0
\]

\[
H_1: \beta_j \text{ not equal to 0 for at least one } j
\] (5.12)

The hypothesis \( H_0 \) confirms that there is no significant regressor variable in the model and therefore the proxy is not good for approximating the response function. The alternate hypothesis \( H_1 \) concludes that there is at least one significant variable and therefore the model is reasonably good enough to model the response accurately.

If \( F_0 \) from the Equation 5.8 exceeds \( F_{a, k, n-k-1} \), which is value from the F-distribution function at the “\( \alpha \)” confidence level that has \( DF_R = p-1 \) and \( DF_E = n-p-k+1 \) degrees of freedom, the null hypothesis \( H_0 \) will be rejected. This means that at least one regressor parameter is significant towards estimation from proxy model.

2. **Test for significance on individual regression coefficients**

Once we have determined at least one of the regressor variables is significant in the proxy model, we also have to test whether each of the regressor coefficient is significant. The non-significant terms can then be dropped from the proxy model. The proxy model with only significant regressor terms is called as reduced proxy model. Similar to the null hypothesis introduced in the previous section, the regressor \( \beta_j \) need to pass a t-statistic test in order to be interpreted as a significant factor in the regression. In other words, the null hypothesis \( H: \beta_j = 0 \) is rejected if \( |t_0| > t_{a/2, n-2} \), where \( t_{a/2, n-2} \) is the quantile of a t-\( n-2 \) distribution at a significant level \( \alpha/2 \) (Montgomery et al., 2006). The t-statistic for this hypothesis is given by:

\[
t_0 = \frac{\hat{\beta}}{\sqrt{\hat{\sigma}^2 c_{ij}}} \tag{5.13}
\]
where \( C_{jj} \) is the diagonal element of the covariance matrix \( (X'X)^{-1} \), which is the covariance between variable \( x_i \) and \( x_j \) corresponding to normalised estimator \( \hat{\beta}_j \), and \( \hat{\sigma}^2 \) is an unbiased estimator of \( \sigma^2 \) (the variance of responses). The unbiased estimator \( \hat{\sigma}^2 \) is given by:

\[
\hat{\sigma}^2 = \frac{SS_E}{n-p}
\]

The p-value for t-statistic is given by Equation 5.15

\[
p\text{-value} = 2 \times T\text{cdf}(-|t|,DF_E)
\]

where \( T\text{cdf} \) is the Student T cumulative distribution value corresponding to \( |-t| \) and with \( DF_E \) degrees of freedom. Smaller p-values indicate significance of the regressor variables and hence these variables are not eliminated from the proxy model. In this work, coefficients with t-statistic values \( > t_{\alpha/2, n-2} \) for \( \alpha = 0.99 \) and p-values \( < 0.01 \) will be considered as significant coefficients and the coefficients not conforming to this criteria will dropped from the proxy models.

Other ways of assessing the goodness of fit of the model, such as using \( R^2 \) and \( R_{adj}^2 \) (Montgomery et al, 2006) can also be evaluated. The coefficient of multiple determinations, \( R^2 \), is defined as

\[
R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}
\]

\( SS_T \) is the total error. \( R^2 \) is a measure of the amount of reduction in the variability of \( y \) obtained by using the regressor variables in the model. It is a descriptive measure between 0 and 1. A value close to 1 implies that most of variability in \( y \) is explained by that response function. Lower \( R^2 \) indicates that the model has more error. Thus, a low \( R^2 \) can warn of imprecise predictions. However, a large \( R^2 \) does not imply that the model is accurate since \( R^2 \) value always increases when an additional regressor variable is added regardless of the value of the contribution of that variable. This is because the errors are assumed to be independent and so the increase in error variance due to error
dependencies is ignored. Also, $R^2$ alone cannot determine if the regressor coefficients in the model are significant. A model with low $R^2$ can possibly have significant regressor coefficients. Therefore, another parameter called an adjusted $R^2$ is used together with the $R^2$ in order to confirm the goodness of fit of a response function.

$$R_{adj}^2 = 1 - \frac{SSE/(n-p)}{SS_{total}/(n-1)} = 1 - \frac{MSE}{SS_{T}/(n-1)} = 1 - \frac{n-1}{n-p} (1 - R^2)$$ (5.17)

$R_{adj}^2$ decreases when an unnecessary regressor variable is added. It will only increase if the addition of a variable to the model reduces the MSE. When $R^2$ and $R_{adj}^2$ differ dramatically, there is a good chance that non-significant terms have been included in the model. In this work, a response surface model with $R^2$ and $R_{adj}^2$ greater than 0.80 will be considered as a reasonable approximation of the response function under study.

Graphical analysis of residuals is also a very effective way to investigate the adequacy of the fit of a proxy model and to check the normality assumptions. For this study, we plotted predicted values of responses $y_i$ versus residuals. If the plot demonstrates that the residuals can be contained in a horizontal band (homoscedasticity), then there are no obvious model biases. We can also look at the normal probability plot of the residuals. The straight line on that plot is usually determined visually; with emphasis on the central values (e.g., the 0.33 and 0.67 cumulative probability points). Substantial departures from a straight line indicate that the distribution of residuals is not normal. The model adequacy using graphical analysis will be evaluated, in the section 5.2, with the response surface models generated for ES-SAGD process.

### 5.1.2 Optimization Techniques

Once the proxy model has been developed as described in the previous sections (Section 5.1.1), the next step is to determine the optimum settings for the control variables that either maximize or minimize the proxy model. Section 5.1.2.1 and 5.1.2.2
describe two optimization approaches for maximizing or minimizing the developed response models.

5.1.2.1 Gradient Based Optimization

This method is a first-order constrained gradient-based optimization technique to find the local minimum or maximum of the function. The path of steepest descent is computed in this technique to find the minimum of the objective function in consideration. This objective function will then be minimized or maximized using gradient based descent to find the optimum settings for the control variables. The work flow of this method starts with a guessed initial vector of control variables $x_0$. This initial guess is moved to a new vector $x_{i+1}$, constrained by the upper and lower limits of variables. The update occurs along a trajectory in the direction $-\nabla f(x_i)$ i.e. in the direction of the local downhill gradient. This process is continued iteratively until the solution finally converges to $x^*$, a local minimizer for the given function. However, if the given response function has several local minima, the result will depend on the starting guess $x_0$. Figure 5.3 presents a simple conceptual representation of gradient descent search.

Figure 5.3: Conceptual representation of gradient descent search algorithm (Ref: wikipedia.org/wiki/Gradient_descent)
5.1.2.2 Multi-objective optimization

In many cases, including the optimization of the ES-SAGD process, it is necessary to simultaneously meet several objectives such as minimize CSOR, maximize recovery, minimize solvent cost. Equation 5.18 shows the generalization of the optimization procedure for the multi-objective problem:

\[ f(x) = [f_1(x), \ldots , f_i(x), \ldots , f_M(x)], \quad (5.18) \]

where \( x \) is a variable vector, \( f(x) \) is an objective vector, \( f_i(x) \) is the \( i^{th} \) objective function, and \( M \) is the number of objective functions.

In the context of the previously discussed gradient-based, multi-objective optimization can be performed by combining the different objectives into a single objective function. This can be accomplished by assigning weights to the different objectives and then linearly combining them. The weights may be obtained based on some expert knowledge about the process. However, alternative multi-objective analyses are advantageous in that their formulation can be made free from an obligation to predetermine the weights, unlike global-objective optimization techniques described above. This capability of multi-objective analyses is important because the optimum for any one objective would be different from the optimum of any other response if the responses are either uncorrelated or negatively correlated. For example, Figures 5.4 a and b illustrate two quadratic objectives, i.e. \( f_1(x) \) and \( f_2(x) \), for which the common variable is \( x \) (\( x \geq 0 \)). As shown in Figure 5.4 a, global-objective optimization regards \( x = 1 \) as the global optimum in the case the two objectives are combined using equal weights although the optimums, which are minimum values of \( f_1(x) \) and \( f_2(x) \) are corresponding to \( x=1 \) and \( x=2 \), respectively.
Figure 5.4: Two quadratic objectives a and b for multi-objective optimization with common variable x

Genetic Algorithm (GA) is an evolutionary process to improve the population from generation to generation. The population in this case is the vector of control variables. The response surface models for the response functions are used for calculating the objective values corresponding to individual members of the population. The individuals are ranked with respect to relative superiority of the objective values using two sorting schemes in NSGA-II: non-dominated sorting for Pareto-ranking (see section 5.1.2.2.1) and crowding-distance sorting for diversity-preservation (see section 5.1.2.2.2). For describing the entire Pareto-optimal front, NSGA-II algorithm prioritizes solutions using two different sorting schemes: non-dominated sorting and crowding-distance sorting. Non-dominated sorting aims to induce solutions to be converged towards the Pareto-optimal front while sorting based on crowding distance distributes solutions uniformly on the Pareto-optimal front.

The population of the next generation is reproduced by breeding the qualified population of the current generation by use of two genetic operators: crossover and mutation (Goldberg, 1989). This stochastic evolution process is terminated if either the generation
reaches the maximum allowed number of iteration or the population stagnates without further improvement.

