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Designing Enhanced Geothermal and Hydraulic Fracturing Systems
Based on Multiple Stages and Proppant

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Designing Enhanced Geothermal and Hydraulic Fracturing Systems
Based on Multiple Stages and Proppant

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Dedication

To my father and late mother
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Abstract

Designing Enhanced Geothermal and Hydraulic Fracturing Systems Based on Multiple Stages and Proppant

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This report consists of two chapters. In the first chapter, designs of Enhanced Geothermal Systems (EGS) with horizontal wells, multiple stages, and proppant are discussed. In EGS, hydraulic stimulation is used to improve well productivity. EGS is typically performed in a nearly vertical well, in one stage, with no proppant. Horizontal wells, multiple stages, and proppant are not used because they are considered not necessary and/or technically infeasible. We found that an EGS design with multiple stages and proppant could give dramatically improved economic performance relative to current designs. We reviewed the literature in order to assess the technical viability of our proposed design. The proposed design would increase cost but deliver substantial improvements in flow rate (and revenue) per well.

The second chapter describes a simulation study of proppant transport with Newtonian fluid in a fully three-dimensional hydraulic fracturing simulator, CFRAC. This model has capability to handle proppant settling due to gravity, proppant migration away from the fracture walls, and fracture closure. In the model, the conservation equations of
fluid and proppant are sequentially solved in a first-order finite difference scheme. A special algorithm is applied to handle proppant packing due to fracture closure. Our simulation results show good agreement with results from other recently published proppant modeling studies. Sensitivity analysis was performed in order to investigate the effect fluid viscosity, proppant density, and proppant size. Finally, simulations of the tip-screen out (TSO) were performed.
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CHAPTER 1: EGS DESIGNS WITH HORIZONTAL WELLS, MULTIPLE STAGES, AND PROPPANT

1.1 INTRODUCTION

1.1.1 Introduction

Hydraulic stimulation was originally developed by the hydrocarbon industry (Montgomery & Smith 2010). In the hydrocarbon industry it is believed that injection causes the initiation and propagation of new hydraulic fractures (Khristianovich & Zheltov 1959; Economides & Nolte 2000). Sand or other particular matter (called proppant) is injected with the fracturing fluid during treatment. The proppant remains in the fractures and allows them to retain transmissivity after pumping stops and the newly formed fractures close.

Hydraulic fractures form perpendicular to the local minimum principal stress (which is typically horizontal), and so in vertical wells, newly formed fractures form axially along the wellbore. The advent of horizontal drilling enabled the creation of transverse fractures. In this design, a horizontal well is drilled in the direction of the minimum principal stress. Fractures form perpendicular to the orientation of the wellbore, which allows multiple fractures to be formed, creating additional contact with the formation.

Rather than injecting simultaneously into the entire reservoir interval of the well, individual sections (called stages) are hydraulically isolated with downhole tools, and pumping is performed sequentially into each stage. The advantage to using multiple stages (rather than injecting into the entire wellbore at once) is that it creates a more uniformly distributed fracture network. Once a fracture has formed at the wellbore, fluid tends to flow into that fracture, reducing wellbore pressure and preventing fractures from forming in
other sections of the wellbore. With multiple stages, fractures are forced to form at each individual stage.

A variety of strategies are used for multiple stage completion in horizontal wells. In "plug and perf" completions, casing is cemented into the lateral prior to stimulation. A section near the "toe" of the wellbore (the section furthest from the wellhead) is perforated with explosive charges that blow holes through the casing (and surrounding cement) and connect the wellbore to the formation. Injection is then performed into the perforated section. Next, a plug is set inside the casing, hydraulically isolating the section that has been stimulated from the rest of the well. Perforation is then performed in the adjacent stage, and the process is repeated until the entire lateral has been stimulated.

In sliding sleeve completions, casing is placed in the production interval but not cemented. Openhole packers are used to isolate the annular sections of the wellbore (around the casing) and a system of valves is used to create isolation within the casing, so that each stage can be pumped sequentially.

The combination of horizontal drilling, multiple stages, and optimization of proppant strategy has created a revolution in the oil and gas industry over the past decade, enabling economically viable production from massive, low permeability oil and gas resources in very low permeability shale (King 2010).

In Enhanced Geothermal Systems (EGS), hydraulic stimulation is used to increase formation permeability, most often in crystalline rock such as granite. In EGS, stimulation is typically performed by injecting fresh water (no proppant) into an openhole section of a vertical wellbore. It is generally believed that injection induces slip on natural fractures, which experience increased transmissivity in response to slip (Tester 2006). One or more production wells are drilled, and fluid is circulated, heating as it flows through the formation.
The objective of EGS is to achieve economic production of energy from low permeability resources, precisely what the oil and gas industry has achieved in shale over the past decade. The strategies that have succeeded for oil and gas should be considered for EGS.

Nearly all EGS wells have been drilled vertical (or somewhat deviated), stimulated in a single stage, and have not used proppant. There are a few exceptions. The Newberry EGS project effectively had multiple stages because a thermally degrading chemical diverter was used (Petty et al. 2013). The Groß Schönebeck project used a nearly horizontal well and used zonal isolation to perform a three stage stimulation in sandstone and volcanic rock (Zimmermann et al. 2010).

For this report, we performed simple "back of the envelope" calculations to demonstrate the potential effects of multiple stages, horizontal drilling, and proppant on the performance of an Enhanced Geothermal System. The goal for economic EGS development is flow rates in the range of 100 kg/s at 200°C, sustained for a decade or more. Our calculations show that multiple hydraulic fracturing stages (and possibly the use of proppant) could lead to these levels of productivity. With enough stages, flow rate may be limited more by pressure gradient in the wellbore than in the reservoir.

EGS projects have not used multiple stages, proppant, and horizontal (or deviated) wells either (1) because it was believed that they would not be effective or (2) because of the technological challenges of operating at high temperature in hard rock. We have reviewed the literature to address these concerns. The review demonstrates the benefit of applying these technologies to EGS, even in granitic rock. Technical feasibility is less certain, but generally we found that the technology exists to apply these concepts at 200°C with current technology, provided the project was designed appropriately. We did not perform calculations on thermal breakthrough, but we expect that using additional stages
would create more pathways for flow and delay thermal breakthrough. This may even allow for closer wellbore spacing, which would make it easier to establish good hydraulic connections between wells and higher flow rate. In practice, the number of stages and wellbore spacing would be chosen based on an optimization of cost and expected future production rate and temperature. In the future, we will repeat the analysis in this report with a model that includes thermal drawdown.

Drilling horizontal wells, using multiple stages, and injecting proppant would increase cost. However, stimulation cost is a small part of the overall cost of an EGS project (Sanyal et al. 2007), and stimulation performance is one of the primary variables in determining project revenue. Drilling costs are a substantial proportion of EGS costs and more study would need to be performed to calculate the incremental cost from using deviated or horizontal wellbores.

The idea of using multiple stages in an EGS well has been proposed by others (Gringarten et al. 1975; Jung 2013). Two new points emphasized in this report are that: (1) completions with cemented casing could make zonal isolation for multiple fracturing stages possible with current technology and (2) the use of proppant should be considered, since field experience indicates it increases fracture transmissivity, even in granite.

1.1.2 Wellbore designs

Multiple stage completions require that each stage is separately by adequate distance in the direction perpendicular to the minimum principal stress (so that the fractures created at each stage do not overlap). Ideally this would be achieved with horizontal wells.

Horizontal drilling is technically challenging in hard rock and high temperature, and while it is technically possible, more study is needed to determine whether it would be cost effective. Steerable drilling systems are available at temperatures up to at least 300°C
Halliburton 2013). Tools that allow precise geosteering are available to temperatures up to 180°C (Schlumberger 2011), though in EGS precise geosteering is probably unnecessary. EGS projects are typically located in hard rock such as granite, but geothermal resources also exist in lithologies that are less challenging to drill.

If horizontal wells were not considered technically feasible, then deviated wells could still permit multiple transverse fractures to be formed. For example, a well deviated at 45° from vertical could achieve a 100 m horizontal spacing between each stage by spacing the stages at 141 m intervals along the wellbore. If necessary, wells could be deviated at shallower elevation than the target depth (where the temperature is lower and the lithology is easier to drill through). In a compressive stress regime, such as at Cooper Basin, hydraulic fractures form horizontally, and multiple transverse fractures could be created in a vertical well. Figure 1.1 shows schematics of multiple stage EGS doublets for (1) deviated wells in a normal or strike slip faulting regime and (2) vertical wells in a reverse faulting regime.
Figure 1.1: Multiple stage EGS concepts. On the left, horizontal fractures (forming in a reverse faulting regime) are stacked vertically. On the right, wells are deviated at an angle from vertical, placed side-by-side, and connected with vertical fractures (normal or strike slip faulting regime).

Ideally, there would be an identical flow rate in each stage. An imbalance could contribute to thermal short-circuiting. However, for the designs in Figure 1.1, a variety of factors may prevent the flow rate from being equal in each stage. Flow paths passing through the stages closest to the wellhead pass through a shorter length of the wellbore, and experience less pressure drop from flow through the wellbore. The stages at lesser depth are at lower temperature and stress. The lower temperature would cause the water circulating in the formation to have higher viscosity (in the cooled region of the formation), but the lower stress would increase the fracture transmissivity (because fracture aperture decreases in response to increasing normal stress). The thermosiphon effect driving flow (caused by the differing temperatures and densities of the fluid in the injector and
production well) would encourage a greater flow rate through the deeper stages. Thermal cooling stresses would develop more rapidly in whichever stages were accepting a greater flow rate. To fully unpack these issues would require a detailed analysis.

A second issue with non-horizontal completions is that technologies for zonal isolation (required for multiple stages) will have a maximum temperature of operation. As deviated or vertical wells are drilled deeper through the producing interval, the formation temperature will increase until the maximum operating temperature for zonal isolation is reached. This issue would limit the practical length of the producing interval and require production from stages shallower (at lower temperature) than the maximum allowable temperature.