5.1.2.2.1 Non-dominated sorting

Pareto-optimality is a state of an optimal allocation of resources in objective space (Min et al., 2014). Mathematically, Pareto-optimality is defined as the best non-domination. The non-domination is a state of equivalence where no solution can be improved with respect to any objective without worsening at least one other objective (Srinivas and Deb, 1994). A variable vector $x_1$ is said to dominate a variable vector $x_2$ only if function of $x_1$ is less than or equal to function of $x_2$:

$$f(x_1) < f(x_2).$$

Figure 5.5 illustrates the non-dominated sorting to prioritize the objective vectors in two-dimensional objective space. Let $n$ be the number of non-dominated fronts $F$. In the figure, $n$ is three: $F_1$, $F_2$, $F_3$. The black circle refers to the best non-dominated objective vectors given the 1st front $F_1$. The objective vector $f(x_1)$ dominates the objective vector $f(x_2)$: $f_1(x_1) < f_1(x_2)$ and $f_2(x_1) < f_2(x_2)$. In the same manner, the objective vector $f(x_3)$ dominates the objective vectors $f(x_4)$, and $f(x_4)$ does $f(x_5)$: $f_1(x_3) < f_1(x_4) < f_1(x_5)$ and $f_2(x_3) < f_2(x_4) < f_2(x_5)$. The objective vectors $f(x_1)$ and $f(x_3)$ are equivalent to each other because any superiority in one objective function accompanies inferiority in the other objective function: $f_1(x_1) < f_1(x_3)$ and $f_2(x_1) > f_2(x_3)$. $f(x_2)$ and $f(x_5)$ are also non-dominated to each other: $f_1(x_2) < f_1(x_5)$ and $f_2(x_2) > f_2(x_5)$. In brief, $f(x_1) ≡ f(x_3)$ in $F_1 < f(x_4)$ in $F_2 < f(x_2) ≡ f(x_5)$ in $F_3$. The objective vectors in the upper fronts have higher probabilities to survive in the population than the objective vectors in the lower fronts. The objective vectors in the $i^{th}$ front $F_i$ (gray rectangular) are inferior to the objective vectors in the upper fronts from $F_1$ (black circle) to $F_{i-1}$, but superior to the objective vectors in the lower fronts from $F_{i+1}$ to $F_n$ (empty triangle).
The implementation of non-dominating sorting therefore proceeds by evaluating the objective function and then sorting the objective function values in order to establish the different fronts $F$ along which the solutions are equivalent. The Pareto-optimal front can then be identified as the most non-dominating one i.e. the front closest to the origin.

5.1.2.2.2 Crowding-distance sorting

Crowding distance is a measure of how close an individual solution is to its neighboring solutions. Figure 5.6 shows how to calculate the crowding-distance, which is a measure of population density (Deb et al., 2002). Equation 5.28 defines the crowding-distance, a measure of population density, for the objective vectors with the same non-dominated front:

$$
\psi^j = \sum_{i=1}^{M} \frac{d_i^j}{f_i^\text{max} - f_i^\text{min}} \quad \forall j = 1, \cdots, N_p
$$

(5.19)
where, $\Psi^j$ is crowding distance value of jth solution, $d_i^j$ is displacement between two neighbors with the jth solution, $f_i^{\min}$ is the minimum Euclidean distance between any two objective vectors in the direction of the ith objective, $f_i^{\max}$ is the maximum Euclidean distance between any two objective vectors in the direction of the ith objective, $N_p$ is the population size and $M$ is number of objective functions. The number of good solutions based on non-dominated sorting increase with number of Pareto-Optimal fronts ($F_k$) starting from first front ($F_1$). Equation 5.19 can also be implied as the reciprocal of population density. For an arbitrary solution $x$, larger the crowding distance value, smaller the density of solutions, which implies that there is a small probability that other solutions are similar to the solution $x$.

A non-dominated front is identical to the rank of the solutions. Therefore, if the cumulative solutions from all the non-dominated solutions fronts exceed the population size, the extra solutions have to be removed. This is done by crowding distance sorting. The crowding distance between the solutions in the last front is computed and sorted in descending order of their magnitude. The solutions with small crowding distance are then eliminated until the sum of the remaining solutions from the last front and solutions from preceding fronts equal the population size. The objective vectors having larger crowding-distance are preferred over the ones with smaller crowding distance to diversify the objective vectors in terms of their dissimilarity and spacing in the Pareto-optimal front. In this way, both non-dominated sorting and crowding distance sorting determine the optimum solutions in each generation. Succeedingly, the algorithm runs until the generation of solutions, in which the desired tolerance for objective function is met. Sparsely-distributed objective vectors with larger crowding-distances are preferred over densely-distributed objective vectors with smaller crowding-distances for diversity-preservation if the objective vectors are given the identical front number.
Figure 5.6: Ranking evaluation of NSGA-II for multi-objective minimization problem using crowding-distance sorting. Here, \( f(x3) \) is preferred over \( f(x1) \) for diversity-preservation because \( f(x3) \) has a larger crowding-distance than \( f(x1) \).

In the context of the example presented in Figure 5.7 below, \( x = [0, 2] \) on the red solid curve are the equivalent Pareto-optimal solutions because the increase of \( f1(x) \) accompanies the decrease of \( f2(x) \) and vice versa (see Figure 5.7 b). It is clear that the global optimum is the Pareto-optimal solution that is the closest solution from the origin under the given weights.

Figure 5.7: Comparison of multi-objective optimization with global-objective optimization for a two-objective minimization problem given in Figure 5.4 a and b.
5.2 Application of Response Surface Methodology for ES-SAGD Optimization

For thermal-solvent processes, the three major physical phenomena that control bitumen production are: heat conduction, mass diffusion/dispersion of solvent and gravity flow. An optimal design of the process is predicted on the addition of just sufficient enough heat and solvent to the native bitumen to make it mobile so that gravity can move it towards the production well. Therefore, out of the numerous operation parameters that determine the recovery rates and energy efficiency of thermal-solvent process in a heavy oil reservoir, injector Bottom Hole Pressure (BHP), duration of steam and solvent injection and amount of solvent injection form the key independent parameters or control variables. These parameters will be classified as primary control variables in this chapter. Other parameters such as steam and solvent injection rates and steam injection temperature are more or less dependent on these three primary control variables. For example, once the injector BHP is fixed, the steam and solvent injection rates depend on inflow potential of reservoir, productivity/injectivity of the well and fraction of solvent in the injected stream. The distance between injector and producer and the location of the wells, which are other two variables, depend upon the connectivity of the reservoir. Our assumption is that the wells have been optimally located based on an assessment of the reservoir heterogeneity. In general, the well pair distance is approximately set between 5-10 m in most SAGD or ES-SAGD operations.

In this work, the ES-SAGD thermal solvent process will be optimized by adjusting the injection pressure, duration of pure steam injection compared to the duration of steam-solvent injection and fraction of solvent in the injected stream. The optimization strategy integrates experimental design, response surface methodology and gradient based optimization, whose theoretical background is provided in previous
sections of this chapter. The optimization algorithm is developed in the MATLAB programming environment and contains two main parts: proxy model development and gradient based optimization. The results are then compared with NSGA-II multi-objective optimization technique developed in C++ programming environment. The functional relationship between an objective function and the primary control variables is represented through the second order proxy model. The overall work flow for optimization is given in Figure 5.8.

5.2.1 Proxy model development for ES-SAGD process

The following form the main components of the Response Surface Model (RSM) for ES-SAGD process:

1. **Control variables**: Injector Bottom Hole Pressure (BHP), steam (without solvent) injection period, solvent fraction (fraction of solvent injected in the injection stream). These parameters are variable inputs for the CMG-STARS™ simulation. The simulation is performed for 4 years with a 5 month pre-heating period. The upper and lower limits of control variables are given in the following Table 5.3.

2. **Experimental design**: In this study, 5 by 4 by 5 levels Multi-level Full Factorial design is found to be most appropriate with i.e. injection pressure and solvent fraction had 5 levels, injection period had 4 levels. This implies 100 runs of the CMG simulator corresponding to the control settings. Because we are interested in predicting responses corresponding to many possible combinations and derive a model that is the best fit for the response variables; multi-level full factorial design seems to be a good choice. This design does not restrict the number of levels for each control variable unlike BBD or CCD designs. The design yields
100 cases with varied combination of factors. The multi-level full factorial scheme covers the entire control variables space including the control limits. The design is detailed in Table A.1 in Appendix.

3. **Response functions:** The information from the simulation including cumulative oil, cumulative steam injected and cumulative mass solvent injected and produced are used to estimate the responses of the quadratic proxy model. The responses for the proxy models are Recovery Factor, cSOR, Solvent Retention. Recovery factor is obtained by dividing cumulative oil produced by the Original Oil Initially in Place (OOIP). In this study involving a 2D cross-sectional model, the OOIP is calculated to be 991 m$^3$. The cumulative Steam Oil Ratio (cSOR) is found by taking the ratio of cumulative steam injected to cumulative oil produced. Solvent Retention (SR) is obtained by taking the ratio of the difference between cumulative mass of solvent injected and cumulative mass of solvent produced to the cumulative injected mass of solvent. All three are important factors that determine the technical success and energy efficiency of ES-SAGD process. The intention of any ES-SAGD process is to maximize the recovery factor of bitumen as quickly as possible, minimize the steam requirement and therefore lower cSOR and minimize solvent retention in the reservoir such that most of the solvent can be recycled during the operation.
Table 5.3: Domain of primary control variables for optimization of ES-SAGD using RSM

<table>
<thead>
<tr>
<th>Primary control variables</th>
<th>Lower limit</th>
<th>Upper limit</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection Pressure ($P_{inj}$)</td>
<td>2000</td>
<td>3800</td>
<td>kPa</td>
</tr>
<tr>
<td>Steam (without solvent) Injection Period ($T_{inj}$)</td>
<td>0.5</td>
<td>2 years</td>
<td>years</td>
</tr>
<tr>
<td>Solvent fraction ($S_{frac}$)</td>
<td>0.05</td>
<td>0.35</td>
<td>dimensionless</td>
</tr>
</tbody>
</table>

For demonstrating the optimization workflow described in this study, a 2D homogenous simulation model with input parameters given in Table 5.4 is constructed in CMG’s thermal simulator STARS™. The simulation grid consists of 150 x 1 x 100 grid blocks with a grid dimension of 1m by 1m by 0.6m. Butane is used as the solvent in this
approach as well due to its suitable PVT properties in the pressure and temperature range for the study (Govind et al, 2008). This model is simulated for 4 years with a 5 month pre-heating period and a steam trap control of 2°C for the producer. Other primary operational constraints such as Injector BHP, steam injection period and solvent fraction in the injected stream are varied based on the experimental design settings. The initial reservoir pressure is set to be 1500 kPa at a reference depth of 270 m. Based on the discussion in Chapter 4 on effectiveness of dynamic refinement to capture such short length scale mass transfer process, dynamic gridding based on mole fraction of butane, with tolerance value of 0.1 is considered for this study.