If horizontal wells can be drilled, we would propose to design an EGS doublet as shown in Figure 1.2. The vertical sections of the wells are on opposite sides of the lateral. With the wells in the doublet oriented from heel to toe, all flow paths through the system will pass through the same length of wellbore, reducing the tendency for unbalanced flow through stages. The entire producing interval is at the same temperature and stress, avoiding the potential consequences for flow imbalance, and the lateral depth could be chosen at the maximum temperature practical for the available zonal isolation technology. The second well would be drilled after the stimulation of the first and could be located to either side of the first well, or even above or below (this could be selected from microseismicity observations). If proppant was used, it may be advantageous to drill the second well below the first well because proppant may tend to settle downward.
1.1.3 Adoption of new EGS designs

Technological barriers are not the only factors preventing EGS projects from using horizontal (or deviated) wells, multiple stages, and proppant. It is widely believed that induced slip on preexisting fractures is crucial to EGS stimulation. Therefore, EGS wells are completed openhole to maximize contact between the wellbore and natural fractures. However, openhole packers are difficult to design because wellbore walls are not smooth and because wellbores often have breakouts that make their shape non-circular and unpredictable (for example, Valley & Evans 2009). Cased hole packers are designed to seal within steel casing and are much easier to engineer and more reliable. Cased hole packers rated to geothermal temperatures are available with current technology (Section 1.4.3). Openhole packers may be possible at high temperature, but the technology carries greater technical risk. Overall, it would be much easier technically to perform multiple stage fracture treatments in a wellbore that has been set with casing and cemented.
Perforating wells in granite at high temperature is not typical, and this may also be an area where research is needed, but at least one service company has developed a tool rated to 230°C or higher (Barker 2013).

In hard formations such as granite, it is unclear how easily new fractures could be initiated through perforations. Field experience injecting into openhole sections in granite indicates that during injection at bottomhole fluid pressure above the minimum principal stress, injection can often open existing natural fractures (even if not oriented perpendicular to the minimum principal stress) rather than initiate new fractures (Baumgärtner & Zoback 1989; Cornet & Descroches 1990; page 74 from Brown et al. 2012).

We do not believe that it is necessary to use openhole completions in order to maximize contact with natural fractures. It has recently been argued that new fractures have formed and propagated through the formation at many EGS projects (McClure 2012; McClure & Horne 2013a; McClure & Horne 2013c; Jung 2013). Whether or not new fractures typically form at EGS projects, new hydraulic fractures would likely form during injection into a perforated wellbore section because the wellbore would have poor connection to the existing natural fractures. These new fractures may themselves create the desired EGS fracture system or they may connect the wellbore to existing fractures. After stimulation of the injector well through perforations, the producer well could be drilled through the microseismic cloud created by stimulation and completed openhole (to maximize connectivity between the newly stimulated fractures and the wellbore).

There appears to be skepticism in the EGS community about using proppants. One argument is that proppants will chemically degrade in the highly reactive environment of an EGS reservoir. Another argument is that proppants are unnecessary because fractures in granite are "self-propping" and have high transmissivity even when not propped.
There is little laboratory data directly studying the transmissivity of propped and unpropped fractures in granite. However, Stoddard et al. (2012) performed laboratory flow tests for propped and unpropped fractures in granite and found that the propped fracture retained significantly greater transmissivity at higher temperature (90°C) and normal stress (3000 psia). In Section 1.4.4, we review the field experience with proppant from EGS in granite and find that proppants have consistently led to improved productivity. In Section 1.4.5, we discuss potential issues associated with chemical degradation and possible mitigation.

A third potential concern about proppant is that it may lead to the formation of thermal short-circuit pathways. Proponents have touted an "advantage" of shear stimulation (induced slip on preexisting fractures, relying on the transmissivity of unpropped fractures to create stimulation) is that it generates distributed fracture networks, which should reduce short-circuiting by distributing flow into a more volumetric fracture network. But these pathways have generally been shown to have poor, unpredictable connectivity and inadequate overall permeability. Furthermore, it seems counterintuitive that we should avoid using proppants because they could create short-circuit pathways. Short-circuit pathways are high transmissivity pathways for flow, and inadequate flow rate is the principal obstacle preventing widespread adoption of EGS. To combat short circuiting, multiple stages could be used. Multiple stages would inhibit short circuiting because it would force fractures to form at each stage. This would distribute stimulation broadly across the entire reservoir interval. With multiple stages, a more viscous fluid could be used during stimulation, which would help transport the proppant and would tend to inhibit fracture network complexity (Cipolla et al. 2010). Network complexity implies a more volumetric fracture network. This may be viewed as good for delaying thermal breakthrough, but it may create tortuous pathways that contain bottlenecks and inhibit high
circulation rates. Flow from perforated intervals might actually inhibit short-circuiting because pressure drop at the perforations would prevent extremely large flow rates from a single zone. Finally, wellbore interventions to plug short circuit intervals would be technically easier in cased, cemented completions.

1.1.4 Technology development strategy

In order to enable very large scale production of geothermal energy, EGS needs to be economically viable in very deep (~5 km) wells, often drilled in hard rock such as granite. However, while not as abundant, significant opportunities exist in lower cost areas, where high temperatures are available at shallower depths and formations that are easier to drill (for example, Chabora et al. 2012). Projects in granite have unique challenges because: (1) drilling is more challenging, (2) fracture initiation is more difficult, and (3) fracturing processes apparently behave differently in granite than in other types of rock.

Projects in less challenging formations could be used to prove the viability of the basic ideas proposed in this report: (1) multiple stage hydraulic fracturing out of a cased, cemented hole, and (2) hydraulic fracturing with the intention of creating new fractures and using proppant. In early projects, direct experiments could be performed -- some stages using proppant, other stages not using proppant. Long term circulation tests could compare the ability of the stages with and without proppant to sustain permeability over time.

Ideally, early projects would be located at existing geothermal fields, on the fringes where temperature is hot but there is minimal natural permeability. These projects would benefit from the existing infrastructure and formation characterization. Low natural permeability would be advantageous because elevated natural permeability could increase fluid loss to the formation. Formations with elevated permeability are more likely to have large, well-developed faults (because large faults tend to be conduits for flow). Large faults
could be induced to slip, which could deform the wellbore if intersecting the fault (Evans et al. 2005). Wellbore deformation due to a shearing fault would be a more significant problem for a cased, cemented hole than for a well completed openhole. Induced seismicity has been an issue at some EGS projects (Majer et al. 2007). An EGS project at an existing field would have the advantage that the induced seismicity hazard in the area has already been well-characterized.
1.2 METHODOLOGY

We conducted a sensitivity analysis to investigate the effect of multiple stages and proppant on the performance of an EGS doublet circulating fluid between two horizontal wells. The fluid circulation rate through the doublet was calculated using different assumptions about (1) the transmissivity of the fracture network created at each fracture stage and (2) the number of fracture stages. The flow rates were then converted to electricity generation rates, and the present value (PV) of the project revenue (neglecting cost) was calculated assuming that the production rate and temperature would be constant for 20 years. Many simplifying assumptions were made in the calculations. The calculations are intended to be used for sensitivity analysis and to demonstrate, roughly, the sort of performance than could be achieved in the system that we propose.

1.2.1 Flow rate calculation

The calculations included pressure gradient in the injection well, the reservoir, and the production well. The injector and producer geometries were assumed to be the same as GPK3 and GPK2 down to the producing depth. At the producing depth, the wells were assumed to be horizontal and oriented toe-to-heel, as shown in Figure 1.2. For simplicity, the temperatures in the injector well, production well, and the reservoir were assumed to be constant (but different in each of the three).

The wellhead pressure of the injector, $WHP_{inj}$, and the wellhead pressure of the producer, $WHP_{prod}$ were specified to be 4 MPa and 0.75 MPa, respectively (following the Soultz circulation test described by Tischner et al. 2006). Flow was also driven by the difference in density between the fluid in the injection well and the production well. Flow rate could have been increased by using a higher injection pressure, but excessively high injection pressure may lead to excessive fluid loss. Flow rate could have been increased by
pumping the production well, but this would involve additional cost and parasitic power loss. In practice, it would probably be economic to pump the production well, and it may be feasible to use a higher injection pressure than we assumed, but we chose these values in order to give conservative estimates.

To simplify the calculation, it was assumed that all fluid entered or exited the wellbores at the bottom of each well at a single location (even though we were modeling wells with multiple stages). The system was modeled as four nodes connected in series: $WHP_{inj}, BHP_{inj}, BHP_{prod},$ and $WHP_{prod}$, where $BHP_{inj}$ and $BHP_{prod}$ are the bottomhole pressure of the injector and the producer wells, respectively. Wellbore pressure drop calculations were used to calculate $\Delta P_{inj}$, equal to $WHP_{inj} - BHP_{inj}$, and $\Delta P_{prod}$, equal to $WHP_{prod} - BHP_{prod}$ (as described in Section 1.2.1.1). Darcy's law was used to calculate $\Delta P_{res}$, equal to $BHP_{inj} - BHP_{prod}$ (as described in Section 1.2.1.2). All fluid in the system was assumed to be single phase, liquid water.

For a given value of reservoir transmissivity, the flow rate through the system was calculated by numerically solving the following nonlinear equation, with flow rate, $q$, as the unknown:

$$WHP_{prod} = WHP_{inj} + \Delta P_{inj(q)} + \Delta P_{res(q)} + \Delta P_{prod(q)}. \quad (1.1)$$

1.2.1.1 Pressure change calculation in the injection well and the production well

This section explains how $\Delta P_{inj(q)}$ and $\Delta P_{inj(q)}$ were calculated. The total pressure gradient $(dp/dz)$ can be calculated as the sum of the frictional gradient $(dp/dz)_F$, the hydrostatic gradient $(dp/dz)_H$, and the accelerational gradient $(dp/dz)_A$ (Hasan & Kabir 2002) and is given by:

$$(dp/dz) = (dp/dz)_F + (dp/dz)_H + (dp/dz)_A. \quad (1.2)$$
From the conservation of momentum, the hydrostatic and the accelerational gradients are represented by:

\[
(dp/dz)_H = -g\rho_f\sin\theta, \tag{1.3}
\]
\[
(dp/dz)_A = -(w/A)\frac{dv}{dz} = -\rho_f v\frac{dv}{dz}, \tag{1.4}
\]

where \(\rho_f\) is the density of fluid, \(\theta\) is the wellbore angle from horizontal line, \(w\) is fluid mass rate, \(A\) is cross-sectional area of casing, and \(v\) is its velocity.

The frictional pressure gradient is:

\[
(dp/dz)_F = -2f v^2 \rho/d_w, \tag{1.5}
\]

where \(d_w\) is well or pipe diameter and \(f\) is friction factor, which depends on the turbulence of the fluid and also on the pipe roughness.