Table 5.4: Input data for reservoir and operating parameters used in CMG-STARS™ simulation for performing optimization of ES-SAGD process.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>API degree of reservoir crude</td>
<td>8</td>
<td>°API</td>
</tr>
<tr>
<td>Reservoir height (H)</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>Porosity (φ)</td>
<td>0.33</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Effective permeability (k)</td>
<td>4.2</td>
<td>Darcy</td>
</tr>
<tr>
<td>Initial oil saturation (S_{oim})</td>
<td>0.85</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Residual oil saturation (S_{or})</td>
<td>0.1</td>
<td>volume fraction</td>
</tr>
<tr>
<td>Molecular weight of oil (M_o)</td>
<td>581</td>
<td>kg/kmole</td>
</tr>
<tr>
<td>Molecular weight of water phase (M_w)</td>
<td>18</td>
<td>kg/kmole</td>
</tr>
<tr>
<td>Thermal conductivity of reservoir rock (K_{rock})</td>
<td>2.85</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Thermal conductivity of reservoir water (K_{water})</td>
<td>0.6</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Thermal conductivity of bitumen (K_{oil})</td>
<td>0.1</td>
<td>W/m/K</td>
</tr>
<tr>
<td>Initial reservoir temperature (T_r)</td>
<td>283</td>
<td>K</td>
</tr>
<tr>
<td>Injected Steam Temperature (T_s)</td>
<td>Based on injector BHP</td>
<td>K</td>
</tr>
</tbody>
</table>

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The regression coefficients of the quadratic model with interaction terms are determined by the least squares error method as described in the preceding sections. The quadratic response surface models for the three response functions namely Recovery factor, cumulative Steam Oil ratio (cSOR) and Solvent Retention (SR) are given by Equations 5.20 through 5.22. In these equations, $A$, $B$ and $C$ are dimensionless control variables evaluated within control domain given in Table 5.3 for $P_{\text{inj}}$, $T_{\text{inj}}$, $S_{\text{frac}}$. The interaction terms in terms of dimensionless form are also given by $AB$, $AC$, and $BC$.

\[
\text{RF} = 0.35 + 0.07A - 0.04B + 0.06C - 0.01AB - 0.01BC - 0.002AC - 0.01A^2 - 0.02B^2 - 0.02C^2
\]

\[
c\text{SOR} = 3.27 + 0.24A + 0.42B - 0.49C + 0.11AB + 0.032AC + 0.05BC - 0.01A^2 - 0.20B^2 - 0.081C^2
\]

\[
\text{SR} = 0.18 - 0.01A + 0.02B - 0.05C - 0.003AB + 0.01BC + 0.002AC + 0.048A^2 - 0.01B^2 - 0.02C^2
\]

The above response surface models need to be tested for model adequacy and significance of the regressor coefficients as per the test statistics described in section 5.1. The model adequacy statistics and goodness of fit for the three proxy models are given in the following Table 5.5. Table shows that the model for RF is best in terms of high values of $R^2$ and $R_{\text{adj}}^2$. However, $R$ and $R_{\text{adj}}^2$ for solvent retention are also reasonably good as they are above the threshold of 0.80. Moreover, $R_{\text{adj}}^2$ and $R^2$ do not differ much in all three models and therefore it can be concluded that there is no control variable that is non-significant. In a complex process such as ES-SAGD, recovery factor, cSOR and solvent retention are non-linear function of various parameters like relative permeabilities, heat loss, temperature and pressure dependent fluid properties, reservoir heterogeneity etc. Therefore, high values of regression statistics for a homogenous 2D
simulation of ES-SGAD process such as those shown in Table 5.5 imply reasonably good response surface models.

Table 5.5: Regression statistics for three response functions modeled using a second-order response surface.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Proxy model</th>
<th>Mean Square Error (MSE)</th>
<th>$R^2$</th>
<th>$R_{adj}^2$</th>
<th>$F$</th>
<th>$P &gt; F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RF</td>
<td>0.0003</td>
<td>0.957</td>
<td>0.953</td>
<td>223.00</td>
<td>1E-10</td>
</tr>
<tr>
<td>2</td>
<td>cSOR</td>
<td>0.0213</td>
<td>0.932</td>
<td>0.925</td>
<td>136.59</td>
<td>1E-8</td>
</tr>
<tr>
<td>3</td>
<td>SR</td>
<td>0.003</td>
<td>0.860</td>
<td>0.850</td>
<td>66.17</td>
<td>8E-5</td>
</tr>
</tbody>
</table>

The $t$-statistic values and $p$-values for the regressor coefficients for each of the model are given in the following Table 5.6 through Table 5.8. The coefficients which are shown in bold and italicized font are the significant ones as they pass the null hypothesis criteria for $t$-statistic test and also have $p$-values less than 0.01.

Table 5.6: $T$-statistic parameters for each regression coefficient corresponding to the response surface model for Recovery Factor (RF)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Control Variables</th>
<th>Regressor Coefficients ($\beta$)</th>
<th>$T$-statistic</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant</td>
<td>0.351833547</td>
<td>96.5</td>
<td>1.2E-92</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>0.064477841</td>
<td>29.1</td>
<td>1.5E-47</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>-0.038253003</td>
<td>-18.2</td>
<td>6.5E-32</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>0.059535797</td>
<td>26.9</td>
<td>9.1E-45</td>
</tr>
<tr>
<td>5</td>
<td>AB</td>
<td>-0.01435167</td>
<td>-4.8</td>
<td>0.00001</td>
</tr>
<tr>
<td>6</td>
<td>AC</td>
<td>-0.001509197</td>
<td>-0.5</td>
<td>0.63124</td>
</tr>
<tr>
<td>7</td>
<td>BC</td>
<td>-0.007951416</td>
<td>-2.7</td>
<td>0.00888</td>
</tr>
<tr>
<td>8</td>
<td>A$^2$</td>
<td>-0.010948434</td>
<td>-2.9</td>
<td>0.00438</td>
</tr>
<tr>
<td>9</td>
<td>B$^2$</td>
<td>-0.020824009</td>
<td>-5.9</td>
<td>0.00000</td>
</tr>
<tr>
<td>10</td>
<td>C$^2$</td>
<td>-0.021846708</td>
<td>-5.8</td>
<td>8.43E-08</td>
</tr>
</tbody>
</table>
Table 5.7: T-statistic parameters for each regression coefficient corresponding to the response surface model for cumulative Steam Oil Ratio (cSOR)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Control Variables</th>
<th>Regressor Coefficients (β)</th>
<th>T-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant</td>
<td>3.267</td>
<td>96.1</td>
<td>1.66E-92</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>0.242</td>
<td>11.7</td>
<td>7.78E-20</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>0.4245</td>
<td>21.6</td>
<td>2.00E-37</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>-0.488</td>
<td>-23.7</td>
<td>2.09E-40</td>
</tr>
<tr>
<td>5</td>
<td>AB</td>
<td>0.112</td>
<td>4.0</td>
<td>0.000112937</td>
</tr>
<tr>
<td>6</td>
<td>AC</td>
<td>0.032</td>
<td>1.1</td>
<td>0.269993894</td>
</tr>
<tr>
<td>7</td>
<td>BC</td>
<td>0.052</td>
<td>1.9</td>
<td>0.063861459</td>
</tr>
<tr>
<td>8</td>
<td>A²</td>
<td>-0.011</td>
<td>-0.3</td>
<td>0.76259789</td>
</tr>
<tr>
<td>9</td>
<td>B²</td>
<td>0.200</td>
<td>6.1</td>
<td>2.56E-08</td>
</tr>
<tr>
<td>10</td>
<td>C²</td>
<td>0.081</td>
<td>2.3</td>
<td>2.32E-02</td>
</tr>
</tbody>
</table>

Table 5.8: T-statistic parameters for each regression coefficient corresponding to the response surface model for Solvent Retention (SR)