Chen (1979) proposed the following equation to calculate the Fanning friction factor:

\[
f = \frac{1}{2\log(\varepsilon/d_w)^{1} - \frac{5.04 \log\Lambda}{Re}}^{2}, \tag{1.6}
\]

where \(\varepsilon\) is pipe roughness, and \(\Lambda\) is the dimensionless parameter given by:

\[
\Lambda = \left(\frac{\varepsilon}{d_w}\right)^{1.1098} + \left(\frac{7.149}{Re}\right)^{0.898} + 1. \tag{1.7}
\]

Fluid properties were chosen for fresh water and are given in Table 1.1. The temperature in the injection well was assumed to be 60°C, and the temperature in the production well was assumed to be 180°C. The surface roughness of casing was assumed to be 150 microns, as estimated for the wellbore casing of GPK2 at Soultz by Mégel et al. (2005) based on measurements of the wellhead and bottomhole pressure during injection.
Table 1.1: Properties used in the flow rate calculations. The reservoir transmissivity shown is the "baseline" transmissivity.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Injector</th>
<th>Reservoir</th>
<th>Producer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>60</td>
<td>190</td>
<td>180</td>
</tr>
<tr>
<td>Fluid density (kg/m³)</td>
<td>983.2</td>
<td>873.9</td>
<td>885.0</td>
</tr>
<tr>
<td>Fluid viscosity (cp)</td>
<td>0.466</td>
<td>0.142</td>
<td>0.151</td>
</tr>
<tr>
<td>Transmissivity (m³)</td>
<td></td>
<td>3.07E-13</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2 shows the geometry of the injection and production wells (based on the Soultz wells GPK2 and GPK3) including depth, casing diameter and inclination (Tischner et al. 2006). Table 1.2 contains the extended laterals on the horizontal wells (which were not present in GPK2 and GPK3). The wellbore lengths increased by 150 m for every stage that was added. The extended horizontal part of the injection well was assumed to be completed with casing and perforations (cased hole completion). The horizontal lateral of the production well was assumed to be openhole. The roughness of the formation in the openhole was assumed to be 2000 microns. Pressure drop in the perforations was not included in the calculation.

Considering the flow geometry shown in Figure 1.2, each molecule of water will pass through the same length of wellbore as it passes through the system (regardless of which stage it flowed through the reservoir), and that length is equal to half of the total length of both the laterals. To account for this effect, every time a stage was added, the length of the horizontal lateral in both the injector and the producer was increased by one half the spacing between stages, 75 m.
This section explains how $\Delta P_{\text{res}}(q)$ was calculated. The pressure change through the reservoir was calculated from Darcy's law, assuming steady-state, linear flow through a fracture with height $h$ (m) and transmissivity $T$ ($\text{m}^3$):

$$\Delta P_{\text{res}} = \frac{q \mu D}{r_T k_f}, \quad (1.8)$$

where $\Delta P_{\text{res}}$ is $BHP_{\text{inj}} - BHP_{\text{prod}}$ (MPa), $q$ is mass flow rate in the reservoir (kg/s), $\mu$ is the viscosity of fluid (cp, or for unit consistency in Equation 1.8, MPa-s), and $D$ is the distance between the injector and producer (m).

The Soultz project was used to make a baseline estimate for the transmissivity of an EGS reservoir. During the 2005 circulation test at Soultz, a 12 kg/s flow rate was
sustained between GPK2 and GPK3. The openhole section of the wells was roughly 500 m and the well separation was roughly 600 m. Using those parameters, and using the reservoir and wellbore properties given in Table 1.1 and Table 1.2 (which were based on the Soultz circulation test), the reservoir transmissivity was estimated to be $3.07 \times 10^{-13}$ m$^3$.

We used this value as our baseline transmissivity for a single stage.

The exact nature and geometry of the fracture network at Soultz (or the network created in our hypothetical EGS system) is not important for the calculation. We are using a single number, reservoir transmissivity, to account for the aggregate flow capacity of the created fracture network, whether it was a single, planar hydraulic fracture, a dense network of stimulated natural fracture, a large, thick, shear stimulated fault zone, or network of both new and preexisting fractures.

In our hypothetical EGS doublet, we assumed $D$ and $h$ to be 200 m. The wellbore spacing is a design parameter. A spacing of 200 m would be too small for a single stage (due to thermal breakthrough) but with enough stages, a smaller spacing could be acceptable. In future work we will perform a more complicated analysis including thermal breakthrough.

When there were multiple stages, we assumed that flow was evenly distributed between each stage. Therefore, the flow rate per stage was $q/S$, where $S$ is the number of stages. In Equation 1.8, this was implemented by making the total system transmissivity equal to $TS$. Table 1.1 shows the fluid properties used in the calculations.

1.2.2 Sensitivity analysis (effect of stages and proppant)

Plots were made of total flow rate versus number of stages. Aside from the baseline transmissivity, $T$ (equal to $3.07 \times 10^{-13}$ m$^3$), we performed calculations for other values of reservoir transmissivity, $T/4$, $2T$, $4T$, and $5T$. The lower value of transmissivity was
included to show the consequences of an inadequate stimulation. The higher values of transmissivity were included to show what could be possible if stimulation technique could be improved (whether through the use of proppant or some other technique). As discussed in Sections 1.1.3 and 1.4.4, proppant use may significantly increase the transmissivity of flow paths created in EGS reservoirs. The Soultz wells, which we used to establish our baseline transmissivity, were not stimulated with proppant.

1.2.3 Conversion to electricity

The rate of thermal energy production at the power plant was calculated with the following equation:

\[ Q = qC_p\Delta T, \] (1.9)

where \( Q \) is the rate of thermal energy production (J/s), \( q \) is mass flow rate (kg/s), \( C_p \) is specific heat capacity of fluid (J/kg/K), and \( \Delta T \) is difference of temperature (K) across the system.

Since we assumed that the temperatures of the injection and production wells were 60°C and 180°C, respectively, the difference of temperature was 120°C. A heat capacity of 4.42 J/g/K was assumed. We assumed that the efficiency of conversion from thermal energy to electricity to be 12.63% (from Equation 7.1 from Tester 2006)) and the price of electricity to be either 5 or 10 cents per kWh.

1.2.4 Present value (PV) of electricity

We calculated the present value (PV) of the revenue from the project. Present value is defined as:

\[ PV(i, N) = \sum_{t=0}^{N} \frac{R_t}{(1+i)^t}, \] (1.10)
where $i$ is the discount rate, $t$ is the time of the cash flow (years), $R_t$ is the net cash flow ($), $N$ is the total number of periods.

We assumed the discount rate to be 16% (Hochwimmer et al. 2013) and that production could be sustained at constant temperature for 20 years.
1.3 RESULTS

1.3.1 Flow rate

Figure 1.3 shows flow rate and number of stages for each transmissivity. The transmissivity $T$ is the baseline transmissivity ($3.07 \times 10^{-13}$ m$^3$). Calculations were performed for each of the five transmissivity values for one to ten stages.

Figure 1.3: Flow rate versus the number of stages.
1.3.2 Pressure drop

Figure 1.4 shows two lines for each calculation. The solid lines give the sum of the frictional and accelerational pressure drop in the injector and producer. The dashed lines give the pressure drop in the reservoir, $BHP_{inj} - BHP_{prod}$.

![Figure 1.4: Pressure drop versus the number of stages. The solid lines represent the sum of the frictional and accelerational pressure drops in the injector and producer. The dashed lines represent the pressure drop in the reservoir.](image)
1.3.3 Power and PV

Figure 1.5 shows the calculated electricity generation for each case. Because flow rate is linearly related to power generation, the shape of curve is identical to Figure 1.3.

Figure 1.5: Power (electricity) versus the number of stages.

Figure 1.6 and Figure 1.7 show the PV of the project revenue (neglecting cost) for electricity prices of 0.05 $/kWhr and 0.10 $/kWhr.
Figure 1.6: PV versus the number of stages at 0.05 $/kWh.
Figure 1.7: PV versus the number of stages at 0.10 $/kWh.
1.4 Discussion

1.4.1 Increasing the number of stages

From Figure 1.3, it can be seen that flow rate initially increases linearly as the number of stages increases. However, the flow rate begins to plateau at high rates, which is due to the pressure drop in the wellbore, which increases nonlinearly. The frictional pressure drop increases with the square of the flow rate (Equation 1.5). The pressure drop in the wellbores also increases as the number of stages increases because the horizontal sections of the wells are becoming longer. As the reservoir transmissivity gets bigger (due to increasing number of stages or increasing transmissivity per stage), the proportion of the overall pressure drop that occurs in the reservoir drops (Figure 1.4). The overall hydraulic head difference during flow through the system is constant, and equal to the difference in the wellhead pressure of the injector and the producer plus the difference in hydrostatic head caused by the temperature (and density) differences of the fluid in the injector and producer.

1.4.2 Increasing the transmissivity of each stage

Five different values were used for transmissivity per stage. As seen from Figure 1.3, the number of stages until flow rate became mostly constant depended on the transmissivity. For larger transmissivity, flow rate reached a constant value with fewer stages and vice versa.

In this study, we assumed that the stages were identical and did not affect each other. However, in the real life, it is possible that fractures from one stage could intersect fractures from another stage. This might decrease the overall transmissivity per stage, an effect that we did not include.
1.4.3 Current technologies for high temperature zonal isolation

Performing a multiple stage fracture treatment requires the ability to hydraulically isolate sections of the wellbore to pump into each stage sequentially. This requires the use of packers or bridge plugs. In this section we provide a nonexhaustive review of technologies available from the service industry.

Cased hole packers and bridge plugs rated to temperatures and pressure typical for EGS projects are available from oilfield service companies and have been demonstrated in the field. Packers Plus has developed packers rated to 300°C and 69 MPa. These packers have been installed successfully at the Cooper Basin EGS project and in high temperature high pressure oil and gas wells (Rivenbark et al. 2011; Rivenbark & Lefsrud 2013; Packers Plus 2013). Schlumberger's Copperhead drillable bridge and flow through frac plugs are rated to 204°C and 103 MPa (Schlumberger 2012). Baker Hughes has developed a cased hole packer rated to 232°C and 138 MPa (Doane et al. 2013). Halliburton developed a packer rated to 232°C and 103 MPa, designed to work even worn casing (Innes et al. 2010). Interwell has a high pressure high temperature retrievable bridge plug rated to 200°C and 103 MPa (Interwell 2011).

Bendall et al. (2014) reviewed several recent EGS projects in Australia. They reported that operators with Petratherm's Paralana project successfully performed a large hydraulic stimulation in granite through perforated casing and with cased hole packers at temperature of nearly 190°C.

Drillable plugs set in cased, perforated hole were used successfully to enable multiple fracture stages in the well GtGrSk4/05 at Groß Shönebeck at 150°C in sandstone and volcanic formations (Zimmermann et al. 2010). The bridge plugs were provided by
Weatherford and were rated to 177°C and 69 MPa (Günter Zimmermann, personal communication).