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Control Variables</th>
<th>Regressor Coefficients (β)</th>
<th>T-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Constant</td>
<td>0.178</td>
<td>102.8</td>
<td>1.99E-62</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>-0.006</td>
<td>36.7</td>
<td>2.51E-02</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>0.023</td>
<td>24.0</td>
<td>3.60E-16</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>-0.053</td>
<td>-57.3</td>
<td>8.87E-37</td>
</tr>
<tr>
<td>5</td>
<td>AB</td>
<td>-0.003</td>
<td>8.9</td>
<td>0.37</td>
</tr>
<tr>
<td>6</td>
<td>AC</td>
<td>0.0021</td>
<td>0.2</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>BC</td>
<td>0.011</td>
<td>-0.9</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>A²</td>
<td>0.0048</td>
<td>2.7</td>
<td>0.25</td>
</tr>
<tr>
<td>9</td>
<td>B²</td>
<td>0.009</td>
<td>8.7</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>C²</td>
<td>-0.020</td>
<td>-3.6</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Figures 5.9 a, b, and c show normal probability plot of residuals and plot of predicted values of responses, $y_i$, versus residuals for respective proxy models. The plot of residuals versus predicted responses form something closer to a horizontal band indicating homoscedasticity and therefore suggesting minimal model defects. The residuals are within 10% of the true values i.e. the fitted data deviate within +/-10% of actual data obtained from CMG simulation. Hence, these proxy models adequately represent the responses from simulation. Moreover, deviations from straight line of normal probability plot are seen corresponding to higher values of RF and Solvent Retention (SR). However, a majority of points lie along a straight line for all three models.
Based on the significance test of each regressor coefficient, the proxy models can be reduced to contain only the significant terms by eliminating the non-significant ones. Therefore, the reduced response surface models with only significant terms are given in Equations 5.23 through 5.25.

\[
\text{RF} = 0.35 + 0.07A - 0.04B + 0.06C - 0.01AB - 0.01BC - 0.01A^2 - 0.02B^2 - 0.02C^2 \quad (5.23)
\]

\[
\text{cSOR} = 3.27 + 0.24A + 0.42B - 0.49C + 0.11AB + 0.05BC - 0.20B^2 - 0.081C^2 \quad (5.24)
\]

\[
\text{SR} = 0.18 - 0.01A + 0.02B - 0.05C + 0.01BC - 0.01B^2 - 0.02C^2 \quad (5.25)
\]

Figure 5.10 through 5.12 show the response surfaces for the three proxy models, RF, cSOR and Solvent Retention (SR) for a range of injection BHP plotted on x-axis, solvent fraction on y-axis corresponding to four values of steam injection period (0.5, 1, 1.5 and 2 years). Higher values of solvent fraction leads to increased solvent dilution and therefore increased bitumen recoveries. Moreover, increasing injection BHP results in higher steam saturation temperature, which in turn favours prolonged steam chamber growth and lower mixture viscosity. This condition also results in a better solvent recovery. However, as expected, RF and solvent retention increase with injection pressure and solvent fraction. In contrast, with higher injection pressures, cSOR also rises with
high values being in the range of 4 - 4.5 at injection pressure of 3800 kPa. It is obvious that higher amount of steam is being injected at higher injection pressures and therefore resulting in an increasing trend of cSOR. Longer duration of steam injection (only steam without solvent) causes decline in bitumen recovery, increase in values of cSOR and solvent retention. Figure 5.13 shows surface plots of all three response functions together for all four values of steam injection period.

Figure 5.10: Response surfaces for Recovery Factor (RF) corresponding to four values of steam injection period (0.5, 1, 1.5 and 2 years)
Figure 5.11: Response surfaces for cSOR corresponding to four values of steam injection period (0.5, 1, 1.5 and 2 years).

Figure 5.12: Response surfaces for Solvent Retention (SR) corresponding to four values of steam injection period (0.5, 1, 1.5 and 2 years).
5.2.2 Optimization of ES-SAGD process

We will now look at implementation of two optimization approaches namely gradient based approach based on a global objective function and hybrid multi-objective optimization in ES-SAGD process. The theoretical backgrounds for the two methods are given in the preceding sections of this chapter.

5.2.2.1 Gradient based optimization strategy of ES-SAGD process

Process optimization fixes the optimum setting of the control variables to produce desirable (maximum or minimum) values of response functions. In this work, gradient based optimization strategy is used that integrates the response surface proxy models for the determination of optimum primary control variables and their corresponding optimum response values. Gradient descent search algorithm is implemented to minimize the objective function. In this method, the search algorithm for optimum primary control
variables, Injection Pressure, Injection Period and solvent fraction, evolve between the constraints specified in Table 5.3. The algorithm stops once the minimum value of objective function is achieved within a tolerance of +/-0.01 of the value. The theory and mathematical background for this approach is described in the Section 5.1. At every iterative step, all three variables are simultaneously varied and optimized within the domain. The algorithm implemented in the MATLAB programming environment. The steps involved in the gradient descent optimization approach are described in brief in the following:

5.2.2.1.1 Objective function

Since we have developed reduced Response Surface models for three response functions namely Recovery Factor (RF), cSOR and Solvent Retention (SR), we will use a single objective function that combines all into one. The main criteria towards a successful optimization in ES-SAGD project are to maximize bitumen recovery, minimize steam requirement and reduce solvent retention in the reservoir. The specified global objective function is presented in Equation 5.26.

\[ G = \sqrt{\left( \frac{RF - RF_{\text{max}}}{RF_{\text{max}}} \right)^2 + \left( \frac{cSOR - cSOR_{\text{min}}}{cSOR_{\text{min}}} \right)^2 + \left( \frac{SR - SR_{\text{min}}}{SR_{\text{min}}} \right)^2} \quad (5.26) \]

Note that because the global objective function has to be defined such that it is either maximized or minimized, the criterion that cSOR be minimized is accommodated by standardizing with respect to cSOR_{\text{min}}. Minimizing the global objective function then yields a cSOR that is closest to cSOR_{\text{min}}. The RF_{\text{max}}, cSOR_{\text{min}} and SR_{\text{min}} are the maximum and minimum values desired for RF, cSOR and Solvent Retention (SR) respectively. The maximum and minimum values of response functions obtained using response surface models within the constraints specified for the control parameters are
given in the following Table 5.9. The results for this optimization approach will be discussed in the succeeding section 5.4.

Table 5.9: Maximum and minimum values of response models within the domain space.

<table>
<thead>
<tr>
<th>Max/Min values</th>
<th>Recovery Factor (RF)</th>
<th>cSOR</th>
<th>Solvent Retention (SR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>48.28%</td>
<td>4.64</td>
<td>36%</td>
</tr>
<tr>
<td>Min</td>
<td>15.83%</td>
<td>2.33</td>
<td>1%</td>
</tr>
</tbody>
</table>

5.2.2.2 Pareto based hybrid multi-objective optimization approach in ES-SAGD process

Here, Pareto based NSGA-II optimization of the reduced response surface models will be discussed. This approach starts with defining the variable vector $x$, which consists of the three primary control variables to be adjusted for optimizing the objective vector $f(x)$:

$$x = [x_1, x_2, x_3] = [\text{Injection Pressure (P}_{\text{inj}}), \text{Steam Injection Period (T}_{\text{inj}}), \text{Solvent volume fraction (S}_{\text{frac}})]$$

(5.27)

The objective function vector $f(x)$ composed of the three responses for the optimization problem is given by Equation 5.28.

$$f(x) = [f_1(x), f_2(x), f_3(x)] = [1 - \text{RF}, \text{cSOR}, \text{SR}]$$

(5.28)

Both RF value and SR value range between zero and unity, while cSOR value increases from zero. It is important to note that the implementation of the multi-objective analysis does not require any objective-normalization or objective-aggregation in order to arrive at a global objective function.

5.3.2.1 Hybrid multi-objective genetic algorithm for ES-SAGD process

Figure 5.14 below shows a flow chart for optimizing the ES-SAGD process using the proposed hybrid approach incorporating NSGA-II with the response surface models.
Figure 5.14: Flow chart to design ES-SAGD processes using hybrid NSGA-II algorithm with the response surface models

5.3 Results and Analysis

5.3.1 Results from gradient based optimization in ES-SAGD process

Figure 5.15 through 5.17 illustrate the surface plots of objective function for range of injection pressures in x-axis and solvent fraction in y-axis at two steam injection periods (0.5 and 1.5 years). All of these figures show concave downward profiles with a maximum in the center, slowly declining towards the minimum values at the flanks. The optimization algorithm tries to achieve the desired goals of high RF, low cSOR and low solvent retention. However, due to the contradicting behavior of RF, solvent retention
(maximum) as compared to cSOR (minimum), the algorithm has to compromise on one of the three primary control variables. Therefore, in this algorithm, solvent fraction tries to attain one of its limiting values, in this case the maximum value, to achieve the desired results from optimization. The Table 5.10 shows the optimum values obtained from gradient based algorithm. The results indicate that short period of steam injection at high solvent fraction results in optimized oil recovery rate and lower steam-oil-ratio.

Figure 5.15: Surface plot for objective function (G) corresponding to various values of RF and cSOR and at two values of steam injection period (0.5 and 1.5 years).
Figure 5.16: Surface plot for objective function (G) corresponding to various values of cSOR and SR and at two values of steam injection period (0.5 and 1.5 years).