Openhole packer technology at high temperature appears to be significantly more limited. A Baker Hughes tool is rated to 316°C but is limited to in or near gauge wellbore. Another Baker Hughes tool is rated to 170°C (Walters et al. 2012). Walters et al. (2012) reports that an openhole packer system developed by Packers Plus and marketed by Schlumberger is rated to 199°C.

Additional review of zonal isolation at high temperatures was given by Walters et al. (2012).

### 1.4.4 EGS field experience with proppant

Proppant has been used at a limited number of EGS projects in granite. In every case, proppant has been placed successfully and increased fracture transmissivity.

The earliest experiment was a small test at the Fenton Hill project in 1974 (page 74 from Brown et al. 2012) involving injection into an openhole section isolated with packers. In the first test, injection was performed, and there was limited fluid recovery during subsequent flowback. Fluid was believed to be entering a natural fracture that was opening in response to injection. Injection was repeated with proppant, and almost full recovery was achieved. The implication is that without proppant, the natural fracture at the wellbore closed during flowback and hydraulically isolated the remaining injection fluid away from the wellbore. With proppant, the fracture retained transmissivity when it closed during flowback, enabling nearly full recovery. Evidently, the natural fracture had much greater transmissivity (post-closure) when filled with proppant.

The first large experiment with proppant at an EGS project was in the well RH15 at Rosemanowes (Bennett & Barker 1989). Proppant was injected with viscous gel at high
rate into an openhole section roughly 150 m long. Production logs showed that after stimulation, the main conduit for flow at the well was a single fracture, believed to be where the injected proppant had gone (Bennertt & Barker 1989). Parker (1999) reported that the proppant placement resulted in a significant lowering of system impedance and water loss during circulation. The proppant was so effective at improving connectivity that it was blamed for worsening a thermal short-circuit.

Cornet & Descroches (1990) described a series of stimulations in the wellbore INAG III-9 at Le Mayet. Injections were performed into four openhole sections isolated with packers. In the upper section, resistivity logs indicated an altered zone, and spinner logs prior to stimulation indicated a flow conduit. The other three zones each contained a fracture identified in a wellbore imaging log but were not flowing. The upper three zones were stimulated with viscous gel at around 70 cp. The lowest zone was stimulated with both viscous gel and proppant. After stimulation, spinner logs indicated flowing zones at the upper zone (where a flow conduit had previously existed) and the lower zone (where proppant was used). There was no flow from the wellbore at the middle zones (where only gel was used). Subsequently, high rate and pressure injection (above the minimum principal stress) was performed in the entire well interval, and spinner logs indicated that the middle sections were major outflow zones. This indicates that during the high pressure injection, natural fractures at the middle zones opened and were highly transmissive. But when bottomhole pressure was lower, the middle fractures closed and had low transmissivity. At lower pressure, only the propped zone and the preexisting flow zone retained transmissivity. The results suggest that proppant had the effect of significantly improving fracture transmissivity.
The well EGS GtGrSk4/05 at Groß Šöhnebeck was stimulated in sandstone and volcanic rock with proppant (Zimmermann et al. 2010). The stimulation was very successful and resulted in a large increase in well injectivity.

Proppant was used at the Fjällbacka project (Wallroth et al. 1999), but we were unable to find additional details.

1.4.5 Chemical stability of proppant at high temperature

Some studies of proppant chemical stability under geothermal conditions have indicated dissolution could be a concern. Brinton et al. (2011) conducted batch experiments on proppant (sand and bauxite) dissolution at 200°C over periods of several months. Evidence was found that dissolution occurred, but dissolution was not directly related to changes in proppant strength or transmissivity. Deon et al. (2013) performed batch experiments with corundum proppants at 150°C for up to 80 days, and found evidence that dissolution occurred. They also did not investigate how the dissolution affected proppant permeability or strength. They found that resin coating did not affect the tendency for dissolution.

Raysoni & Weaver (2013) performed batch experiments with aluminum-based proppant at up to 450°F for 60 days or more. They tested the retained strength and permeability of the proppant materials before and after treatment. They found significant loss of strength and permeability in uncoated proppants, but slight or nonexistent loss or strength and permeability in proppants that were coated with hydrophobic surface-modification agents. These results are not necessarily inconsistent with the results of Deon et al. (2013) because it is likely that a different proppant coating was used.

In reality, it is impossible to replicate field conditions in the laboratory. The laboratory results suggest dissolution may be a concern, but the degree to which it may
reduce fracture transmissivity is not really known. The only way to truly test proppant performance would be with long-term circulation tests in an actual EGS reservoir. Proppant material (and coating) could be selected on the basis of laboratory batch experiments using site-specific fluid samples, rock types, and temperatures. Even with degradation, proppant may still be able to increase system performance. In practice, if reservoir transmissivity was found to degrade over time (and it was suspected that proppant degradation was responsible), wells could be refractured with additional proppant. Refracturing may be cost effective because drilling and surface facilities are the dominant drivers of cost for EGS, not stimulation cost (Sanyal et al. 2007).
1.5 CONCLUSION

Our simple calculations demonstrate that using multiple stages in an EGS well could result in substantially higher flow rates, electricity production, and revenue generation. Multiple stage completions have not generally been used in EGS because of the technical challenge of using openhole packers. We believe that cased hole completions could be effective (for the well that is stimulated, not the production well drilled subsequently). Cased hole zonal isolation at high temperature is possible with current technology. Not only would the use of multiple stages allow higher flow rates, it would inhibit thermal short-circuiting by distributing flow over a larger number of fractures.

The ideal wellbore geometry for an EGS doublet would be two horizontal wells, oriented toe-to-heel. Horizontal drilling is technically feasible at temperatures seen in most EGS projects, but we are uncertain about the incremental cost, especially in the hard rock formations where EGS is often attempted. As an alternative to horizontal well completion, deviated wells could be used, or vertical wells in a reverse faulting regime.

Our review of the literature shows that in the few cases where proppant has been used in EGS projects in granite, it has clearly improved fracture transmissivity. Laboratory investigations of proppant dissolution at high temperature are inconclusive, but at least one study indicated that coated proppants are resistant to degradation.

Deviated or horizontal EGS wells with cemented, cased production intervals, multiple stage fracturing treatments, and proppant would have higher completion and drilling cost. But much of the cost of an EGS project is from drilling to the target depth, building surface facilities, and long term operation. These technologies could increase project revenue by several multiples.
There are technical challenges in applying these technologies, but they offer a clear path forward for designing EGS projects with radically improved economic performance.
CHAPTER 2: SIMULATION OF PROPPANT TRANSPORT IN A THREE-DIMENSIONAL HYDRAULIC FRACTURING SIMULATOR WITH PROPPANT SETTLING AND FRACTURE CLOSURE

2.1 INTRODUCTION

Hydraulic fracturing is performed by injecting fluid into the subsurface at high pressure, opening and propagating fractures through the rock. In the majority of fracturing treatments, particulate matter called proppant is pumped in a slurry with the injection fluid. After injection is stopped, fluid pressure decays and the fractures close. The proppant holds open the fractures and allows them to be effective conduits for flow even after fluid pressure has drawn down.

Numerical simulation of proppant transport is challenging because of the complex interactions between the fluid, particles, and fracture walls. Moreover, in the modeling of hydraulic fracturing, a variety of other physical processes are occurring simultaneously: fluid flow, stress induced by fracture deformation, complex fluid rheology, and fracture propagation (Adachi et al. 2007).

Several different approaches have been used for numerical simulation of fluid-solid two-phase systems. The two most common frameworks are Eulerian-Eulerian and Eulerian-Lagrangian (Hu et al. 2001; Zhang & Chen 2007). In the Eulerian-Eulerian technique, the particles and fluid are both treated with an Eulerian framework. Each component is governed by conservation equations in stationary control volumes (Clifton & Wang 1988; Ouyang et al. 1997; Mobbs & Hammond 2001; Adachi et al. 2007; Weng et al. 2011; Dontsov & Peirce 2015). In the Eulerian-Lagrangian technique, proppant transport is described in a Lagrangian way, with tracking of the location of individual
particles, and fluid flow is described with an Eulerian framework (Tsai et al. 2013; Tomac & Gutierrez 2015).

For describing slurry flow, it is necessary to calculate an effective fluid viscosity (Adachi et al. 2007). The earliest major contribution was the theory of dilute suspensions of particles (Einstein 1906). For concentrated suspensions of particles, one of the simplest expressions was introduced by Mooney (1951). For the modeling of proppant transport, an expression similar to Krieger-Dougherty equation (Krieger 1959) is usually used (Adachi et al. 2007). In this study, we followed the method of Dontsov & Peirce (2014), who used the constitutive model introduced by Boyer et al. (2011).

The slip velocity vector expresses the difference in average velocity between the particles and fluid. There is a tendency for transverse particle migration from near the fracture walls, where shear stress is maximum, to the center of flow channel, where shear stress is lowest. This phenomenon causes higher proppant concentration at the center of channel, where fluid velocity is highest (Economides & Nolte 2000). Some models assume that proppant distribution is uniform across the aperture, and so the velocity difference between fluid and proppant is caused only by gravity (Adachi et al. 2007). Other models account for proppant migration away from the fracture walls. Mobbs & Hammond (2001) performed simulations of proppant transport, taking into account the migration effect with an assumed proppant distribution across the aperture. Boronin & Osiptsov (2014) performed a similar analysis with a different assumed particle distribution and achieved good agreement with experiment results. Eskin & Miller (2008) developed a model accounting for micro-level particle dynamics from the kinetic theory using the key parameter, granular temperature. Dontsov & Peirce (2014) derived the distribution of proppant velocity across the fracture aperture as a function of averaged proppant concentration using the empirical constitutive model introduced by Boyer et al. (2011).
Taking into account the slip velocity and performing a boundary layer calculation, Dontsov & Peirce (2014) were able to capture the complete transition from Poiseulle to Darcy flow that occurs as proppant concentration increases. Using their model, Dontsov & Peirce (2015) performed simulations of proppant transport with Khristianovich-Geertsma-de Klerk (KGD) and pseudo-three dimensional (P3D) hydraulic fracture models using Carter’s leakoff model (Howard & Fast 1957).

In this study, using the approach introduced by Dontsov & Peirce (2014), we performed simulations of proppant transport with a fully three-dimensional hydraulic simulator, CFRAC (McClure & Horne 2013b; McClure et al. 2015). We performed simulations of the entire injection and post-injection period, simulating both fracture propagation and fracture closure. We have extended the framework of Dontsov & Peirce (2014) to capture the process of fracture closure after pumping has stopped.