Figure 5.17: Surface plot for objective function (G) corresponding to various values of RF and SR and at two values of steam injection period (0.5 and 1.5 years).
Table 5.10: Optimum values obtained from gradient based approach

<table>
<thead>
<tr>
<th>Injection BHP (Kpa)</th>
<th>Steam injection period (months)</th>
<th>Solvent volume fraction (%)</th>
<th>Recovery Factor (RF) (%)</th>
<th>cSOR (m³/m³)</th>
<th>Solvent Retention (fraction of solvent injected) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,451</td>
<td>6.9</td>
<td>34.1</td>
<td>46.0</td>
<td>2.70</td>
<td>8.0</td>
</tr>
</tbody>
</table>

5.3.2 Results from hybrid multi-objective optimization approach in ES-SAGD process

Figures 5.18 through 5.21 depict the scatter plots that project the Pareto-optimal solutions and the global-optimum in two-dimensional variable space and objective space. Solutions represent a conflict between the bitumen recovery and the thermal energy efficiency in terms of cSOR similar to gradient based approach, as higher injection pressures and solvent fractions increase RF while higher injection pressures and lower solvent fractions increase cSOR. Longer steam injection (without solvent) deteriorates the injection efficiency. Figures 5.18 and 5.19 show the domain of control variables that have resulted from the final 100 Pareto-optimum solutions. The final 100 Pareto-optimal solutions indicate that the values of steam injection period range between 5-9 months and hence 100 points (indicated by green diamonds in the plot) on the plots for steam injection period overlap each other and are cluttered between 5<sup>th</sup> and 9<sup>th</sup> month in Figure 5.19. It is evident that low values of steam injection period and high values of solvent fraction at wide range of injector BHP would require low volumes of steam injection resulting in improved bitumen recoveries and solvent recoveries. Figures 5.20 and 5.21 show the Pareto optimal solution front for the response functions, RF, CSOR and SR. The final 100 Pareto optimal solutions (indicated by green diamonds in the plots) form somewhat of a curvilinear profile. It should be noted that the Pareto-optimal solution is in
reality in the 3D space defined by Injection Pressure, Injection Period and Solvent Fraction. The figures below show a projection of the Pareto-optimal solution in 2D space. The solutions obtained from gradient based approach (indicated by blue squares in the plots) also lie on these curves, but at one of the extreme ends of the curve.

Figure 5.18: Figure showing final 100 Pareto optimal solutions and the initial population of 100 combination of injection period and injection pressure obtained from hybrid multi-objective optimization. It also shows the optimum solution obtained from global objective gradient based approach.
Figure 5.19: Figure showing cross plot of final 100 Pareto optimal solutions and the initial population of 100 combinations of solvent fraction, steam injection period and injection pressure obtained from hybrid multi-objective optimization. It also shows the global solution obtained from single objective gradient based approach.

Figure 5.20: Figure showing final 100 Pareto optimal solutions and the initial population corresponding to 100 combinations of the control variables. This plot is in terms of the responses RF and cSOR obtained from hybrid multi-objective optimization. It also shows the optimum solution obtained from the global objective gradient based approach.
Figure 5.21: Figure showing final 100 Pareto optimal solutions and the initial population corresponding to 100 combinations of the control variables. This plot is in terms of the responses Solvent Retention and RF obtained from hybrid multi-objective optimization. It also shows the optimum solution obtained from single objective gradient based approach.

5.3.3 Results from NPV Analysis

Most of the optimization techniques available for ES-SAGD process involve some sort of cost function set to a desirable value (minimization or maximization). For example, Gates and Chakrabarty, 2006 demonstrated optimization of solvent assisted SAGD process using cost function based on energy efficiency. The cost function is a single objective function that combines process responses such as RF and cSOR. It is indeed interesting to see if the optimum solutions obtained from gradient based approach and hybrid multi-objective approach result in economical ES-SAGD process. Therefore, with this intention, a cost function determining Net Present Value (NPV) of project is formulated combining all three response variables as given in Equation 5.29.

\[ NPV = \sum_{n=1}^{N} (Revenue)/(1 + r)^n - CAPEX \]  

(5.29)

where Revenue = [RF × OIIP × Oil price]
\[
- [RF \times OIIP \times cSOR \times \text{Steam handling cost}]
- [RF \times OIIP \times cSOR \times \text{Solvent fraction} \times \text{Solvent cost}]
+ [RF \times OIIP \times cSOR \times \text{Solvent fraction} \times \text{Solvent cost} \times (1-\text{Solvent Retained})]
\]

It should be noted that solvent injected volume is obtained as the fraction (denoted in terms of solvent fraction) of steam injected volume, which in turn is a fraction, measured by cSOR values, of cumulative oil produced. Therefore, Equation 5.29 contains cSOR and RF terms which are used to obtain solvent injected volumes.

The parameters used for revenue calculation have been obtained from data given in the latest publication by Galvão et al. (2014) and are as follows:

- **CAPEX** is assumed as cost per well pair, which is $8000/well-pair/m. Since, 2D CMG simulation is performed using a single well pair, therefore, the CAPEX of the project is set to $8000/well-pair/m.
- **Rate of return (r) = 1\%**
- **Oil price = $80/bbl or $503/m^3**
- **Steam factor = 0.16 (i.e. cost of steam generation unit is 0.16 times that of oil price).** Generally, steam cost is available in terms of $MM/Btu. However, in this calculation, a factor that compares with cost of oil per barrel is used for steam cost (Galvão et al., 2014).
- **Solvent factor = 1.9 (i.e. cost of solvent injection is 1.9 times that of oil price – the lighter solvent fractions are available at a premium in the market).**

Figures 5.22 a and b show plots of NPV versus optimum response variables, RF and cSOR for gradient based approach and initial and final Pareto solutions obtained from hybrid multi-objective optimization. NPV values computed from output of CMG-STARSTM simulation performed using a few optimum primary variables obtained from
the final 100 Pareto optimal solutions are also indicated in both the plots. It is evident that these NPV values from CMG simulation lie close to that obtained from final 100 Pareto-optimal solutions. The conclusions from these plots are:

- The final 100 solutions from Pareto-optimality give higher NPVs as compared to the initial population.
- The NPV obtained from a single objective function optimization such as the gradient based approach (indicated by blue squares in the plots) is close to the maximum value. However, gradient based approach does not always give the maximum value. Moreover, Pareto-based optimization adds flexibility to the optimum operating parameters resulting in high values of NPV.

Figure 5.22: Plot showing NPV of the ES-SAGD process for optimum cSOR values obtained using hybrid multi-objective optimization, gradient based optimization and CMG simulation.
5.4 Chapter Summary

The response surface analysis yields a good representation of the variation of key response variables with primary controller variables. The response surface methodology reduces the cost of full physics reservoir simulation at every iterative step during the execution of optimization algorithm. However, there is an initial expense of running several flow simulations in order to calibrate the response surface. The usefulness of hybrid multi-objective optimization over a single objective gradient-based approach is evident from the results in this work. Multi-objective optimization yields a wide solution space within which operating parameters can be varied in order to obtain the desired optimum responses of the process. The full potential of solvent-aided thermal process for bitumen recovery can then be realized. Conventional gradient based approach yields a single optimum solution, that perhaps belongs to one of the solutions obtained from multi-objective optimization. This is suggestive of the fact that solutions obtained from multi-objective optimization that conform to Pareto-optimality also contain the global optimum values. The results from this exercise indicate that addition of solvent to steam yields lower values of cSOR and results in an energy efficient process. Without having to develop an optimization algorithm specifically to evaluate cost effectiveness of the process, the two approaches for optimization, as discussed in this chapter, give optimum solutions which also reflect high NPV in addition to desired response values.

In conclusion, implementing optimization strategies as described in this chapter in ES-SAGD process could prove beneficial for technical success and economic efficiency of the process.
Chapter 6: Optimization of ES-SAGD process using the improved semi-analytical model

Chapter 3 presents the development of the improved semi-analytical model for better understanding of the ES-SAGD process. We then explored two methods of process optimization, in Chapter 5, using the proxy models built based on Response Surface Analysis. While optimization using the response surface is faster than running the full physics flow simulation at each step of the optimization process, the initial cost of calibrating the response surface is significant. In order to alleviate this expense, we present in this chapter the optimization of ES-SAGD process using the improved semi-analytical model.

The improved semi-analytical model weaves together some key physical phenomenon like solvent partitioning, diffusion and bitumen dilution through mathematical equations providing output in terms of cumulative volume of bitumen production, steam and solvent injection volumes. It should be noted that unlike CMG simulator, the solvent recovery is not modelled in the semi-analytical approach. These output variables can be utilized in an efficient optimization design to determine optimum operating conditions delivering an improved energy efficient ES-SAGD process. The two approaches for optimization as described in Chapter 5 namely gradient-based optimization of a total objective function and hybrid multi-objective optimization are integrated with the improved semi-analytical model. The results from these methods are then compared in order to understand the working of the optimization process. The principle of super-position is implemented in order to model the effective pressure at the steam-solvent-bitumen interface corresponding to variable injection pressure scenarios. This effective pressure is used to retrieve the value of the equilibrium constant for the solvent and that in turn determines the partitioning of the solvent into the liquid and
vapor phases. All these modifications together would render the semi-analytical useful towards developing a complete understanding and efficient design of ES-SAGD process.

6.1 Modifications To The Improved Semi-analytical Model To Incorporate Optimization Schemes

The mathematical framework for the improved semi-analytical model is the same as discussed in Chapter 4. However, a key modification to enable modeling the effect of variable injection pressure with time has been implemented through the theory of superposition. This modification in the model utilizes the linear superposition theorem to compute the effective steam chamber interface pressure corresponding to fluctuations in injector pressure. Although the behavior of thermal-solvent process in the presence of variable injection pressure is complex, we employ the assumption that variations in pressure due to injection pressure changes can be linearly superimposed in order to obtain the effective pressure at any location on the steam chamber interface.