This chapter provides the details of our model, including governing equations, the method for handling fracture closure with proppant, and the method of solution. Simulations of injection into a single planar fracture are provided, with key parameters varied for sensitivity analysis. The results demonstrate that the model is capable of seamlessly describing the complex tip-screen out (TSO) process.
2.2 METHODOLOGY

2.2.1 Model setup and assumption

In this study, a full three-dimensional hydraulic fracturing simulator, CFRAC, was extended to perform proppant flow calculations, including: pressure-driven convection, gravity settling, and fracture closure. In CFRAC, fluid flow and fracture deformation (opening and sliding) are fully and implicitly solved using iterative coupling. The discontinuity displacement method, a boundary element method, is used for the calculation of stresses induced by fracture deformation, assuming an elastically homogeneous and isotropic formation, linear elastic deformation, and small strain (Okada 1992). The simulations are isothermal.

For simplicity, it is assumed that the proppant consists of non-colloidal spherical particles and all of the particles have the same size. Proppant particles are assumed to be incompressible in flow calculation, but when the proppant packs into a porous bed, the compressibility of the porosity is taken into account (Section 2.2.3). The pure fluid is slightly compressible and Newtonian. Following Dontsov & Peirce (2014), this model focuses on the case where:

\[ Pe \rightarrow \infty, Re \rightarrow 0, \]  

(2.1)

where \( Pe \) and \( Re \) are Péclet and Reynolds numbers, respectively. Under these conditions, fluid flow is laminar and proppant diffusion is negligible.

The simulations are fully three-dimensional, and so the fractures are meshed in both the vertical and horizontal directions. The fractures are not meshed across their aperture, though cross-aperture variation in flow velocity is implicitly considered by the constitutive equations. All fractures in the model are assumed to be vertical, and so gravity is applied only in the vertical direction. The coordinate system is shown in Figure 2.1.
Both fluid and proppant particles are assumed to be a continuous medium (Eulerian-Eulerian approach) and fluid and proppant are governed by mass conservation equations, solved with the finite volume method. The mass balance equations are sequentially solved (iterative coupling) simultaneously with mechanical equations. The primary variables at each element are (1) fluid pressure in fracture, (2) fracture aperture, (3) fracture sliding displacement, and (4) proppant mass concentration.

The model allows the leakoff of fracturing fluid from fracture into the surrounding rock. A one-dimensional leakoff model is used that takes into account the varying fracture pressure over time (Vinsome & Westerveld 1980).

2.2.2 Flow equations

The unsteady fluid mass balance equation and fluid flux are:

$$\frac{\partial ([\rho_f E (1-C/\rho_p])] }{\partial t} = -\nabla \cdot (q_{f,flux} E) - q_{leakoff} + s_f,$$

(2.2)
\[ q_{f,flux} = -\rho_f \frac{k}{\mu} \tilde{Q}_s \left(1 - \chi \tilde{Q}_p\right) \nabla P, \]  \hspace{1cm} (2.3)

where \( E \) is the aperture, \( \rho_f \) is fluid density, \( C \) is proppant mass concentration, \( \rho_p \) is proppant density, \( \nabla \) is gradient operator, \( q_{f,flux} \) is fluid mass flux (mass flow rate per cross-sectional area), \( \mu \) is fluid viscosity, \( q_{\text{leakoff}} \) is fluid mass leakoff rate per fracture surface area, \( k \) is fracture permeability, \( P \) is fluid pressure, \( s_f \) is a fluid source term, \( \tilde{Q}^s \) and \( \tilde{Q}^p \) are dimensionless functions controlling flow as a function of proppant concentration and aperture, and \( \chi \) is a blocking function (described later in this section). In some versions of CFRAC, a distinction is made between the void aperture \( E \) (fluid volume per surface area), and the hydraulic aperture \( e \) (used for calculating the fracture transmissivity), but in the present work, these apertures were assumed the same.

The unsteady proppant mass balance equations and proppant flux are:
\[
\frac{\partial (CE)}{\partial t} = -\nabla \cdot (q_{p,flux} E) + s_p, \hspace{1cm} (2.4)
\]
\[
q_{p,flux} = -\rho_p \frac{k}{\mu} \tilde{Q}_s \chi \tilde{Q}_p \nabla P - \frac{d^2}{4\cdot12\mu} (\rho_p - \rho_f) \rho_p g e \chi \tilde{G}_p, \hspace{1cm} (2.5)
\]

respectively, where \( q_{p,flux} \) is proppant mass flux, \( s_p \) is source term of proppant, \( d \) is proppant diameter, \( g \) is gravitational acceleration, and \( \tilde{G}_p \) is the dimensionless function controlling flow due to gravity as a function of proppant concentration and aperture.

Fracture permeability is defined as:
\[
k = \frac{E^2}{12}. \hspace{1cm} (2.6)
\]

Transmissivity is defined as a product of permeability and aperture and the cubic law results (Witherspoon et al. 1980):
\[
T = k E = \frac{E^3}{12}. \hspace{1cm} (2.7)
\]
\( \tilde{Q}_s, \tilde{Q}_p, \) and \( \tilde{G}_p \) are the dimensionless functions introduced by Dontsov & Peirce (2014) and expressed as:
\[
\tilde{Q}_s = Q_s + \frac{d^2}{4E^2} \tilde{D}_b, \hspace{1cm} (2.8)
\]
\[
\tilde{Q}_p = \frac{4E^2 Q_p}{4E^2 Q_s + d^2 \tilde{D}_b}. \hspace{1cm} (2.9)
\]
\[
\tilde{G}_p = G_p - \frac{4E^2G_\phi Q_p}{4E^2Q_s + d^2\bar{\phi}D_b},
\]  

(2.10)

where \(Q_s, Q_p, G_s,\) and \(G_p\) are dimensionless functions numerically calculated as a function of normalized proppant concentration \(\bar{\phi} = \frac{c}{\rho_p\phi_m}\), and \(D_b\) is a constant related to packed particles \(\left(\frac{8(1-\phi_m)\beta}{3\phi_m}\right)\). The expression \((1 - \phi_m)^\beta\) in \(D_b\) originally comes from the normalized settling rate, which is hindered due to interaction between particles, as a function of volume fraction of particles (Morris & Boulay 1999). \(\phi_m\) and \(\beta\) are the maximum volume fraction of proppant and a constant, respectively. They are chosen to be 0.585 and 4.1, respectively, in this study following Dontsov & Peirce (2014).

Figure 2.2 shows the functions \(\bar{Q}_s, \bar{Q}_p\) and \(\tilde{G}_p\), following Dontsov & Peirce (2014). \(\bar{Q}_s\) captures the transition from Poiseuille to Darcy flow of slurry. The first term of Equation 2.8 represents the inverse of the effective viscosity of slurry. The viscosity of slurry is higher than the viscosity of pure fluid because of interactions between particles and interactions between particle and the fluid. The second term captures flow in porous media related to Darcy’s law, and so its effect is only significant when the normalized proppant concentration is close to 1 (its maximum). \(\bar{Q}_p\) and \(\tilde{G}_p\) describe flowing volume fraction of proppant due to pressure difference and gravity, respectively.
Figure 2.2: The dimensionless functions $\bar{Q}_s$, $\bar{Q}_p$, and $\bar{G}_p$ introduced by Dontsov & Peirce (2014) as functions of normalized proppant concentration $\bar{\phi}$ for difference values of the parameter $E/d$. Normalized proppant concentration is equal to the volumetric fraction of proppant divided by $\phi_m$. The black, blue, and red lines represent cases with $E/d = \{50, 5, 3\}$, respectively. $\bar{Q}_s$ captures the transition from Poiseuille to Darcy flow. In other words, it simplifies to the cubic law when proppant concentration is low, and simplifies to Darcy’s law when proppant concentration is high. $\bar{Q}_p$ and $\bar{G}_p$ describe pressure-driven proppant convection and gravity settling, respectively.
The blocking function $\chi$ is expressed as:

$$
\chi = \begin{cases} 
1 & \text{if } E \geq N_{\text{max}}d \\
\frac{E-N_{\text{min}}d}{(N_{\text{max}}-N_{\text{min}})d} & \text{if } N_{\text{min}}d \leq E < N_{\text{max}}d \\
0 & \text{if } E < N_{\text{min}}d 
\end{cases} \quad (2.11)
$$

In order to capture the bridging of proppant, this function allows proppant to flow into or out of a fracture element only when a fracture is open and also its aperture is greater than $N_{\text{min}}$ times the proppant diameter. The ratio of proppant flow allowed linearly increases from 0 to 1 as aperture changes from $N_{\text{min}}d$ to $N_{\text{max}}d$. $N_{\text{min}}$ and $N_{\text{max}}$ are chosen to be 3 and 4, respectively, in this study. The value of $\chi$ is determined based on Equation 2.11 at the beginning of a timestep and kept constant during a single timestep, as shown in Figure 2.3.

In order to obtain stable scheme, it is important to select appropriate “wind” for each flux. The wind for each connection of adjacent fracture elements is determined by the sign of the derivative of proppant flux with respective to the product of aperture and normalized proppant concentration and using shock velocities following Dontsov & Peirce (2015). The “wind” is determined for each connection at the beginning of each timestep and fixed during the timestep. If the signs of both derivatives at adjacent elements are positive, the flux is upstream weighted. If negative, it is downstream weighted. If the signs are different, the shock velocity is used to define the “wind.” If the sign of shock velocity is positive, it is upstream weighted. If negative, it is downstream weighted. In this case, the shock velocity is defined as:

$$
V_{sh} = \frac{(q_{p,flux})_i - (q_{p,flux})_j}{\phi_i - \phi_j}, \quad (2.12)
$$

where $i$ and $j$ are two adjacent elements, located upstream and downstream, respectively.
2.2.3 Fracture aperture

We define an "open" fracture as a fracture where the fluid pressure has reached the normal stress, the fracture walls are separated, and no compressive stress is transmitted between the walls by the proppant.

The aperture is decomposed into three components: $E_0$, representing the roughness of the fracture walls, $E_p$, representing the contribution to aperture from the volume occupied by the proppant if it were in a packed bed, and $E_{open}$, the additional separation of the fracture walls. The aperture of an open element is defined as:

$$E = E_0 + E_p + E_{open}. \quad (2.13)$$

A fracture is defined as open if $E_{open}$ is greater than 0. There is a certain maximum capacity for proppant that could become lodged within the "roughness" dominated portion of the aperture, represented by $E_0$. If the fracture contains less than that maximum capacity, then $E_p$ is set to zero. Specifically, this is calculated by evaluating the following:

If $E \frac{C}{\rho_p \phi_m} = E \bar{\phi} \leq E_0$, then $E_p = 0. \quad (2.14)$

In this case, the aperture at closure will be $E_0$, and the volumetric fraction of proppant at closure will be equal to the maximum volume fraction of proppant $\phi_m$ or less. It is a simplifying assumption that the proppant would be able to efficiently fill the roughness-generated aperture of the fracture up to a volumetric fraction of exactly $\phi_m$.