6.1.1 Implementation of super-position theorem for variable injection pressure

6.1.1.1 Theoretical background

The superposition theorem for the analysis of build-up and draw down data was first applied by van Everdingen and Hurst (1949). The principle of superposition states that for all linear systems the net response at a given position or time caused by two or more stimuli is the sum of the responses, which would have been caused by each stimulus individually. Mathematically the superposition theorem states that the sum of individual solutions of a second order linear differential equation is also a solution of the equation. Adding solutions to a linear differential equation results in a new solution to that differential equation but corresponding to different boundary conditions. The most fundamental fluid flow equation that integrates mass conservation, Darcy’s law and fluid
PVT properties is the diffusivity equation. The linearity of the diffusivity equation allows problems involving multiple rate or multi-well scenarios to be solved using the principle of superposition in space and time. In this work, we will focus only on superposition in time as the improved semi-analytical model is developed for one injector producer-well pair. However, by modeling multiple well pairs in the same improved semi-analytical model, superposition in space can also be implemented for multi-well production/injection.

6.1.1.1 Superposition in time

Superposition in time is required in order to analyze the effect of variable rate or variable injection pressure. Since, the well inflow equations relates well differential pressure to well rate, the superposition in time available for multi-rate test have to be converted for variable pressure situations. The constant of proportionality between differential pressure ($\Delta P$) and well flow rate ($q(t)$) for a steady state flow is given by well productivity or injectivity ($J$). Equation 6.1 defines the Inflow Performance Relationship (IPR).

$$J = \frac{q(t)}{\Delta P} = \frac{2\pi kh}{\mu \ln \frac{r_e}{r_w}}$$

(6.1)

where, $k$ is formation permeability, $h$ is pay zone thickness, $r_e$ is radius of the reservoir drainage boundary, $r_w$ is well bore radius.

Therefore, in order to determine an effective pressure at the steam chamber interface, the variable injector pressure sequence has to be decomposed into a sequence of pressure changes similar to a multi-rate test. The pressure used for each step is the difference between the current pressure and the previous pressure.

Duhamel’s principle to solve a linear homogenous Partial Differential Equation (PDE) corresponding to an observed rate function involves computation of the
convolution of the input pressure drop function and the derivative of the constant-pressure rate solution. Assuming a constant well injectivity (J), the superposition theorem for injection rate is given by Equation 6.2

\[ q = J \sum_{k=1}^{n} (\Delta p_k - \Delta p_{k-1})(t - t_{k-1}) \]  

(6.2)

where, \( q \) is the volume of fluid flow over a time period \( t \), \( \Delta p_k \) is the differential pressure corresponding to the \( k^{th} \) step in the injection rate, \( t_k \) is time period of injection at the rate \( q_k \). Substituting Equation 6.1 into Equation 6.2, for steady state fluid flow, we can write Equation 6.2 in terms of the effective pressure over a total time (T) of injection as:

\[ \frac{k h \Delta P_{eff}}{\mu ln \frac{r_e}{r_w}} = \frac{2\pi k h}{T \mu ln \frac{r_e}{r_w}} \sum_{k=1}^{n} (\Delta p_k - \Delta p_{k-1})(t - t_{k-1}) \]  

(6.3)

Simplifying the equation and substituting \( t/T \) as \( t_D \), we get Equation 6.4 for effective pressure (\( \Delta P_{eff} \)) as:

\[ \Delta P_{eff} = \sum_{k=1}^{n} (\Delta p_k - \Delta p_{k-1})(t_D - t_{Dk-1}) \]  

(6.4)

The pressure changes in Equation 6.4 are the drops from the initial reservoir pressure in conventional well test analysis. However, consistent with the assumptions of the semi-analytical model, the calculations here are done purely based on operating pressure inside the steam chamber environment without any interactions with reservoir pressure. In such situations, assuming differential pressure as the difference between injector pressure (\( P_k \)) and producer pressure (\( P_p \)) and holding producer pressure (\( P_p \)) constant, Equation 6.4 can be modified to obtain effective pressure (\( P_{eq} \)) as:

\[ P_{eff} - P_p = \sum_{k=1}^{n} ((P_k - P_p) - (P_{k-1} - P_p))(t_D - t_{Dk-1}) \]  

(6.5)

This assumption holds true in SAGD or ES-SAGD process, where difference between injector and producer well pressures are small in order of 200 kPa and the process is sustained by primarily heat supplied by injector dictated by water-solvent thermal
properties at steam-chamber interface pressure for gravity driven oil drainage. This effect of variable injection pressure on interface pressure can be assumed to be varying linearly. Moreover, oil rate in such processes are constant during horizontal growth period and is a function of viscosity lowering with temperature and solvent dilution, which in turn are dependent on pressure and temperature of the steam chamber. Hence, because the semi-analytical model itself being a simplified prototype of complex phenomena taking place in ES-SAGD process, this assumption of linear behavior also adds to its simplicity. However, by modeling effect of reservoir pressure through solutions to diffusivity equation, implementation of super-position theorem becomes accurate.

From Equation 6.5, the Equation 6.6 forms the basis to model effective interface pressure for a variable injection pressure case in the improved semi-analytical model.

\[ P_{eff} = \sum_{k=1}^{n} (P_k - P_{k-1})(t_D - t_{Dk-1}) \] (6.6)

In order to demonstrate that the performance of the semi-analytical model with injection pressure variations gives, at least qualitatively, the same type of response as a simulator, the results from superposition of pressures is validated against numerical simulation. The validation exercise, in form of simple application, is described in the following text and the results are then analyzed in section 6.4.

To compare the results obtained by modeling variable injection pressure in improved semi-analytical model, a 2D homogenous simulation model is constructed. The input reservoir properties are the same as that summarized in Tables 3.1 of Chapter 3. The viscosity data for bitumen and solvent, used in this exercise, is given in the following Table 6.1. The same input data is used in the improved semi-analytical model also. The simulation grid consists of 150 x 1 x 100 grid blocks with grid dimension of 1m by 1m by 0.6m. Since it is a 2D model, the dimension in the y direction is assumed to be of unit length. This model is simulated for 2 years with 3 months of pre-heating period. The
initial reservoir pressure is set at 1500 kPa at a reference depth of 270 m. The Solvent Steam Ratio (SSR) of 0.25 is used as an additional constraint. The improved semi-analytical model is also evaluated for single pressure cases, without the application of superposition principle, to see if solution obtained by superposition of varying injection pressures indeed lies somewhere amongst the single injection pressure cases.

Table 6.1: Input data for bitumen and butane viscosity used for validating the superposition of pressures in semi-analytical method with CMG-STARS™ simulation.

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Bitumen viscosity cp</th>
<th>Butane viscosity cp</th>
</tr>
</thead>
<tbody>
<tr>
<td>282.15</td>
<td>1648266</td>
<td>0.191</td>
</tr>
<tr>
<td>292.15</td>
<td>307989</td>
<td>0.178</td>
</tr>
<tr>
<td>302.15</td>
<td>73413</td>
<td>0.166</td>
</tr>
<tr>
<td>312.15</td>
<td>21410</td>
<td>0.156</td>
</tr>
<tr>
<td>322.15</td>
<td>7388</td>
<td>0.146</td>
</tr>
<tr>
<td>332.15</td>
<td>2935</td>
<td>0.138</td>
</tr>
<tr>
<td>342.15</td>
<td>1314</td>
<td>0.131</td>
</tr>
<tr>
<td>352.15</td>
<td>650</td>
<td>0.125</td>
</tr>
<tr>
<td>372.15</td>
<td>203</td>
<td>0.113</td>
</tr>
<tr>
<td>397.15</td>
<td>68</td>
<td>0.102</td>
</tr>
<tr>
<td>422.15</td>
<td>30</td>
<td>0.093</td>
</tr>
<tr>
<td>447.15</td>
<td>16</td>
<td>0.086</td>
</tr>
<tr>
<td>472.15</td>
<td>10</td>
<td>0.080</td>
</tr>
<tr>
<td>497.15</td>
<td>7</td>
<td>0.075</td>
</tr>
<tr>
<td>522.15</td>
<td>5</td>
<td>0.071</td>
</tr>
<tr>
<td>547.15</td>
<td>4</td>
<td>0.067</td>
</tr>
<tr>
<td>572.15</td>
<td>3</td>
<td>0.064</td>
</tr>
<tr>
<td>1272.15</td>
<td>1</td>
<td>0.035</td>
</tr>
</tbody>
</table>

The variation in injection pressure in both improved semi-analytical model and reservoir simulator is implemented as given in Table 6.2
Table 6.2: Injection pressure scheme implemented for validation with CMG simulation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Injection Pressure</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{inj} ) at the 1(^{st} ) injection period</td>
<td>3500 kPa</td>
<td>8 months</td>
</tr>
<tr>
<td>( P_{inj} ) at the 2(^{nd} ) injection period</td>
<td>3000 kPa</td>
<td>4 months</td>
</tr>
<tr>
<td>( P_{inj} ) at the 3(^{rd} ) injection period</td>
<td>2600 kPa</td>
<td>12 months</td>
</tr>
</tbody>
</table>

6.1.1.2 Implementation of super-position theorem in semi-analytical model.

In the improved semi-analytical model, the phase equilibrium properties are calculated based on pressure and temperature conditions at the steam-chamber interface. With variation in injection pressure with time, the effective pressure at the interface can be computed through the superposition theorem as described above. The model will calculate the effective interface pressure using the Equation 6.6 for the corresponding time periods and the phase equilibrium properties will be calculated at this pressure. In other words, the entire calculations as described in section 3.1 of Chapter 3 are performed on the basis of this effective interface pressure.