If a fracture element contains a sufficiently large amount of proppant, then when the fracture closes, there will be a layer of proppant separating the fracture walls, and the aperture at closure will be greater than $E_0$. In this case, the aperture at closure is equal to $E_0 + E_p$, where $E_p$ is the additional contribution to the aperture at closure due to the proppant. The following equation is used to update $E_p$:

If $E \frac{C}{\rho_p \phi_m} = E \bar{\phi} > E_0$, then $E_p = E \bar{\phi} - E_0. \quad (2.15)$
With these definitions in place, it is possible to define aperture evolution through fracture opening and closure in a consistent way for any proppant concentration, either very high or very low.

Equation 2.14 or 2.15 is applied explicitly by updating \( E_p \) only at the beginning of every timestep. When \( E_p \) is updated, the value of \( E_{open} \) is changed by the opposite amount, ensuring that \( E \) remains constant. Because \( E_p \) is updated only at the beginning of the timesteps, the conditions in Equations 2.14 and 2.15 can sometimes be slightly violated, but this effect is minor.

The aperture of a closed element is defined as:

\[
E = \frac{E_0}{1 + 9 \sigma_n'/\sigma_{n,ref}} + E_p \exp(-c_p \sigma_n'),
\]

(2.16)

where \( \sigma_n' \) is the effective normal stress, \( \sigma_{n,ref} \) is the effective normal stresses required to cause a 90% reduction in aperture, \( c_p \) is the compressibility of the part of the aperture filled with proppant. The effective normal stress \( \sigma_n' \) is defined as (Jaeger et al. 2007):

\[
\sigma_n' = \sigma_n - P,
\]

(2.17)

where \( \sigma_n \) is normal stress. The \( E_0 \) term represents the natural compressibility of the fracture due to deformation of the asperities in the fracture wall. The \( E_p \) term represents the compressibility of the porosity of the proppant bed. This equation is somewhat ad-hoc, but we are unaware of a more rigorously derived constitutive law for fracture aperture that accounts for the compressibility of both the fracture wall roughness and the compressibility of the proppant porosity.

The \( E_0 \) term in Equation 2.16 was first used by Willis-Richards et al. (1996) to describe joint closure, based on the work of Barton et al. (1985). In the original formulation, the value of \( \sigma_{n,ref} \) was a constant, considered a property of the fracture. However, if proppant occupies the fracture, we would expect that the fracture would become stiffer, due to the presence of a stiff, bridged, immobile solid phase wedged between the fracture
walls. Therefore, we chose to make $\sigma_{n,\text{ref}}$ a function of effective proppant concentration at closure. In the absence of any literature quantifying this phenomenon, we use a simple expression relating these values to proppant concentration:

$$
\sigma_{n,\text{ref}} = (\sigma_{n,\text{ref, max}} - \sigma_{n,\text{ref, min}}) \bar{\phi} + \sigma_{n,\text{ref, min}},
$$

(2.18)

where $\sigma_{n,\text{ref, max}}$ and $\sigma_{n,\text{ref, min}}$ are the maximum and minimum values of the effective normal stresses required to cause a 90% reduction in aperture.

Because of the bridging of the proppant between the fracture walls, proppant is not permitted to flow into or out of a closed fracture element.

For preexisting fractures, $E_0$ is treated as a constant. However, a special algorithm must be used to define $E_0$ for hydraulic fracture elements (McClure et al. 2014). When a hydraulic fracture element is initiated, it is given an initial value of $E_0$ of 0.1 microns. The element is filled with fluid and water is not taken from an adjacent element to compensate, which does not strictly conserve mass, but the mass balance error is small because the initial aperture is so small. The element cannot be initialized with a larger $E_0$ because it would result in a more significant mass balance error. But the residual aperture of a typical joint should be on the order of 100s of microns. Therefore, an algorithm is needed to allow $E_0$ to grow larger as the element opens. $E_0$ can only increase when the element is open. The algorithm sets $E_0$ to be equal to 90% of $E$ up until a maximum value, $E_{hf,max,rest}$. When $E_0$ is updated, $E_{open}$ is decreased by an equal amount in order to keep $E$ constant. $E_0$ is not permitted to decrease. This algorithm mimics the natural process of fracture roughness development as a fracture forms and opens. The algorithm in Equations 2.14 and 2.15 is applied after the updating of $E_0$. 
2.2.4 Wellbore boundary conditions

Injection can be performed at specified rate or pressure. The fluid flow calculation does not include frictional pressure drop in the wellbore.

When proppant is injected at the surface, it does not enter the formation until all of the fluid in the well has first flowed into the formation. This process is included in the model. The wellbore is meshed into a series of elements of constant volume. The fluid velocity is assumed constant along the entire well and the flowing volume fraction of proppant is assumed equal to the actual volume fraction of proppant.

2.2.5 Equations for fracture sliding displacement

To satisfy force equilibrium, the effective normal stress of open elements is enforced to be equal to zero (Crouch & Starfield 1983):
\[ \sigma'_n = 0. \]  
(2.19)

The shear stress on open elements is also enforced to be equal to zero:
\[ |\tau - \eta \nu_s| = 0. \]  
(2.20)

For closed elements, the Coulomb failure criterion with a radiation damping term is enforced (Jaeger et al. 2007):
\[ |\tau - \eta \nu_s| = \mu_f \sigma'_n - S_0, \]  
(2.21)

where \( \tau \) is shear stress, \( \nu_s \) is the sliding velocity, \( \eta \) is the radiation damping coefficient (Rice 1993), \( \mu_f \) is the coefficient of friction, and \( S_0 \) is fracture cohesion.

If the shear stress is less than the frictional resistance to slip, the fracture is assumed to be not sliding. If the fracture is sliding, then equality is enforced in Equation 2.21. If the left-hand side of Equation 2.21 is less than the right-hand side, the fracture sliding velocity is assumed to be zero.
2.2.6 Solving the coupled equations

The system of equations is coupled either with sequential coupling or explicit coupling with adaptive timestepping based on coupling error. In the sequential coupling, the code sequentially solves: (1) the shear traction equations, (2) the fluid flow and normal traction equations, and (3) the proppant equations (Figure 2.3). In the scheme, each system of equations is solved while holding the primary variables from the other systems of equations constant. The process is repeated until all equations are simultaneously satisfied within a certain tolerance (Kim et al. 2011). Each of the systems of nonlinear equations is solved with a modified version of Newton-Raphson iteration in first-order finite difference scheme. This approach is the extension of the method described by McClure & Horne (2013b)
Figure 2.3: Summary of the iterative coupling approach for a single time step. After updates of some constants, three systems of the equations are separately solved.

When proppant concentration becomes very high in an element, the coupling between proppant concentration and the fluid flow equations becomes very strong and convergence of the iterative coupling scheme becomes poor. To avoid this problem, when dimensionless proppant concentration goes above 0.8 anywhere in the simulation, the code switches to "explicit coupling," in which the cycle shown in Figure 2.3 is terminated after a single timestep (Kim et al. 2011).
The danger to explicit coupling is that it could lead to significant coupling error in which the residual of Equations 2.2, 2.19, 2.20 and 2.21 are far from being satisfied after solving the proppant system of equations. To prevent this, adaptive timestepping is used in which the residuals to Equations 2.2, 2.19, 2.20 and 2.21 are enforced to be below a certain tolerance. If they are too high, the timestep is aborted and repeated with a smaller $dt$. If the timestep is accepted, adaptive timestepping is performed to keep the coupling error near a certain target value. The new timestep size is selected using the method suggested by Grabowski et al. (1979):

$$
d t^{n+1} = d t^n \min \left( \frac{(1+\omega) \eta_t}{\delta_i^n + \omega \eta_t} \right),
$$

(2.22)

where $\delta_i^n$ is residual of Equation 2.2 or 2.19 at element $i$, $\eta_t$ is a user specified target for the maximum change, which is one fourth of a user specified tolerance for explicit coupling in this case, and $\omega$ is a factor that can ranges from zero to one ($\omega$ is set to one in this study). If $\delta_i > 4 \eta_t$ for any element, the timestep is discarded and repeated with a smaller value of $dt$.

When the iterative coupling scheme is used (and is converging efficiently), coupling error can be driven down to very low levels within a few iterations. But with the explicit scheme, coupling error is difficult to drive to zero without taking extremely small timesteps. Therefore, the convergence tolerance is loosened to 10 times larger than its original one. We have tested the effect of this loosening by performing global mass balance calculations on the fluid in the problem domain. We evaluate:

$$
R_f = \{ \text{Mass of fluid injected} \} + \{ \text{Mass in fracture and well at the beginning} \} - \{ \text{Mass in fracture and well at the end} \} - \{ \text{Mass of fluid leaked off} \}
$$

(2.23)

The calculations show that in a typical simulation, global mass balance error does not exceed more than 1-3%.
2.3 RESULTS AND DISCUSSION

Several simulations were performed using the algorithm described in the previous section. For validation, we first performed a simulation with the same settings of the simulation performed by Dontsov & Peirce (2015). Next, we performed a “base case” simulation of injection into a planar fracture with the settings shown in Table 2.3 and Table 2.4. The results were studied on the basis of proppant distribution at the end of pumping and after closure. Then, sensitivity analysis was performed by changing variables such as proppant density, proppant diameter, and viscosity of fluid injected. Finally, a simulation was performed in which tip-screen out (TSO) occurred.

2.3.1 Validation

The validation simulation was performed with the settings and the pumping schedule shown in Table 2.1 and Table 2.2, respectively. In Table 2.1, the values in red letters are same as the setting used in Dontsov & Peirce (2015). Note that they defined $\rho_p - \rho_f$ and Young’s modulus as input values, not $\rho_p, \rho_f, G,$ and $v$, separately. Some settings required by our simulator were not defined by Dontsov & Peirce (2015), and so we made reasonable choices for these variables, shown in black. For the pumping schedule, following Dontsov & Peirce (2015), clean fluid was first injected for 1,000 seconds, and then slurry (fluid and proppant) was injected for another 3,000 seconds with the normalized proppant concentration $\bar{\phi} = 0.2$ in the fluid injected. The volumetric flow rate was constant, 0.04 m$^3$/s for the entire simulation. Note that the slurry flow rate is doubled compared to Dontsov & Peirce (2015) because our simulation creates a bi-wing fracture. Two notable differences between our model and Dontsov & Peirce (2015) is that they used a pseudo-3D (P3D) model assuming vertically uniform fluid pressure, following Adachi et al. (2010), and used Carter’s leakoff model, which assumes constant pressure in the fracture
(Howard & Fast 1957). Our simulation was fully 3D and used the Vinsome & Westerveld (1980) 1D leakoff model, which accounts for the changing fracture pressure over time. Also, the numerical methods used to solve the problem differed considerably.