6.1.2 Other modifications to the proxy

Optimization in ES-SAGD requires determination of key operating parameters such as solvent fraction, injector pressure. Therefore, solvent fraction, injector pressure and time period for variable injector pressure were introduced as variables in the semi-analytical model. The output parameters that are optimized are cumulative bitumen production and cumulative Steam to Oil Ratio (cSOR). The semi-analytical model does not provide solvent recovery as one of the output parameters. The solvent retention can be estimated by modeling relative permeabilities and variations in oil saturations with production. Since these phenomena are not modeled in the semi-analytical model, determination of solvent retention is not possible. Other modifications that are introduced in the model are as follows:
1. In the semi-analytical approach discussed in the Chapter 3, the amount of solvent required for injection is determined as an output variable in the form of mass rate of solvent \( m_{\text{sol}} \) in equation. This variable is computed based on the evaluation of solvent flow and oil-viscosity integrals given in Equations 3.10 and 3.11 of Chapter 3. As the algorithm in the model proceeds, the amount of steam required for injection is calculated based on heat loss and heat transfer equations. With these variables, Solvent Steam Ratio (SSR) is determined, which is obtained by taking ratio of amount of solvent injected to the amount of steam injected. However, to introduce the objectives of the optimization scheme, SSR is specified as an input control variable in this chapter. The semi-analytical model runs until the SSR determined from solvent integral evaluation and heat loss calculations attains the value equal the SSR specified as input. In this manner, solvent volume fraction in terms of SSR is introduced as one of the primary control variables in the optimization.

2. In order to introduce variable injection pressure in the optimization scheme, three variables for injector pressure corresponding to three time steps were introduced. Three time steps were introduced to model the initial injection ramp up (Time period 1), plateau period (Time period 2) and injection decline (Time period 3). This was done to test and implement the superposition theorem for variable injection pressure scheme. However, the optimization can also be performed for more than three time periods. The optimization algorithm determines the length of each time period while reaching the desired responses, cumulative volumes of bitumen production and cSOR. The length of each time step \( \Delta T_k \) is varied such that Equation 6.7 is satisfied:

\[
\sum_{k=1}^{n} \Delta T_k = \text{Total time of simulation.} \tag{6.7}
\]
6.2 Optimization Scheme Using the Improved Semi-Analytical Model

Two methods of optimization, gradient based approach and hybrid multi-objective optimization are explored using the semi-analytical model. The theoretical background and the computational procedure for these two methods have been discussed in the preceding Chapter 5. The parameters common to both the methods are given below:

1. **Demonstration example:**

   A synthetic reservoir case was put together in the improved semi-analytical model to demonstrate the application of optimization using the improved semi-analytical formulation. The relevant input data for this case are the similar, with some modifications, to that summarized in Tables 3.1 of Chapter 3. The variation of solvent and bitumen properties with temperature is modeled using available correlations that are presented in Table 3.2 of Chapter 3. Outputs from a 2 year simulation of this synthetic reservoir in the improved semi-analytical model are used in both the methods of optimization.

2. **Control variables:** Solvent volume fraction ($S_{frac}$) in the form of SSR, injector pressure at three time steps ($P_{inj}$ for the 1st injection period, $P_{inj}$ for the 2nd injection period, $P_{inj}$ for the 3rd injection period), length of the three time steps (Time period1, Time period 2 and Time period 3) are the control variables in both the optimization schemes.

3. **Objective functions or outputs from the model:** Cumulative volume of bitumen and cSOR are selected as two objective functions that have to be optimized. The goal of the optimization process is to maximize the cumulative oil and minimize cSOR.

4. **Control variable domain:** The constraints for varying the control variables are given in the following Table 6.3:
Table 6.3: Range of the control variables used in both methods of optimization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Lower limit</th>
<th>Upper limit</th>
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</thead>
<tbody>
<tr>
<td>$P_{inj}$ at the 1$^{st}$ injection period</td>
<td>kPa</td>
<td>1,800</td>
<td>3,800</td>
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<tr>
<td>$P_{inj}$ at the 2$^{nd}$ injection period</td>
<td>kPa</td>
<td>1,800</td>
<td>3,800</td>
</tr>
<tr>
<td>$P_{inj}$ at the 3$^{rd}$ injection period</td>
<td>kPa</td>
<td>1,800</td>
<td>3,800</td>
</tr>
<tr>
<td>Time period length</td>
<td>months</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>$S_{frac}$</td>
<td>dimensionless</td>
<td>0.05</td>
<td>0.35</td>
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6.2.1 Gradient-based optimization approach

Here, gradient descent search algorithm was implemented and its computational procedure is same as discussed in Chapter 5. Maximizing cumulative oil production and minimizing steam injection are the main goals of the optimization using the improved semi-analytical model. As mentioned in previous section, the model in unable to model solvent retention and therefore will not be used in optimization work flow. The objective function (G), given by Equation 6.8, determines the relative difference between the predicted response and the desired response (maximum or minimum). This difference is then minimized as the optimization algorithm proceeds.

$$G = \sqrt{\left(\frac{CumOil - CumOil_{max}}{CumOil_{max}}\right)^2 + \left(\frac{cSOR - cSOR_{min}}{cSOR_{min}}\right)^2}$$ (6.8)

The maximum value of cumulative oil and minimum value for cSOR is found by evaluating the semi-analytical model for many combinations of control variables, the injector pressure and SSR. The search algorithm in gradient based method starts with an initial guess for three control variables and the objective function is minimized along the negative direction of the derivative of the function. At every iterative step, all three variables are simultaneously varied and optimized within the domain. This method is implemented in MATLAB programming environment. The computational steps involved in gradient descent optimization approach are described in Chapter 5.
6.2.2 Hybrid multi-objective optimization

The theoretical background and workflow for hybrid multi-objective optimization based on Pareto Optimality is given in Chapter 5. Here, we will discuss its implementation in optimization using the improved semi-analytical model.

The multi-objective optimization process evaluates two objectives, that are cumulative volumes of bitumen production and cSOR, by adjusting five control variables over a 2-year duration. The domain space for control variables used in this optimization, given in Table 6.3, is same as that used in gradient based optimization approach. Genetic Algorithm (GA) configurations are presented in Table 6.4. In Genetic Algorithm (GA), the number of iterations for the algorithm to achieve the goal of optimization is referred to as generation of algorithm. The sub-set of control variables in each generation that satisfies the optimization criteria is termed as solutions per generation A total of 2,000 runs are executed which belong approximately to 0.002% of the total 115,819,875 production scenarios that can be obtained from various combinations of control variables within the constraints given in Table 6.3. In this work, after 20 generations with 100 solutions per generation, the tolerance for the goal of the objective function is achieved. The generation number, which is 20 in this work, for the termination of algorithm is user defined. Hence, the 20th generation gave final 100 Pareto-optimal solutions that form the optimum solution set for this case study.

From generation to generation, recombination of qualified solutions by means of two genetic operators, i.e. crossover and mutation, yielded more improved solutions. Crossover indicates that two randomly selected solutions exchange their variables partially, while mutation is a change of variables in one randomly selected solution. Both crossover and mutation are different ways to generate new solutions by recombining old (existing) solutions. In general, the probabilities of crossover and mutation are user-
specified values based on the expert knowledge. In fact, for most of Genetic Algorithms, the probability of crossover is larger than the probability of mutation. The probability of crossover of 0.9 is a widely used value in global or multi-objective genetic algorithm. In this study, the probabilities of crossover and mutation are 0.9 and 0.1, respectively.

Table 6.4: Genetic Algorithm (GA) configurations for hybrid multi-objective optimization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Generation</td>
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<tr>
<td>Population</td>
<td>100</td>
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<tr>
<td>Probability of crossover</td>
<td>0.9</td>
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<tr>
<td>Probability of mutation</td>
<td>0.1</td>
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6.3 RESULTS AND ANALYSIS

6.3.1 Validation of application of superposition in semi-analytical model with CMG simulation

Here, we analyze and discuss the quality of match between the improved semi-analytical model and that of CMG-STARSTM simulation and validate whether results using effective interface pressure obtained from superposition of injection pressures presents trends similar to that from CMG simulation. Figure 6.1 shows the variation of injector Bottom Hole Pressure and oil flow rate with time. Low oil flow rates are achieved corresponding to low injection pressures. The initial increase in the oil rate is due to steam chamber vertical rising period. The oil rate then becomes constant during steam chamber horizontal growth period once the steam chamber intercepts the overburden. The transition from steam chamber vertical rise period and horizontal growth period occurs in December 2013 (See Figure 6.1) in the CMG simulation. Since, the improved semi-analytical model computes oil and steam rates only for horizontal growth period, comparison between the CMG simulation and semi-analytical method, hence
forth, will be analyzed only for that phase. Figure 6.2 shows the comparison of variation of oil rate between the improved semi-analytical model and 2D CMG simulation. The result from the improved semi-analytical model for effective injection pressure is closer to that obtained from CMG simulation. It can also be seen that oil rate from effective injection pressure ($P_{\text{eff}}$) forms an intermediate solution between oil rates obtained from three single injection pressure cases (indicated by dotted lines in the figures). Since, injection pressure of 2600 kPa was held for longest time period, out of the three injection pressures, the solution from effective pressure ($P_{\text{eff}}$) lies close to this case. Figure 6.3 shows similar comparison for cSOR. The same conclusion can be derived from this figure as well. In conclusion, that implementation of superposition theorem under the assumption of isolated steam chamber to model variable pressure case gives relatively closer solution to that of CMG simulation.

Figure 6.1: Variation of oil rate with time for variable injector Bottom Hole Pressure (BHP) in CMG-STARS™ simulation.
Figure 6.2: Comparison of bitumen rate predicted using superposition of variable injection pressure in the improved semi-analytical model and CMG-STARS™ simulation with variable injection pressure.

Figure 6.3: Comparison of cSOR predicted using superposition of variable injection pressure in the improved semi-analytical model and CMG-STARS™ simulation with variable injection pressure.
Judging from the results presented comparing the CMG results to those obtained by applying superposition, it is clear that direct superposition of pressure using linear assumptions may be insufficient to capture the full effects of fluctuations in injection conditions. It is quite likely that rate-superposition that is to find the effective pressure at the steam chamber interface by superimposing the pressure fluctuations corresponding to rate fluctuations might be more appropriate and there are analytical solutions from the well test literature to apply such a procedure. This is recommended in the future.

### 6.3.2 Results from gradient based optimization

The optimum solution from gradient based approach is given in Table 6.5. Longer duration of higher injection pressures and smaller values of solvent fraction, around 13%, resulted in the optimized values for recovery factor and cSOR.

Table 6.5: Optimum solution obtained from the improved semi-analytical model using gradient-based approach.

<table>
<thead>
<tr>
<th>Injection pressure (kPa)</th>
<th>Time period (months)</th>
<th>Optimum SSR (fraction)</th>
<th>Optimum cumulative oil produced (m³)</th>
<th>Optimum cSOR (m³/m³)</th>
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<tbody>
<tr>
<td>$P_{\text{inj}}$ at the 1st injection period</td>
<td>3450.5</td>
<td>8.2</td>
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<tr>
<td>$P_{\text{inj}}$ at the 2nd injection period</td>
<td>2615.2</td>
<td>3.25</td>
<td>0.13</td>
<td>51.23</td>
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<tr>
<td>$P_{\text{inj}}$ at the 3rd injection period</td>
<td>3752.0</td>
<td>12.54</td>
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### 6.3.3 Results from hybrid multi-objective optimization

Figure 6.4 shows the scatter plots showing the final 100 Pareto-optimal solutions and the global-optimum in two-dimensional variable space and objective space. The Pareto optimal solutions between two objectives namely cSOR and cumulative oil production show a concave upwards profile (red squares). These solutions satisfy the optimization criteria – maximum bitumen production at the lowest steam requirement.
Figure 6.5 and Figure 6.6 show the optimum values for the control variables, which are effective interface pressure, solvent fraction and time period that result in Pareto-optimal solutions for the objective functions. Figure 6.5 indicates that relatively high solvent fractions at lower effective interface pressures result in high recoveries and low cSOR. On the contrary, for higher values of effective interface pressures, low values of solvent fractions leads to higher recoveries and low cSOR. The optimum values obtained from gradient-based optimization (indicated by blue triangle in the plots) lie somewhat close to the optimum solutions obtained through multi-objective optimization. Intermediate values of solvent fraction ranging 0.1 to 0.2, for almost the entire range of effective interface pressures, result in high bitumen recoveries and relatively low cSOR. From Figure 6.6, it can be concluded that longer duration of ramp up (Time Period 1) and decline (Time period 2) periods at wide range of effective interface pressures are favorable for optimized Recovery Factor and cSOR. In addition, Pareto-optimal solutions indicate a wide range of the optimum operational parameters.
Figure 6.4: Plot showing Pareto-optimal solutions for 1st (grey circles) and 20th-final generation (red squares) along with solution obtained applying the gradient-based approach for optimizing cSOR and cumulative oil production.

Figure 6.5: Plot showing the effective pressures ($P_{eq}$) and solvent fraction for three time periods corresponding to the 1st (grey circles) and 20th generation (red squares) of the algorithm. The final (20th) generation represents the Pareto optimal solution. The solution obtained applying the gradient based approach (blue triangle) is also shown.

Figure 6.6: Plot showing the 1st (grey squares) and 20th generation (red squares) together with the solution obtained using the gradient based approach (blue triangle). These results are for the effective pressures ($P_{eq}$) and the duration of the three time periods corresponding to the Pareto-optimal solutions.
6.4 Chapter Summary

The improved semi-analytical model, not only models the important physical phenomena involved in ES-SAGD process, but also proves useful in performing process optimization studies. The superposition theorem is used to model the effects of variable injection pressure on the effective pressure at the steam chamber interface. The optimization demonstrated in this chapter gives a simple application of super-position theorem under the assumption that variation in pressure due to injection pressure changes, in the absence of any interactions with reservoir pressure, can be linearly superimposed to obtain the effective pressure at any location. The two optimization methods discussed in this chapter can be integrated with the semi-analytical model in addition to optimization using Response Surface proxy models discussed in Chapter 5. The multi-objective optimization procedure yields a Pareto-optimal solution space within which operating parameters can be varied to obtain desirable outputs from the process. The solution from gradient based approach lies close to one of the Pareto optimal solutions.
Chapter 7: Conclusions and Future Work

This research sheds insights on the formulation of an improved semi-analytical approach that represents the basic and significant physical processes that take place during the ES-SAGD process. With the integration of the improved semi-analytical model together with optimization algorithms, an effective process design and control scheme can be developed. An important insight from this work is that short length processes such as associated with solvent diffusion into oil coupled with thermal diffusion process can be modeled efficiently using simple models.

7.1 Summary and Conclusions

The key summary and conclusions from this study are as follows:

- In Chapter 2, we carefully reviewed the available methods to study ES-SAGD performance and found that experimental and simulation studies outnumbered the existing analytical models. However, Das and Butler (1996), Sharma et al., 2011, Azom and Srinivasan (2012) came up with analytical models that were good for some specific problems but employed some impractical assumptions. However, Rabiei et al., 2014 came up with an unsteady state model, which is a good start to delve into the characteristics of the transient ES-SAGD mechanisms. A recent work by Gupta and Gittins (2012) based on modeling steam heating and solvent dilution effects together with the phase behavior of solvent becomes the basis for the improved model for the ES-SAGD process presented in this research. Moreover, it is well iterated throughout the study that there is a need for some simple analytical models for ES-SAGD process to replace the already available fit-for purpose models and to circumvent the issue of computational expensive full flow simulations at least during the initial stages of any ES-SAGD project.
• Improvements and amendments were introduced into Gupta and Gittins’ (2012) semi-analytical model to alleviate some assumptions that affect the solvent mass transfer into bitumen to a greater extent. The variation of overall solvent diffusion coefficient with concentration and viscosity using simple correlations were incorporated into the model. These modifications gave better results for bitumen recovery and eliminated the need for the assumption of a sharp interface between vapour chamber and bitumen and consequently the use of a constant value for overall solvent diffusion coefficient (D).

• Numerical dispersion and grid orientation effects that occur in numerical simulations tend to mask the effect of the actual small scale mass-transfer mechanism that take place at the steam chamber edge. Dynamic gridding based on solvent mole fraction along the solvent front at the steam chamber edge seemed to be a feasible option to mitigate such numerical artifacts. Finer grid blocks at the steam chamber edge gave optimistic results as compared to coarse grid simulation. The process of dynamic refinement of moving steam chamber interface together with straight line relative permeability curves gave a better match with improved semi-analytical model for bitumen rates, solvent rates, SSR and SOR.

• The application of multi-objective optimization using proxy models is presented in Chapter 5. This technique arrives at a wide range of combination of key operating parameters that maximize oil recovery while minimizing steam and solvent requirement. Furthermore, the multi-objective optimization solution also contains the global optimum solution obtained using a gradient-based approach.
High NPV values obtained from these Pareto-optimal solutions helped in analyzing the cost effectiveness of the process. Proxy model development through response surface models acts as one of the effective ways to approximate process variables without always relying on complicated and computation intensive numerical models during the process of optimization.

- Higher solvent fraction and longer duration of steam–solvent injection period gave best results in terms of bitumen recovery and energy efficiency of the process overall. Similar trend was also seen in optimization using semi-analytical model, where relatively lower solvent fractions (around 0.2-0.25) coupled with injection BHPs ranging from 2500-4000 kPa resulted in good oil recoveries and low cSOR.

Last but not least, the inclusion of optimization techniques during the development of the semi-analytical model fulfills the main objective of building a simple and robust model for ES-SAGD process. The improved semi-analytical approach can provide a preliminary estimate of the trends of fluid rates, SOR and SSR for ES-SAGD project and thereby help in project screening and process optimization studies.

### 7.2 Recommendations for Future Work

In order to fully emulate the physics of ES-SAGD process, modelling of multi-phase flow within the semi-analytical approach becomes important for accurately predicting production characteristics and optimizing operation parameters. Relative permeability curves and models for sweep efficiency, built into the semi-analytical model, could solve this problem. For a non-isothermal process like SAGD or ES-SAGD, the concept of temperature dependent relative permeability can also be explored.
The movable oil saturation is assumed to be constant in the semi-analytical model over the duration of production. Some variation in this parameter would mimic the fluid flow behavior similar to that observed in numerical simulation. These modifications might bring the match between the semi-analytical model and numerical simulation even closer.

The other aspect to be looked at is incorporating the effects of reservoir pressure into the semi-analytical model. This would enable better understanding of pressure-saturation interaction of the process, similar to the actual reservoir behavior.

Studying and accommodating all the above extensions to this work in a 3-Dimensional environment would make the semi-analytical model a bit complex, but more accurate.
Table A.1: Multi-level factorial design matrix

<table>
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<tr>
<th>Coded form</th>
<th>True variable</th>
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<tr>
<td>Injection pressure (A)</td>
<td>Steam injection period (B)</td>
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References


Das, S. K., and Butler, R. M., 1995. Diffusion Coefficients of Propane and Butane in Extraction of Heavy Oil and Bitumen Using Solvents at Reservoir Pressure. Technical Meeting/Petroleum Conference of South Saskatchewan Section, 16-18 October, Regina SK., Canada.


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