Table 2.1: Simulation settlings for validation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_p )</td>
<td>2300 kg/m³</td>
<td>( k_{\text{leak}} )</td>
<td>0 m²</td>
</tr>
<tr>
<td>( d )</td>
<td>0.0008 m</td>
<td>( G )</td>
<td>10000 MPa</td>
</tr>
<tr>
<td>(800 microns)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_p )</td>
<td>0.00145 MPa⁻¹</td>
<td>( v )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \phi_m )</td>
<td>0.585</td>
<td>( K_{\text{lc}} )</td>
<td>1 MPa · m¹/²</td>
</tr>
<tr>
<td>( N_{\text{min}}, N_{\text{max}} )</td>
<td>3, 4</td>
<td>( E_{\text{h,fmax,redid}} )</td>
<td>0.0005 m</td>
</tr>
<tr>
<td>( \sigma_{n,maxref} )</td>
<td>300 MPa</td>
<td>( \sigma_{xx} )</td>
<td>35 MPa</td>
</tr>
<tr>
<td>( \sigma_{n,minref} )</td>
<td>20 MPa</td>
<td>( \sigma_{yy} )</td>
<td>20 MPa</td>
</tr>
<tr>
<td>( \mu )</td>
<td>10⁻⁷ MPa · s</td>
<td>( h )</td>
<td>100 m</td>
</tr>
<tr>
<td>(100 cp)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_{\text{f,0}} )</td>
<td>1000 kg/m³</td>
<td>( H )</td>
<td>25 m</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>17 MPa</td>
<td>( \Delta \sigma )</td>
<td>2.5 MPa</td>
</tr>
<tr>
<td>( \phi_f )</td>
<td>0 MPa⁻¹</td>
<td>( V_w )</td>
<td>1 m³</td>
</tr>
</tbody>
</table>

Table 2.2: Pumping schedule for validation.

<table>
<thead>
<tr>
<th>stage number</th>
<th>Normalized proppant concentration [-]</th>
<th>Slurry rate [m³/s]</th>
<th>Stage time [s]</th>
<th>Cumulative time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.04</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.04</td>
<td>3000</td>
<td>4000</td>
</tr>
</tbody>
</table>
Figure 2.4 shows the schematics of the P3D fracture used in Dontsov & Peirce (2015). Following the geometry, we also set the same the reservoir layer height $H$, 25 m, and pressure difference due to stress barriers $\Delta \sigma$, 2.5 MPa. Figure 2.5 shows the comparison between our simulation result and the one in Dontsov & Peirce (2015). The fracture elements were discretized with rectangular mesh, and the size of each element was 5 m $\times$ 4.27 m. Note our result shows only one-wing of the result in the figure. They show the same trend even though they are different physical models.

Figure 2.4: Schematics of the P3D fracture. Here, $h$, $H$, $\Delta \sigma$, $l$ and $w$ represent fracture height, reservoir layer height, additional stress, horizontal length of the fracture, and aperture, respectively. Note that the notations of $l$ and $w$ are different in this report. Figure from Dontsov & Peirce (2015).
2.3.2 Base case

The base case simulation was performed with the settings and pumping schedule shown in Table 2.3 and Table 2.4, respectively. First, clean fluid was injected for 1,000 seconds, which is called the “pad.” After the pad injection, slurry (clean fluid with proppant) was injected for 2,000 seconds. Then clean fluid was injected again for another 1,000 seconds to sweep proppant out of the wellbore. After the 4,000 second injection, the well was shut-in. For the entire injection period, the volume injection rate of clean fluid or slurry was constant, 0.04 m$^3$/s. The amount of clean fluid injected for the stage 1 and 3 is same as the volume of wellbore (40 m$^3$). The normalized proppant concentration $\Phi$ was 0.2 in the slurry injected. After the well was shut-in, the fluid pressure decreased due to fluid leakoff, and eventually the fracture closed. The size of each rectangular element was 5 m $\times$ 4.27 m. This element size was consistently used for the simulations in Section 2.3.2 and 2.3.3.
Table 2.3: Simulation settings for the base case.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_p )</td>
<td>2500 kg/m(^3)</td>
<td>( k_{\text{leak}} )</td>
<td>( 10^{-18} , \text{m}^2 )</td>
</tr>
<tr>
<td>( d )</td>
<td>0.0008 m</td>
<td>( G )</td>
<td>15000 MPa</td>
</tr>
<tr>
<td>( c_p )</td>
<td>0.00145 MPa(^{-1})</td>
<td>( v )</td>
<td>0.25</td>
</tr>
<tr>
<td>( \phi_m )</td>
<td>0.585</td>
<td>( K_{\text{fc}} )</td>
<td>3.5 MPa \cdot m(^{1/2})</td>
</tr>
<tr>
<td>( N_{\min}, N_{\max} )</td>
<td>3, 4</td>
<td>( E_{\text{hyst,red}} )</td>
<td>0.0005 m</td>
</tr>
<tr>
<td>( \sigma_{\text{n,maxref}} )</td>
<td>300 MPa</td>
<td>( \sigma_{xx} )</td>
<td>35 MPa</td>
</tr>
<tr>
<td>( \sigma_{\text{n,minref}} )</td>
<td>20 MPa</td>
<td>( \sigma_{yy} )</td>
<td>20 MPa</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( 10^{-7} , \text{MPa} \cdot \text{s} )</td>
<td>( h )</td>
<td>100 m</td>
</tr>
<tr>
<td>( \rho_{f0} )</td>
<td>1000 kg/m(^3)</td>
<td>( H )</td>
<td>100 m</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>17 MPa</td>
<td>( \Delta \sigma )</td>
<td>0 MPa</td>
</tr>
<tr>
<td>( c_f )</td>
<td>0.0004 MPa(^{-1})</td>
<td>( V_w )</td>
<td>40 m(^3)</td>
</tr>
</tbody>
</table>

Table 2.4: Pumping schedule for the base case.

<table>
<thead>
<tr>
<th>stage number</th>
<th>Normalized proppant concentration [-]</th>
<th>Slurry rate [^{m^3/s}]</th>
<th>Stage time [s]</th>
<th>Cumulative time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.04</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.04</td>
<td>2000</td>
<td>3000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.04</td>
<td>1000</td>
<td>4000</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>996000</td>
<td>1000000</td>
</tr>
</tbody>
</table>
Figure 2.6 and Figure 2.7 show the simulation results, including proppant distribution, aperture, and fluid pressure in the fracture at the end of pumping and after closure, respectively. Here, the fracture is vertical and planar, and the fracture height is limited to 100 m. The wellbore connection to the fracture is located at \((x, z) = (0, 0)\). Note that the simulation results caused a bi-wing fracture geometry but only one wing is shown in the figures, and the figures show the proppant distribution in the two different scales. As can be seen in Figure 2.6, proppant accumulated at the fracture tip (the tip of the region proppant can enter) because the flowing volume fraction of proppant is about 20% higher than volume fraction of proppant in each element unless concentration is very high (see \(\tilde{Q}^p\) in Figure 2.2). Also, fluid leakoff into the formation causes proppant concentration to increase. At shut-in, the effect of gravity settling was subtle due to high viscosity (100 cp) of the fluid (Figure 2.6). Much later, after fracture closure, all the proppant had settled to the bottom of the fracture, except at the tip of the proppant filled-region (Figure 2.7). There is a significant region of the fracture that proppant never reached because aperture was too small. At the tip of the proppant-filled region, proppant was packed (the normalized proppant concentration reached to 1) and became immobile. The fracture propagated even after the well was shut-in. The region of propped fracture is seen in aperture in Figure 2.7. The total mass of fluid and proppant injected was 150,640 and 23,400 kg, respectively. These values are same for the simulations for sensitivity analysis except the case in which proppant density is changed. The total mass of fluid leakoff was 4,028 kg at the end of pumping (Figure 2.6) and 106,365 kg at the end of pumping and after closure (Figure 2.7).
Figure 2.6: Simulation result at the end of pumping for the base case. The figure shows the distribution of propant concentration in the two different scales.
2.3.3 Sensitivity analysis

Three more simulations were performed by changing fluid viscosity, proppant density, and proppant size. Other settings were the same as in Table 2.3. Figure 2.8 and Figure 2.9 show the results at the end of pumping and after closure for fluid viscosity of 10 cp. Because viscosity was lower than in the base case, the effect of gravity settling was more significant. Proppant was not accumulated at the tip because the proppant settled before it reached the tip, which means the gravity effect dominated the convection. The fracture is longer, and aperture is smaller than in the base case because of lower viscosity. The total mass of fluid and proppant injected was 150,640 and 23,400 kg, respectively. The
total mass of fluid leakoff was 5,454 kg at the end of pumping (Figure 2.8) and 107,593 kg after closure (Figure 2.9).

Similarly, Figure 2.10 and Figure 2.11 show the results when fluid viscosity is 1 cp. In this case, the proppant could not enter the fracture and accumulated in the wellbore for the entire simulation. This occurred because aperture was never more than $N_{min}$ (= 3) times proppant diameter. In other words, the blocking function (Equation 2.11) was always zero in the simulation. The total mass of fluid and proppant injected was 150,640 and 23,400 kg, respectively. The total mass of fluid leakoff was 7,010 kg at the end of pumping (Figure 2.10) and 125,221 kg after closure (Figure 2.11).

Figure 2.8: Simulation result at the end of pumping when fluid viscosity is 10 cp.
Figure 2.9: Simulation result after closure when fluid viscosity is 10 cp.
Figure 2.10: Simulation result at the end of pumping when fluid viscosity is 1 cp.
Figure 2.11: Simulation result after closure when fluid viscosity is 1 cp.

Figure 2.12 and Figure 2.13 show the results for a simulation with proppant density of 1054 kg/m$^3$. This is the lightest proppant the author has found in literature (The Hole Solution Company 2015). Since the proppant was only slightly denser than the fluid, the effect of gravity settling was negligible at the end of injection (Figure 2.12). Proppant settling was apparent after closure (Figure 2.13), but it was still subtle. The total mass of fluid and proppant injected was 150,640 and 9,865 kg, respectively. The total mass of fluid leakoff was 4,027 kg at the end of pumping (Figure 2.12) and 104,163 kg after closure (Figure 2.13).
Figure 2.14 and Figure 2.15 show the results with proppant diameter of 200 microns. In this case, proppant can enter the parts of the fracture with small aperture, and so proppant reaches the “true” tip of the fracture (Figure 2.14). The fracture did not propagate much after the well was shut in (Figure 2.15). This is because the fracture was plugged by proppant, which prevented fracture from propagating further. This is effect is called tip-screen out (TSO). The total mass of fluid and proppant injected was 150,640 and 23,400 kg, respectively. The total mass of fluid leakoff was 4,019 kg at the end of pumping (Figure 2.14) and 84,854 kg after closure (Figure 2.15), respectively.

Figure 2.12: Simulation result at the end of pumping with proppant density of 1,054 kg/m$^3$. 
Figure 2.13: Simulation result after closure with proppant density of 1,054 kg/m³.
Figure 2.14: Simulation result at the end of pumping when diameter is 200 microns.
2.3.4 Tip-screen out (TSO)

A simulation was performed to study the effect of TSO. In order to create a scenario in which significant TSO happens, a schedule was designed with injection of slurry for 4,000 seconds without clean fluid injection as seen in Table 2.5. The normalized proppant concentration in the fluid injected was 0.4, which is higher than the base case, and the volumetric injection rate of slurry was constant, 0.04 m$^3$/s. Proppant density and diameter were 1,054 kg/m$^3$ and 200 microns, respectively, and other settings were the same as in Table 2.3.

Figure 2.15: Simulation result after closure when diameter is 200 microns.
Figure 2.16 shows the simulation result at the end of pumping. Figure 2.17 shows the result of an identical simulation, in which proppant injection was not performed. It can be clearly seen from the two figures that TSO significantly affects fracture propagation, aperture, and fluid pressure. Figure 2.18 shows time elapsed and pressure above closure, which is defined as bottomhole pressure (BHP) minus minimum horizontal stress \( \sigma_{yy} \), in log-log scale for the both simulations. The pressure above closure of the case with proppant injection is higher than the one without proppant. The total mass of fluid and proppant injected was 122,560 and 39,462 kg, respectively, for the TSO simulation (Figure 2.16), and they were 160,000 and 0 kg for the simulation without proppant injection (Figure 2.17). The total mass of fluid leakoff was 3,235 and 4,045 kg for Figure 2.16 and Figure 2.17, respectively.

Table 2.5: Pumping schedule for tip-screen out.

<table>
<thead>
<tr>
<th>stage number</th>
<th>Normalized proppant concentration [-]</th>
<th>Slurry rate ([m^3/s])</th>
<th>Stage time ([s])</th>
<th>Cumulative time ([s])</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.04</td>
<td>4000</td>
<td>4000</td>
</tr>
</tbody>
</table>
Figure 2.16: Simulation result with proppant injection at the end of pumping.
Figure 2.17: Simulation result without proppant injection at the end of pumping.
Figure 2.18: Time elapsed versus pressure above closure in log-log scale. “Fluid” and “TSO” are the simulations without and with proppant in fluid injected, respectively.
Simulations of proppant transport were performed in a three-dimensional hydraulic fracturing simulator, CFRAC. The flow equations (fluid and proppant) and the mechanical equations are fully coupled so that the effect of proppant on fracture propagation can be calculated. Proppant self-migration away from the fracture walls was taken into account using the approach introduced by Dontsov & Peirce (2014). The average velocity of proppant is mostly higher than that of fluid, which can cause tip-screen out (TSO) even when formation permeability is zero (no fluid leakoff). Also, the simulator has capability to calculate proppant distribution after fracture closure by introducing the algorithm described in Section 2.2.3.

In the comparison of the simulation results, our three-dimensional result showed a good agreement with the result in pseudo-3D model shown by Dontsov & Peirce (2015), considering the differences between the models. The sensitivity analysis showed the effects of fluid viscosity, proppant density, and proppant size. It was shown that TSO reduces fracture propagation because only small amount of fluid can flow through the packed proppant at the tip.
GLOSSARY

A: cross-sectional area of casing [m²]

$BHP_{inj}$: bottomhole pressure of injector [MPa]

$BHP_{prod}$: bottomhole pressure of producer [MPa]

$c_f$: fluid compressibility [MPa⁻¹]

$c_p$: compressibility of the part of the aperture filled with proppant [MPa⁻¹]

$C$: proppant mass concentration [kg/m³]

$C_p$: specific heat capacity of fluid [J/kg/K]

d: proppant diameter [m]

$d_w$: well diameter [m]

$dp/dz$: total pressure gradient [MPa]

$(dp/dz)_A$: accelerational pressure gradient [MPa]

$(dp/dz)_F$: frictional pressure gradient [MPa]

$(dp/dz)_H$: hydrostatic pressure gradient [MPa]

$D$: distance between injector and producer [m]

$D_b$: term related to the permeability of the packed particles [-]

$D_{eff}$: effective cumulative sliding displacement [m]

$D_{eff,max}$: maximum cumulative sliding displacement [m]

$e$: hydraulic aperture [m]

$E$: void aperture [m]

$E_0$: initial void aperture [m]

$E_{hfmax,redid}$: maximum value of residual aperture of hydraulic fracture [m]

$E_p$: part of mechanical separation between walls of an open element [m]

$E_{open}$: part of mechanical separation between walls of an open element [m]
\( f \): friction factor [-]
\( g \): gravitational acceleration [m/s\(^2\)]
\( G \): shear modulus [MPa]
\( G_p \): numerical function [-]
\( G_s \): numerical function [-]
\( \bar{G}_p \): function controlling flowing volume fraction of proppant in flow due gravity [-]
\( h \): fracture height or highest possible fracture height [m]
\( H \): reservoir layer height [m]
\( i \): discount rate [-]
\( k \): fracture permeability [m\(^2\)]
\( k_{leak} \): formation permeability [m\(^2\)]
\( K_{fc} \): fracture toughness [MPa-m\(^{1/2}\)]
\( N \): total number of periods [-]
\( N_{man}, N_{min} \): number of proppant particles used for the blocking function [-]
\( P \): fluid pressure [MPa]
\( P_0 \): initial fluid pressure [MPa]
\( Pe \): Péclet number [-]
\( PV \): present value [U.S. $]
\( \Delta P_{inj} \): well pressure drop of injector [MPa]
\( \Delta P_{prod} \): well pressure drop of producer [MPa]
\( \Delta P_{res} \): pressure drop through reservoir [MPa]
\( q \): mass flow rate [kg/s]
\( q_{f,flux} \): fluid mass flux (mass flow per cross-sectional area) [kg/m/s]
\( q_{p,flux} \): proppant mass flux (mass flow per cross-sectional area) [kg/m/s]
\( q_{leakoff} \): fluid mass leakoff rate per fracture surface area [kg/m/s]
\( Q \): rate of thermal energy production [J/s]
\( Q_s \): numerical function [-]
\( Q_p \): numerical function [-]
\( \overline{Q}_s \): function representing effective viscosity and transition of flow [-]
\( \overline{Q}_p \): function controlling flowing volume fraction of proppant in pressure-driven flow [-]

\( Re \): Reynolds number [-]
\( R_t \): net cash flow [U.S. $]
\( s_f \): source term of fluid [kg/m\(^2\)/s]
\( s_p \): source term of proppant, [kg/m\(^2\)/s]
\( S_0 \): fracture cohesion [MPa]
\( T \): transmissivity [m\(^3\)]

\( \Delta T \): difference of temperature [K]
\( t \): time [second or year]
\( \nu \): fluid velocity [m/s]
\( \nu_s \): sliding velocity [m/s]
\( V_{sh} \): shock velocity [kg/m/s]
\( V_w \): wellbore volume [m\(^3\)]
\( w \): fluid mass rate [kg/s]
\( WHP_{inj} \): wellhead pressure of injector [MPa]
\( WHP_{prod} \): wellhead pressure of producer [MPa]

\( \delta_t \): variable used for adaptive time step [various units]
\( \varepsilon \): pipe roughness [-]
\( \eta \): radiation damping coefficient [MPa/(m/s)]
\( \eta_{\text{arg}} \): one fourth of a user specified tolerance [-]
\( \theta \): wellbore angle from horizontal line [°]
\( \mu \): fluid viscosity [cp or MPa-s]  
\( \mu_f \): coefficient of friction [-]  
\( \rho_f \): fluid density [kg/m\(^3\)]  
\( \rho_{f0} \): initial fluid density [kg/m\(^3\)]  
\( \rho_p \): absolute density of proppant [kg/m\(^3\)]  
\( \sigma_n \): normal stress [MPa]  
\( \sigma'_n \): effective normal stress [MPa]  
\( \sigma_{n,\text{ref}} \): effective normal stresses required to cause a 90\% reduction in aperture [MPa]  
\( \sigma_{n,\text{maxref}} \): user-defined maximum value of \( \sigma_{n,\text{ref}} \) [MPa]  
\( \sigma_{n,\text{minref}} \): user-defined minimum value of \( \sigma_{n,\text{ref}} \) [MPa]  
\( \sigma_{xx} \): initial principal stress in the x-direction [MPa]  
\( \sigma_{yy} \): initial principal stress in the x-direction [MPa]  
\( \Delta \sigma \): additional stress in stress barrier [MPa]  
\( \tau \): shear stress [MPa]  
\( \nu \): Poisson’s ratio [-]  
\( \phi_{\text{redit}} \): shear dilation angle for hydraulic aperture [\(^\circ\)]  
\( \phi_m \): maximum volume fraction of proppant [-]  
\( \bar{\phi} \): normalized proppant concentration [-]  
\( \chi \): blocking function [-]  
\( \omega \): factor for adaptive time step [-]
REFERENCES


Doane, J. et al., 2013. Pushing the boundary of ultra-HPHT completion technology: the first 20,000 psi, 470°F permanent production packer for as-rolled casing. paper presented at the International Petroleum Technology Conference, Beijing, China.


Grabowski, J. et al., 1979. A fully implicit general purpose finite-difference thermal model for in situ combustion and stream. paper presented the 54th Annual Fall Technical Conference and Exhibition of the Society of Petroleum Engineers of AIME, Las Vegas, NV.


Innes, R. et al., 2010. New 9-7/8-in HPHT permanent packer with new back-up system provides North Sea operator with an effective solution for worn casing. paper presented at the Offshore Technologies Conference, Houston, TX.


